# TRIGONOMETRY

BY

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AND

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# PREFACE

There are two methods of teaching elementary trigonometry in this country. One method begins with the general angle in the endeavor to give a more unified course with a saving of time. In the other method, the trigonometry of the acute angle is considered first and the general angle taken up later. Both authors of this text have found from experience that this latter method was best for the majority of students. It seems to be the more natural method. New ideas are taken up one at a time and spread throughout the course. Many students trace their difficulties in trigonometry to the great number of new concepts thrust upon them during the first few days. Mature teachers of mathematics often forget the difficulties of the beginner in comprehending general notions without a background of detail.

The book aims to give a simple straightforward text in trigonometry adaptable to courses of various lengths. In many institutions, a rather short course is given — often two hours per week for one semester. One of the objects of this text is to make the selection of material for a two- or three-hour course as easy as possible.

Since numerical computation plays a considerable rôle in trigonometry, special attention has been paid to the subject of approximate calculation. Many problems involving the practical use of trigonometry are given, but all are within the range of experience of the student.

In making up the exercises and problems, two complete sets were drawn up. One set marked with odd numbers has the answers in the back of the book. The set marked with even numbers is without answers. However, an answer book with answers to both sets is furnished upon request.

The following is the outline of a course in which thirty assignments are given, thus leaving some time for review and written quizzes, if the course is two hours per week for one semester.

| *Assignment* | *Chapter* | *Articles* | *Exercises* |
|---|---|---|---|
| 1 | I | 1 | 1–19 |
| 2 | I | 2 | 1–3, 7–9 |
| | I | 3 | 1–3, 15–18 |
| 3 | I | 4 | 1–6 |
| | I | 5 | 1–12, 23–28 |
| 4 | I | 6, 7 | 1–15 |
| 5 | I | 7 | 19–40 |
| 6 | I | 7 | 41–60 |
| 7 | I | 8 | 1–15 |
| 8 | I | 8 | 16–46 |
| 9 | II | 9 | 1–10 |
| | II | 10 | 1–4 |
| | II | 11 | 1–15 |
| 10 | II | 12 | 1–4 |
| | II | 13 | 1–10 |
| | II | 14 | 1–3, 10–12 |
| 11 | II | 15 | 1–12 |
| | II | 16 | 1–6 |
| 12 | II | 17 | 1–4 |
| 13 | II | 18, 19 | 1–15 |
| 14 | III | 20, 21 | 1–5 |
| | III | 22 | |
| 15 | III | 23–25 | |
| 16 | III | 25 | 1–20 |
| 17 | III | 26 | |
| | III | 27 | 1–15 |
| 18 | III | 28 | 1–10 |
| 19 | IV | 33 | 1–8 |
| | IV | 34 | 1–4 |
| | IV | 35 | |
| 20 | IV | 36, 37 | |

| *Assignment* | *Chapter* | *Articles* | *Exercises* |
|---|---|---|---|
| | IV | 38 | 1–10 |
| | IV | 39 | 1–5 |
| 21 | IV | 40 | 1–10 |
| | V | 41 | Ten selected problems |
| 22 | V | 42, 43, 44 | 1–5 |
| | V | 45 | 2–10 |
| 23 | V | 46, 47 | 2–7 |
| 24 | V | 48 | 1–5 |
| | V | 49 | 1–6 |
| 25 | V | 50 | 1–4 |
| | V | 51 | 1–6 |
| 26 | V | Selected miscellaneous problems on the Solution of Triangles. | |
| 27 | V | | |
| 28 | V | | |
| 29 | VII | 67-70 | 1-5, 11-13, 15, 16 |
| 30 | VII | 70 | 24-28, 30-34, 41-45 |

The time for this course could easily be lengthened and the work adapted to a high school course.

A three-hour course would be made up by lengthening the above schedule by the addition of more problem work and by adding the content of Chapters VI, VII, and VIII.

A longer course, or one following a very elementary course would include the chapters on spherical trigonometry.

A. R. C.
E. B. L.

University of Illinois, 1930.

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# TRIGONOMETRY

## CHAPTER I

### TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES

**1. The trigonometric functions.** In Fig. 1, let $A$ be a given angle. By dropping perpendiculars to one side from points on the other side we obtain a number of right triangles, three of which are shown in the figure; namely, $ABC$, $AB'C'$ and

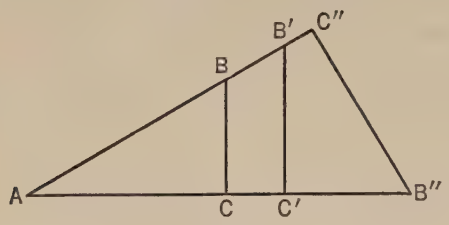


FIG. 1

$AB''C''$. These triangles are similar triangles. From plane geometry we know that the ratios of corresponding sides of similar triangles are equal. From the figure, we may then write

$$\frac{CB}{AB} = \frac{C'B'}{AB'} = \frac{C''B''}{AB''}, \tag{1}$$

$$\frac{AC}{AB} = \frac{AC'}{AB'} = \frac{AC''}{AB''}, \tag{2}$$

$$\frac{CB}{AC} = \frac{C'B'}{AC'}, = \frac{C''B''}{AC''}. \tag{3}$$

These ratios bear such an important relation to the angle $A$ and are so useful in certain mathematical work that they have been given special names, the **sine of $A$** (written sin $A$), the

**cosine of $A$** (written $\cos A$), and the **tangent of $A$** (written $\tan A$), respectively. These three ratios are called the **principal trigonometric functions.** If the angle $A$ is fixed, these ratios remain the same no matter what may be the size of the right triangle.

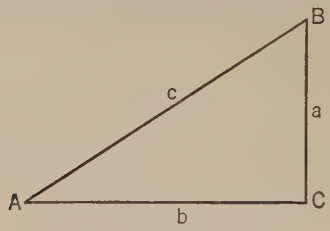


FIG. 2

We may then define the principal trigonometric functions as follows: Let $ABC$ (Fig. 2) be any right triangle containing the given angle $A$. We may then write

$$\sin A = \frac{CB}{AB} = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}, \tag{4}$$

$$\cos A = \frac{AC}{AB} = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}, \tag{5}$$

$$\tan A = \frac{CB}{AC} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}. \tag{6}$$

There are three other ratios which may be formed from the right triangle $ABC$. They are $\frac{c}{a}, \frac{c}{b}, \frac{b}{a}$, named **cosecant of $A$** (written $\csc A$), **secant of $A$** (written $\sec A$), and **cotangent of $A$** (written $\cot A$), respectively. Referring again to Fig. 2, we may write

$$\csc A = \frac{AB}{CB} = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}}, \tag{7}$$

$$\sec A = \frac{AB}{AC} = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}}, \tag{8}$$

$$\cot A = \frac{AC}{CB} = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}}. \tag{9}$$

The definitions of the trigonometric functions given in equations (4), . . ., (9) should be memorized.

These six trigonometric functions are numbers depending upon the size of the angle and not upon the size of the triangle or upon the unit used in measuring the sides. For example, in the familiar 3, 4, 5 triangle (Fig. 3) we find

$$\sin A = \tfrac{3}{5}, \qquad \csc A = \tfrac{5}{3},$$
$$\cos A = \tfrac{4}{5}, \qquad \sec A = \tfrac{5}{4},$$
$$\tan A = \tfrac{3}{4}, \qquad \cot A = \tfrac{4}{3}.$$

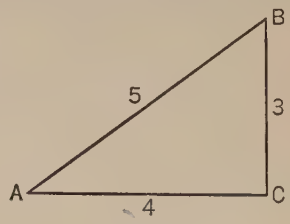

FIG. 3

## EXERCISES

**1.** Construct a square of side 3 inches. Draw a diagonal and measure its length in inches. From this figure find the six trigonometric functions of 45°. Repeat, using a square 4 inches to the side.

**2.** Construct an equilateral triangle of side 3 inches. From one vertex drop a perpendicular to the opposite side. From the figure find the six trigonometric functions of 60°. Repeat, using a triangle 5 inches to the side.

**3.** The sides of a right triangle are 3, 4 and 5 feet (Fig. 3). If the sides are measured in inches, find the six trigonometric functions of the larger acute angle. Repeat by measuring the sides in centimeters. (One foot = 30.48 centimeters.)

**4.** The sides of an equilateral triangle are 18 inches in length. Find the six trigonometric functions of 30°, measuring the sides and perpendicular in feet. In centimeters.

**5.** Which of the trigonometric functions are never greater than 1, no matter what size the angle may be? Which are never less than 1? What functions may be sometimes greater than 1, sometimes less than 1?

**6.** Assume that the angle $A$ increases from a very small angle to one nearly 90°. What can you say of the change in each of the trigonometric functions?

**7.** The hypotenuse of a right triangle with one angle 15° is 10 inches. The sides of the triangle are 9.66 inches and 2.60 inches. Find the six trigonometric functions of 15°.

**8.** The base of a right triangle is 1. The angle at the base is 67°. The hypotenuse and vertical side are 2.56 inches and 2.36 inches, respectively. What are the trigonometric functions of 67°?

**9.** Given $\cos A = \frac{2}{3}$. Construct geometrically the angle $A$.

SOLUTION: From a point $A$ in a horizontal line, lay off a length 2 to the point $C$ (Fig. 4). At $C$ erect a perpendicular. With $A$ as a center and radius 3 construct an arc cutting the perpendicular in the point $B$. The angle $CAB$ is the acute angle whose cosine is $\frac{2}{3}$.

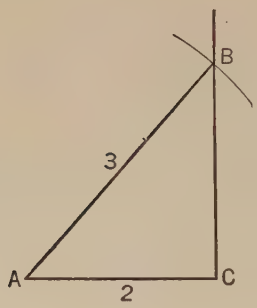


FIG. 4

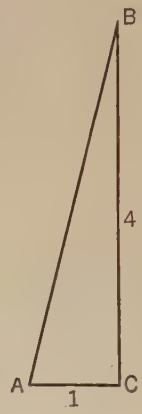


FIG. 5

**10.** Given $\tan A = 4$. Construct geometrically the angle $A$.

SOLUTION: Let $AC$ (Fig. 5), be of length 1. At $C$ erect a perpendicular of length 4. Draw the line $AB$. The angle $CAB$ is the acute angle whose tangent is 4.

**11.** Given $\cos A = \frac{1}{2}$. Construct geometrically the angle $A$.

**12.** Given $\cos A = \frac{3}{7}$. Construct geometrically the angle $A$.

**13.** Given $\sin A = \frac{1}{3}$. Construct geometrically the angle $A$.

**14.** Given $\sin x = \frac{3}{4}$. Construct geometrically the angle $x$.

**15.** Given $\tan B = 2$. Construct geometrically the angle $B$.

**16.** Given $\tan z = \frac{1}{3}$. Construct geometrically the angle $z$.

**17.** Given $\sec y = 3$. Construct geometrically the angle $y$.

**18.** Given $\csc x = 3$. Construct geometrically the angle $x$.

**19.** Find the trigonometric functions of $A$ when the side opposite $A$ is $3x$ and the adjacent side is $4x$.

**20.** The sides and hypotenuse of a right triangle are $a = nx$, $b = ny$, $c = n\sqrt{x^2 + y^2}$, respectively. Find the trigonometric functions of the angle opposite the side $a$.

**21.** If $x$ is an acute angle, show that $\sin x$ is less than $\tan x$ and that $\tan x$ is less than $\sec x$.

**22.** A tree casts a shadow 83 feet long. The angle between the ground and the imaginary line connecting the tip of the shadow and the top of the tree is 49°. If $\tan 49° = 1.15$, what is the height of the tree to the nearest foot?

**2. The trigonometric functions of 45°, 30° and 60°.** If one angle of a right triangle is 45°, then the triangle has two equal sides. Let each of the equal sides be 1; then, since in any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagorean Theorem, pronounced Py-thag′-ō-rē′-an), we have from Fig. 6

$$c^2 = a^2 + b^2 = 1 + 1 = 2,$$
$$c = \sqrt{2}.$$

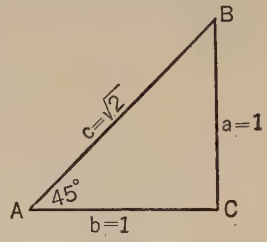

FIG. 6

From the definitions of the trigonometric functions, we then find

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}, \qquad \csc 45° = \sqrt{2},$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}, \qquad \sec 45° = \sqrt{2},$$

$$\tan 45° = 1, \qquad \cot 45° = 1.$$

In Fig. 7, an equilateral triangle, with sides equal to 2, is divided into two equal triangles by the perpendicular from one vertex. One of these triangles is lettered $ABC$. It is a right triangle with angles 60° and 30°. The hypotenuse is of length 2 and the base of length 1. From the Pythagorean theorem, the length of the side $a$ is given by

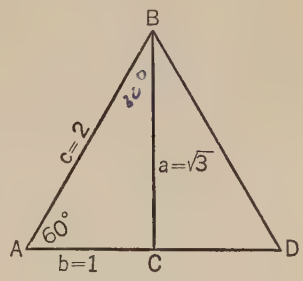

FIG. 7

$$a = \sqrt{c^2 - b^2} = \sqrt{4 - 1} = \sqrt{3}.$$

For the trigonometric functions of 60°, we find

$$\sin 60° = \frac{1}{2}\sqrt{3}, \qquad \csc 60° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3},$$

$$\cos 60° = \frac{1}{2}, \qquad \sec 60° = 2,$$

$$\tan 60° = \sqrt{3}. \qquad \cot 60° = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}.$$

To find the trigonometric functions of an angle of 30°, we again use the triangle $ABC$ in Fig. 7, but consider $BC$ as the base. We easily find

$$\sin 30° = \frac{1}{2}, \qquad \csc 30° = 2,$$

$$\cos 30° = \frac{\sqrt{3}}{2}, \qquad \sec 30° = \frac{2}{3}\sqrt{3}$$

$$\tan 30° = \frac{1}{3}\sqrt{3}, \qquad \cot 30° = \sqrt{3}.$$

Since we often meet 30°, 45° and 60° angles, and the sides of triangles involving these angles also involve $\sqrt{2}$ and $\sqrt{3}$, it is advisable to memorize their approximate values, or at least to write them on a flyleaf of the text where they can be easily found. These approximate values are

$$\sqrt{2} = 1.414, \qquad \sqrt{3} = 1.732.$$

We give for ready reference a table showing the exact and approximate values to four decimal places of the trigonometric functions of 30°, 45° and 60°.

| Angle | sin | cos | tan | cot | sec | csc |
|---|---|---|---|---|---|---|
| 30° | $\frac{1}{2}$ = .5000 | $\frac{1}{2}\sqrt{3}$ = .8660 | $\frac{1}{3}\sqrt{3}$ = .5774 | $\sqrt{3}$ = 1.7321 | $\frac{2}{3}\sqrt{3}$ = 1.1547 | 2.0000 |
| 45° | $\frac{1}{2}\sqrt{2}$ = .7071 | $\frac{1}{2}\sqrt{2}$ = .7071 | 1.0000 | 1.0000 | $\sqrt{2}$ = 1.4142 | $\sqrt{2}$ = 1.4142 |
| 60° | $\frac{1}{2}\sqrt{3}$ = .8660 | $\frac{1}{2}$ = .5000 | $\sqrt{3}$ = 1.7321 | $\frac{1}{3}\sqrt{3}$ = .5774 | 2.0000 | $\frac{2}{3}\sqrt{3}$ = 1.1547 |

## EXERCISES

By actual substitution, verify the following:

1. $\tan 30° = \dfrac{\sin 60°}{1 + \cos 60°}$.

2. $\tan 60° = \dfrac{2 \tan 30°}{1 - \tan^2 30°}$.

3. $\cos 60° = \cos^2 30° - \sin^2 30°$.

*Note.* It is customary to write $\cos^2 x$ for $(\cos x)^2$.

4. $\tan 45° + \sin 60° = (\sin 30° + \cos 30°)^2$.

5. $\cos 60° = \dfrac{1 - \tan^2 30°}{1 + \tan^2 30°}$.

6. $\csc 60° = \dfrac{\tan 30° + \cot 30°}{2}$.

For $x = 30°$, verify:

7. $\sin 2x = 2 \sin x \cos x$.

8. $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$.

For $x = 60°$, verify:

9. $\cos \frac{1}{2} x = \sqrt{\dfrac{1 + \cos x}{2}}$.

10. $4 \cos^2 \dfrac{x}{2} = (\sqrt{1 + \sin x} + \sqrt{1 - \sin x})^2$.

For $x = 30°$, $y = 30°$ verify:

11. $\sin (x + y) = \sin x \cos y + \cos x \sin y$.

12. $\tan (x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$.

**3. Computation of the trigonometric functions when the value of one is given.** If we have one trigonometric function given, the others can be found. For example, let there be given, $\sin A = \dfrac{3}{5}$. Construct a right triangle with side 3 and hypotenuse 5 as shown in Fig. 8. From the Pythagorean theorem, we find the other side to be

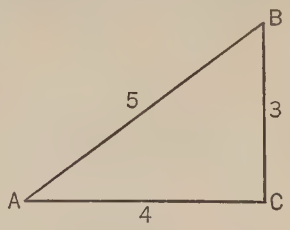

FIG. 8

$$b = \sqrt{25 - 9} = 4.$$

It follows easily that

$$\sin A = \frac{3}{5}, \qquad \csc A = \frac{5}{3},$$

$$\cos A = \frac{4}{5}, \qquad \sec A = \frac{5}{4},$$

$$\tan A = \frac{3}{4}, \qquad \cot A = \frac{4}{3}.$$

Again, suppose we have given

$$\tan A = 3.$$

Construct a triangle of sides $a = 3$, $b = 1$, as in Fig. 9.

From the Pythagorean theorem, we find

$$c = \sqrt{9 + 1} = \sqrt{10},$$

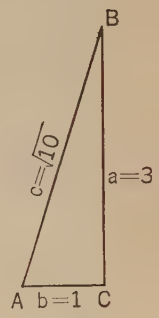


FIG. 9

and we have

$$\sin A = \frac{3}{\sqrt{10}} = \frac{3}{10}\sqrt{10}, \qquad \csc A = \frac{\sqrt{10}}{3},$$

$$\cos A = \frac{1}{\sqrt{10}}, = \frac{1}{10}\sqrt{10}, \qquad \sec A = \sqrt{10},$$

$$\tan A = 3, \qquad \cot A = \frac{1}{3}.$$

## EXERCISES

In the following exercises, the value of one trigonometric function is given. Find the values of the others.

1. $\sin A = \frac{3}{4}$.
2. $\sin A = \frac{1}{3}$.
3. $\cos A = \frac{1}{4}$.
4. $\cos A = \frac{2}{3}$.
5. $\tan A = 2$.
6. $\tan A = 7$.
7. $\cot A = 1.5$.
8. $\cot A = \frac{1}{2}$.
9. $\sec A = 3$.
10. $\sec A = 1.2$.
11. $\csc A = 4$.
12. $\csc A = 10$.
13. $\sin x = a$.
14. $\tan x = b$.

The following exercises refer to a triangle $ABC$ lettered as in Fig. 2, page 2,

**15.** Given $\sin A = \frac{1}{2}$, $b = 1$, find $a$.

**16.** Given $\sin A = \frac{1}{2}$, $a = 1$, find $b$.

**17.** Given $\cos A = \frac{1}{2}$, $a = 1$, find $c$.

**18.** Given $\cos A = \frac{1}{2}$, $c = 1$, find $b$.

**19.** Given $\tan A = 1$, $c = 2$, find $a$.

**20.** Given $\tan A = 1$, $a = 1$, find $c$.

**21.** Given $\cot A = 2$, $b = 1$, find $c$.

**22.** Given $\cot A = 2$, $a = 1$, find $c$.

**23.** Given $\sec A = 2$, $b = 1$, find $a$.

**24.** Given $\sec A = 2$, $a = 1$, find $b$.

**25.** Given $\csc A = 3$, $a = 1$, find $b$.

**26.** Given $\csc A = 3$, $c = 1$, find $b$.

**4. Trigonometric functions of complementary angles.** It should be noticed that up to this point we have usually arranged the six trigonometric functions in three pairs as follows:

$$\begin{array}{ll} \sin A, & \csc A, \\ \cos A, & \sec A, \\ \tan A, & \cot A. \end{array}$$

These pairs are called reciprocal pairs because the functions in each pair are reciprocals of each other; that is, their product is 1. For example, from Fig. 2, we have

$$\sin A = \frac{a}{c},$$

$$\csc A = \frac{c}{a};$$

hence,

$$\sin A \cdot \csc A = 1,$$

and so on for the other pairs.

We may arrange the trigonometric functions in three pairs in another way; namely,

$$\begin{array}{ll} \sin A, & \cos A, \\ \tan A, & \cot A, \\ \sec A, & \csc A. \end{array}$$

In this arrangement, the two functions in each pair are called the **co-functions** of each other. Referring again to Fig. 2, we see that

$$\begin{aligned}
\sin B &= \frac{b}{c} = \cos A, \\
\cos B &= \frac{a}{c} = \sin A, \\
\tan B &= \frac{b}{a} = \cot A, \\
\cot B &= \frac{a}{b} = \tan A, \\
\sec B &= \frac{c}{a} = \csc A, \\
\csc B &= \frac{c}{b} = \sec A.
\end{aligned} \tag{1}$$

From plane geometry, two angles are called complementary if their sum is 90°. Hence, the angles $A$ and $B$ of the right triangle in Fig. 2 are complementary, and we may write $(90° - A)$ in place of $B$ and then formulas (1) may be written in the form:

$$\begin{aligned}
\sin (90° - A) &= \cos A, & \csc (90° - A) &= \sec A, \\
\cos (90° - A) &= \sin A, & \sec (90° - A) &= \csc A, \\
\tan (90° - A) &= \cot A, & \cot (90° - A) &= \tan A.
\end{aligned}$$

These six formulas can all be summed up in the statement: *Any trigonometric function of the complement of an acute angle $A$, is equal to the co-function of $A$.*

We have already had a verification of the foregoing statement in the tabulation of the values of the trigonometric functions of 30° and 60°. Examination of these functions gives

$$\begin{aligned}
\sin 30° &= \cos 60°, & \csc 30° &= \sec 60°, \\
\cos 30° &= \sin 60°, & \sec 30° &= \csc 60°, \\
\tan 30° &= \cot 60°, & \cot 30° &= \tan 60°.
\end{aligned}$$

## EXERCISES

**1.** Express the following as functions of their complementary angles: $\sin 15°$, $\cos 44°$, $\tan 57°$, $\cot 17°25'$, $\sec 85°45'$, $\csc 45°$.

**2.** Express the six functions of 74° as functions of a smaller angle.

**3.** Find an acute angle $A$ for which

$$\sin A = \cos (45° + A).$$

SOLUTION: $$\sin A = \cos (90° - A).$$

Hence, $$\cos (90° - A) = \cos (45° + A),$$

and, since two acute angles with the same cosine are equal to each other, we have

$$90° - A = 45° + A,$$
$$A = 22°30'$$

**4.** Find an acute angle $x$ for which

$$\tan x = \cot x.$$

**5.** Find an acute angle $x$ for which

$$\sin x = \cos 2x.$$

**6.** For what value of $x$ is

$$\sin 3x = \cos (x + 45°)?$$

**7.** For what value of $x$ is

$$\sec 3x = \csc 4x?$$

**8.** Find an angle $x$ for which

$$\sin \tfrac{1}{2}x = \cos x.$$

For what values of $A$ are the following equations true?

**9.** $\tan A = \cot 2A$.

**10.** $\csc A = \sec 4A$.

**11.** $\sin (A + 30°) = \cos (A + 45°)$.

**12.** $\tan 2A = \cot \frac{A}{2}$.

**5. Trigonometric tables.** While it is quite easy to find the values of the trigonometric functions of some angles, as we have seen in the case of 30°, 45° and 60°, accurate methods of finding the values of the functions for other angles are not very simple and require some knowledge of mathematics beyond our elementary courses. It is, of course, easy to draw a triangle with a given angle and by actual measurement to find the approximate values of the trigonometric functions of the given angle, but this method is too inaccurate for most purposes, so tables giving the values of the functions are used. These tables are

available to various degrees of accuracy and are usually known as two-place, three-place, four-place tables . . . according to the number of places of decimals given. Four- and five-place tables are usually used, but we shall content ourselves at this point with a three-place table.

In the table adjoining, the values of the six trigonometric functions for each degree from 1° to 89° are tabulated. For angles less than 45°, the headings at the tops of the columns should be used, while for angles greater than 45° the headings are at the bottoms of the columns. For example, we find from the table sin 36° = .588, sin 75° = .966, cos 25° = .906, cos 81° = .156.

For angles not included in the tables, the process known as **interpolation by proportional parts** must be used. Some illustrations will explain the method.

EXAMPLE 1. Find the value of sin 37° 30′ from the table on page 13.

The angle 37° 30′ is not found in the table, but we find sin 37° = .602, and sin 38° = .616. Since 37° 30′ is just half way between 37° and 38°, we assume that sin 37° 30′ is just half way between .602 and .616, or sin 37° 30′ = .609. While this assumption is only approximately true, the resulting error is very small.

EXAMPLE 2. Find the value of cos 51° 24′ from the table on page 13.

Obviously cos 51° 24′ comes between cos 51° and cos 52°. The following form exhibits the corresponding tabular differences:

$$60'\left\{\begin{array}{l} 24'\left\{\begin{array}{ll}\cos 51^\circ & = .629\\ \cos 51^\circ 24' & = y\end{array}\right\}x \\ \qquad\quad \cos 52^\circ \quad = .616\end{array}\right\}.013$$

For a constant rate of change, these corresponding differences would be proportional. For small intervals in a table, we may assume the rate of change to be constant with small error and write the proportion

$$\frac{x}{.013} = \frac{24'}{60'}.$$

Solving for $x$, we get

$$x = \frac{24}{60}\,.013 = .0052,$$

and noting that the cosine is decreasing in this part of the table, we obtain

$$y = .629 - .0052 = .6238.$$

Since we are using only a three-place table, we write finally

$$\cos 51^\circ 24' = .624.$$

| $A$ | sin $A$ | cos $A$ | tan $A$ | cot $A$ | sec $A$ | csc $A$ | |
|---|---|---|---|---|---|---|---|
| **0°** | .000 | 1.000 | .000 | —— | 1.000 | —— | **90°** |
| 1° | .017 | 1.000 | .017 | 57.290 | 1.000 | 57.299 | 89° |
| 2° | .035 | .999 | .035 | 28.636 | 1.001 | 28.654 | 88° |
| 3° | .052 | .999 | .052 | 19.081 | 1.001 | 19.107 | 87° |
| 4° | .070 | .998 | .070 | 14.301 | 1.002 | 14.336 | 86° |
| **5°** | .087 | .996 | .087 | 11.430 | 1.004 | 11.474 | **85°** |
| 6° | .105 | .995 | .105 | 9.514 | 1.006 | 9.567 | 84° |
| 7° | .122 | .993 | .123 | 8.144 | 1.008 | 8.206 | 83° |
| 8° | .139 | .990 | .141 | 7.115 | 1.010 | 7.185 | 82° |
| 9° | .156 | .988 | .158 | 6.314 | 1.012 | 6.392 | 81° |
| **10°** | .174 | .985 | .176 | 5.671 | 1.015 | 5.759 | **80°** |
| 11° | .191 | .982 | .194 | 5.145 | 1.019 | 5.241 | 79° |
| 12° | .208 | .978 | .213 | 4.705 | 1.022 | 4.810 | 78° |
| 13° | .225 | .974 | .231 | 4.331 | 1.026 | 4.445 | 77° |
| 14° | .242 | .970 | .249 | 4.011 | 1.031 | 4.134 | 76° |
| **15°** | .259 | .966 | .268 | 3.732 | 1.035 | 3.864 | **75°** |
| 16° | .276 | .961 | .287 | 3.487 | 1.040 | 3.628 | 74° |
| 17° | .292 | .956 | .306 | 3.271 | 1.046 | 3.420 | 73° |
| 18° | .309 | .951 | .325 | 3.078 | 1.051 | 3.236 | 72° |
| 19° | .326 | .946 | .344 | 2.904 | 1.058 | 3.072 | 71° |
| **20°** | .342 | .940 | .364 | 2.747 | 1.064 | 2.924 | **70°** |
| 21° | .358 | .934 | .384 | 2.605 | 1.071 | 2.790 | 69° |
| 22° | .375 | .927 | .404 | 2.475 | 1.079 | 2.669 | 68° |
| 23° | .391 | .921 | .424 | 2.356 | 1.086 | 2.559 | 67° |
| 24° | .407 | .914 | .445 | 2.246 | 1.095 | 2.459 | 66° |
| **25°** | .423 | .906 | .466 | 2.145 | 1.103 | 2.366 | **65°** |
| 26° | .438 | .899 | .488 | 2.050 | 1.113 | 2.281 | 64° |
| 27° | .454 | .891 | .510 | 1.963 | 1.122 | 2.203 | 63° |
| 28° | .469 | .883 | .532 | 1.881 | 1.133 | 2.130 | 62° |
| 29° | .485 | .875 | .554 | 1.804 | 1.143 | 2.063 | 61° |
| **30°** | .500 | .866 | .577 | 1.732 | 1.155 | 2.000 | **60°** |
| 31° | .515 | .857 | .601 | 1.664 | 1.167 | 1.942 | 59° |
| 32° | .530 | .848 | .625 | 1.600 | 1.179 | 1.887 | 58° |
| 33° | .545 | .839 | .649 | 1.540 | 1.192 | 1.836 | 57° |
| 34° | .559 | .829 | .675 | 1.483 | 1.206 | 1.788 | 56° |
| **35°** | .574 | .819 | .700 | 1.428 | 1.221 | 1.743 | **55°** |
| 36° | .588 | .809 | .727 | 1.376 | 1.236 | 1.701 | 54° |
| 37° | .602 | .799 | .754 | 1.327 | 1.252 | 1.662 | 53° |
| 38° | .616 | .788 | .781 | 1.280 | 1.269 | 1.624 | 52° |
| 39° | .629 | .777 | .810 | 1.235 | 1.287 | 1.589 | 51° |
| **40°** | .643 | .766 | .839 | 1.192 | 1.305 | 1.556 | **50°** |
| 41° | .656 | .755 | .869 | 1.150 | 1.325 | 1.524 | 49° |
| 42° | .669 | .743 | .900 | 1.111 | 1.346 | 1.494 | 48° |
| 43° | .682 | .731 | .933 | 1.072 | 1.367 | 1.466 | 47° |
| 44° | .695 | .719 | .966 | 1.036 | 1.390 | 1.440 | 46° |
| **45°** | .707 | .707 | 1.000 | 1.000 | 1.414 | 1.414 | **45°** |
| | cos $A$ | sin $A$ | cot $A$ | tan $A$ | csc $A$ | sec $A$ | $A$ |

EXAMPLE 3. Find the angle $A$ whose sine is .382 by the use of the table on page 13.

From the table, we see that .382 lies between the table values .375 and .391; hence, $A$ must lie between 22° and 23°. The following form exhibits the corresponding tabular differences:

$$60'\left\{\begin{array}{l} x\left\{\begin{array}{l}\sin 22^\circ = .375\\ \sin A \;\; = .382\end{array}\right\}.007 \\ \quad\;\; \sin 23^\circ = .391 \end{array}\right\}.016$$

If the rate of change were constant, these corresponding differences would be proportional. Since the intervals are small, we assume the rate of change to be constant and write the proportion

$$\frac{x}{60'} = \frac{.007}{.016}.$$

Solving for $x$, we get

$$x = \frac{.007}{.016}\,60' = 26' \text{ to the nearest minute}$$

and finally have

$$A = 22^\circ\, 26'.$$

## EXERCISES

From the table on page 13, find:

**1.** The six trigonometric functions of 33°.

**2.** The six trigonometric functions of 9°.

**3.** The six trigonometric functions of 76°.

**4.** The six trigonometric functions of 47°.

**5.** sin 27° 15′; cos 62° 45′.

**6.** cos 32° 45′; sin 57° 15′.

**7.** tan 15° 10′; cot 74° 50′.

**8.** cot 18° 20′; tan 71° 40′.

**9.** sec 70° 5′; csc 19° 55′.

**10.** csc 31° 6′; sec 58° 54′.

**11.** sin 44° 13′.

**12.** sin 76° 19′.

**13.** cos 19° 7′.

**14.** cos 77° 17′.

**15.** tan 4° 28.

**16.** tan 79° 47′.

**17.** cot 5° 5′.

**18.** cot 53° 53′

**19.** sec 31° 51′.

**20.** sec 62° 1′.

**21.** csc 23° 56′.

**22.** csc 81° 11′.

Find the angles corresponding to the following values in the table:

**23.** sin ($A$) = .511.

**24.** sin ( ) = .948.

**25.** cos ( ) = .215.

**26.** cos ( ) = .795.

**27.** tan ( ) = .225.

**28.** tan ( ) = 2.121.

**29.** cot ( ) = 5.223.

**30.** cot ( ) = .315.

**31.** sec ( ) = 1.111.

**32.** sec ( ) = 2.675.

**33.** csc ( ) = 4.511.

**34.** csc ( ) = 1.129.

**6. Solution of right triangles.** If we know any two sides, or a side and an acute angle, of a right triangle, we can always find the other parts of the triangle. The finding of the unknown parts of a triangle when some parts are known is called the **solution of the triangle.** The solution of right triangles by means of trigonometric tables is illustrated in the following examples.

EXAMPLE 1. For the right triangle in Fig. 10, we have given $a = 17$, $b = 29$, to find $c$, $A$ and $B$.

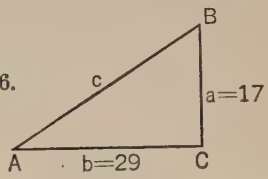

FIG. 10

From the definition of the tangent, we have

$$\tan A = \frac{a}{b} = \frac{17}{29} = .586.$$

From the table on page 13, we find that

$$\tan 30° 22' = .586;$$

hence,

$$A = 30° 22' \text{ approximately},$$

and

$$B = 90° - 30° 22' = 59° 38'.$$

To find $c$, we can use the relation

$$\frac{a}{c} = \sin A, \quad \text{or} \quad c = \frac{a}{\sin A} = \frac{17}{.506} = 33.6.$$

The hypotenuse $c$ could have been found from the formula

$$\frac{b}{c} = \cos A,$$

whence

$$c = \frac{b}{\cos A} = \frac{29}{.863} = 33.6,$$

or the formulas

$$\frac{c}{b} = \sec A, \text{ or } \frac{c}{a} = \csc A$$

could have been used. The Pythagorean theorem could have been used to find $c$, thus giving

$$c = \sqrt{a^2 + b^2} = \sqrt{289 + 841} = \sqrt{1130} = 33.6,$$

but this method is usually more tedious.

EXAMPLE 2. Given the sides $a = 25$, $c = 43$; to find $b$, $A$, and $B$.

From the relation

$$\sin A = \frac{a}{c} = \frac{25}{43} = .581,$$

we find

$$A = 35° 30', \; B = 90° - 35° 30' = 54° 30'.$$

To find $b$, we can use the formula

$$\frac{b}{a} = \cot A; \text{ or } b = a \cot A = 25 \cdot 1.402 = 35.05.$$

There are other formulas that can be used to find $b$; for example:

$$\frac{b}{c} = \cos A; \text{ or } b = c \cos A = 43 \cdot .814 = 35.02.$$

These two values of $b$ do not check exactly. This is to be expected, for all the numbers used in the calculation are only approximately correct.

EXAMPLE 3. Given $c = 241$, $A = 52° 14'$; to find $a$, $b$ and $B$.

$$B = 90° - 52° 14' = 47° 46'.$$

From

$$\frac{a}{c} = \sin A,$$

we get

$$a = c \sin A = 241 \cdot .791 = 191.$$

From

$$\frac{b}{c} = \cos A,$$

we have

$$b = c \cdot \cos A = 241 \cdot .613 = 148.$$

**7. Area of a right triangle.** From geometry, we know that the area of a triangle is given by one-half the product of the base by the altitude. In the right triangle, this rule takes on the simple form

$$\text{Area} = \tfrac{1}{2}ab.$$

From this formula, we can derive many others. For example, since $a = c \sin A$, we have

$$\text{Area} = \tfrac{1}{2}bc \sin A.$$

Again replacing $b$ by $a \cot A$, we have

$$\text{Area} = \tfrac{1}{2}a^2 \cot A.$$

## EXERCISES AND PROBLEMS

In each of the following right triangles, two parts are given. Compute the remaining parts and the area. Check the lengths of the sides by computing the lengths in two ways. Give the angles to tenths of degrees.

**1.** $a = 3$, $b = 4$.

**2.** $a = 143$, $b = 72$.

**3.** $a = 45$, $c = 62$.

**4.** $a = 11$, $c = 12$.

**5.** $a = 112$, $A = 25.2°$.

**6.** $a = 7.13$, $A = 65.4°$.

| | | | |
|---|---|---|---|
| **7.** $b = .13$, | $A = 9.5°$. | **13.** $b = 100$, | $c = 150$. |
| **8.** $b = 171$, | $A = 81.5°$. | **14.** $b = 19$, | $c = 20$. |
| **9.** $a = 36$, | $B = 26.4°$. | **15.** $c = 27$, | $A = 41.6°$. |
| **10.** $a = 145$, | $B = 73.3°$. | **16.** $c = 115$, | $A = 49.1°$. |
| **11.** $b = 8.6$, | $B = 12.6°$. | **17.** $c = 7$, | $B = 6.6°$ |
| **12.** $b = 61.2$, | $B = 51.6°$. | **18.** $c = 51$, | $B = 64.7°$. |

**19.** A ladder 20 feet long is placed against a wall. The foot of the ladder is 4 feet from the base of the wall and in the same horizontal plane. How high will the ladder reach up the wall?

**20.** A tin roof rises 4 inches to the horizontal foot. What angle does the roof make with the horizontal?

**21.** A tin roof rises 4 inches to a foot measured along the roof. What angle does the roof make with the horizontal?

**22.** The *pitch* of a roof is $\frac{BC}{AD}$ (Fig. 11). If the pitch of a roof is $\frac{1}{2}$, what angle does the roof make with the horizontal?

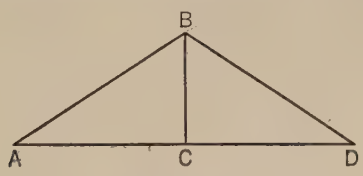


FIG. 11

**23.** If the pitch of a roof is $\frac{1}{3}$, what angle does the roof make with the horizontal?

**24.** At what angle does a railroad incline to the horizontal if the grade is $\frac{1}{100}$ (that is, a rise of 1 foot in 100 feet measured horizontally)?

The angle which the line of sight from an observer to a point makes with a horizontal line in the same vertical plane is called the *angle of elevation* if the point is higher than the observer (Fig. 12). If the point is lower than the observer the angle is called the *angle of depression* (Fig.13).

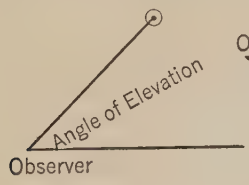


FIG. 12

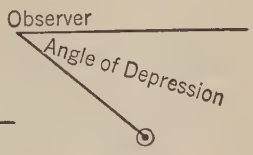


FIG. 13

**25.** At a point 212 feet away from the base of a tower and in the same horizontal plane with the base, the angle of elevation of the top was found to be 40° 30′. Find the height of the tower.

**26.** From the top of a mountain 2716 feet above a hut in the valley, the angle of depression of the hut is found to be 35°. What is the straight line distance from the top of the mountain to the hut?

**27.** At the time when the angle of elevation of the sun is 41°, the length of the shadow of a tree is found to be 83 feet. What is the height of the tree?

**28.** From the top of a tower 250 feet high the angle of depression of the top of a flagpole 75 feet high is found to be 51° 20′. What is the distance from the flagpole to the tower?

**29.** From the top of a mountain 2315 feet above a level plain, the angles of depression of two rocks in the same vertical plane as the observer are observed to be 36° and 40°. How far apart are the rocks?

**30.** An observer is situated on a hill with a straight level road running directly away from the foot of the hill. He finds the angles of depression of two successive milestones on the road to be 61° and 15°. How high is the observer?

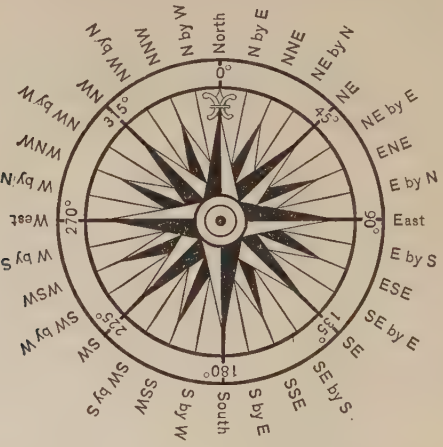


FIG. 14

**Mariner's Compass.** A mariner's compass is shown in Fig. 14. It will be noticed that the circumference is divided into 32 parts giving 32 points on the circumference. The North, East, South and West points, are called the cardinal points. The other directions are given by the initials on the circumference. The directon of one point from a second point is called the

**bearing** with respect to the second point. For example, the bearing of the point $P$ in Fig. 15 from the point $O$ is south-east by east (SE by E).

**31.** Two motor boats leave the same wharf at the same time. One goes NE by E, the other SE by S at rates 10 and 12 miles per hour, respectively. At the end of 30 minutes how far apart are the boats and what is the bearing of the second from the first?

**32.** At 9 A.M. a boat leaves a dock and steams NE by N at a rate of 19 miles per hour. At 10 A.M. a second boat leaves the same dock and steams due east. At 11:10 A.M. the second boat bears due south from the first boat. Find the distance between the two boats at 11:10 A.M. and the rate of the second boat.

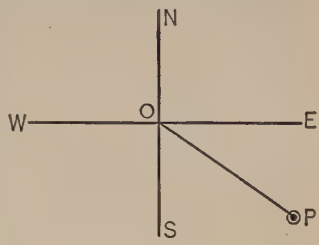

FIG. 15

**33.** A ship $A$ bears SE from a ship $B$. From a lighthouse on the coast the ship $A$ bears NE and the ship $B$ bears N by W. If the ships are 12 miles apart how far is each from the lighthouse?

**34.** A ship sails due east at a rate of 9 miles per hour. At midnight a lighthouse is observed SSE. Three quarters of an hour later the lighthouse is observed due south. Find the distance from the ship to the lighthouse at one o'clock.

**Projection.** The projection of a point $P$, Fig. 16, on a line is the foot $M$ of the perpendicular dropped from $P$ to the line. If there is a second

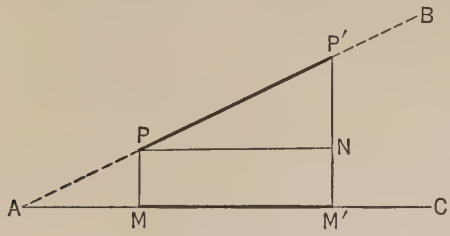

FIG. 16

point $P'$, with the projection $M'$, the segment $MM'$ is called the projection of the segment $PP'$ on the line $AC$. If $A$ is the angle between the lines containing the segments $PP'$ and $MM'$, then we have

$$\frac{MM'}{PP'} = \frac{PN}{PP'} = \cos A,$$

or

$$MM' = PP' \cos B.$$

Line segments are often used to represent quantities such as velocities and forces that have direction as well as magnitude. The projection of the line segment upon another line is often called the **component** in the direction of the line. For example, a boat is moving NE by E at a rate of 9 miles per hour (Fig. 17). The velocity can be represented by a directed line segment nine units long making an angle of $33\frac{3}{4}°$ with the horizontal. The component of the velocity in the east direction, that is the rate at which the boat is moving east, is given by the projection of the segment on the horizontal. This projection is

$$9 \cdot \cos 33° 45' = 9 \cdot .831 = 7.479.$$

**35.** Find the projection of a line segment 12 inches in length on a horizontal line, the angle between the line segment and the horizontal being 35°.

**36.** Find the projection of the line segment in problem 35 on a vertical line.

**37.** Find from Fig. 17 the rate at which the boat is moving north.

**38.** From Fig. 17 find the rate at which the boat is moving north-east.

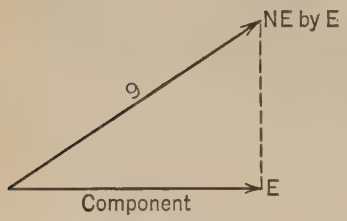


FIG. 17

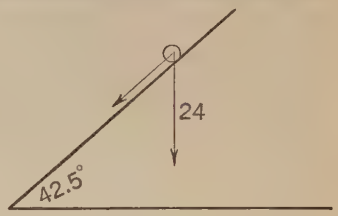


FIG. 18

**39.** A motor boat is moving at a rate of 15 miles per hour in direction NNE. At what rate is the boat moving north and at what rate is it moving east?

**40.** A smooth weight of 24 pounds lies on a smooth inclined plane making an angle of 42.5° with the horizontal (Fig. 18). What force acting in the direction of the plane will just keep the weight at rest?

**41.** A force of 15 pounds is just sufficient to keep a smooth block at rest on a smooth plane making an angle of 37.6° with the horizontal. What is the weight of the block?

**42.** Find the projection of a line segment 14.2 inches long upon a line making an angle of 15° with the horizontal, if the line segment makes an angle of 66.2° with the horizontal. Find two solutions.

**43.** From a point directly east of an inaccessible peak, the angle of elevation of the top is found to be 25°. From another point on the same

level and directly north of the peak, the angle of elevation of the top is found to be 38°. The distance between the two points of observation is 5700 feet. What is the height of the peak?

**44.** A surveyor marking off an irregular field (Fig. 19), runs a line NE 263 feet, thence ESE 465 feet, thence S by E 379 feet, thence back to the starting point. Find the bearing and distance of the starting point from the last station, also the area of the field.

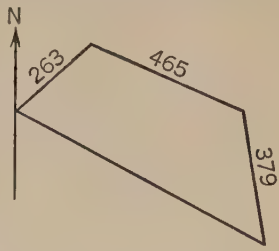

FIG. 19

**45.** A surveyor starts from a point $A$ and runs a line due north 431 feet to $B$, thence due east 965 feet to $C$, thence SSE 840 feet to $D$, thence back to $A$. Find the area of the field around which the line is run.

**46.** The shadow of a man 5 feet 8 inches high standing directly north of a revolving searchlight is 28 feet long. After walking due east 300 feet he finds his shadow to be 33 feet long. How high is the searchlight?

**47.** Find the angle between two diagonals of a cube.

**48.** A flagstaff 20 feet high stands on top of a building. What angle is subtended by the flagstaff at a point on a roof 37 feet lower than the roof with the flagstaff? The horizontal distance between the foot of the flagstaff and the point of observation on the roof is 525 feet.

**49.** A tower stands on a level plane. From a point $W$ due west of the tower the angle of elevation of the top of the tower is $A$. From a point $S$ due south of the tower the angle of elevation is found to be $B$. If the distance $SW$ is $c$, show that the height of the tower is

$$\frac{c}{\sqrt{\cot^2 A + \cot^2 B}}.$$

**50. Distance of the visible horizon.** In Fig. 20, let the circle represent the circumference of the earth, $C$ its center, and $r$ its radius. $P$ represents a point above the surface of the earth and $PH$ represents the distance from an observer at $P$ to his horizon. Let $h$ represent the height of $P$ above the earth's surface. Prove that

$$\overline{PH} = \sqrt{2rh + h^2}.$$

**51.** A man stands on a rock near the seashore so that his eye is 10 feet above sea level. Assuming the radius of the earth to be 3960 miles, how many miles out is his horizon?

**52.** Two men stand on the seashore. Their eyes are exactly five and six feet above the sea level. How much farther out can the taller man see?

**53.** Assuming $h = 10$ feet and $r = 3960$ miles, what is the effect upon the value of $PH$, (Fig. 20), if the $h^2$ is dropped and the simple formula $PH = \sqrt{2rh}$ is used?

**54.** If $r$ is the radius of a circle, show that the area of the circumscribed regular polygon of $n$ sides is given by the formula

$$\text{Area} = nr^2 \tan \frac{180°}{n}.$$

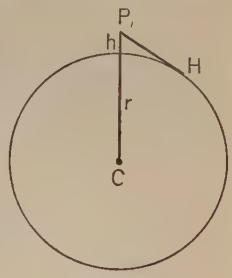

FIG. 20

**55.** In the oblique triangle $ABC$ (Fig. 21) we have given $A = 37°$, $B = 73°$, $CB = 220$ feet. Find the remaining parts of the triangle, and the area.

HINT: Drop a perpendicular from the vertex $C$ to the side $AB$. This perpendicular divides the triangle $ABC$ into two right triangles $BDC$ and $ADC$. In the right triangle $BDC$, we have given the angle at $B$ and the side $CB$. Hence, we can find $DB$ and $CD$. Then in the right triangle $ADC$, we can find $AD$ and $AC$.

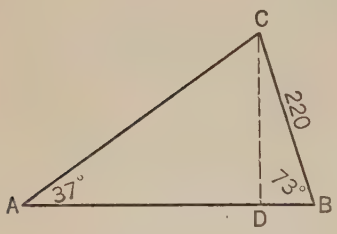

FIG. 21

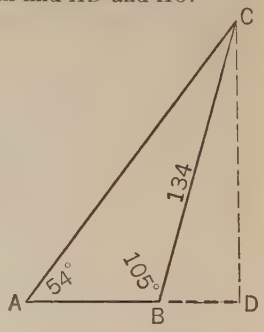

FIG. 22

A better method of solution of such triangles is given in Art. 44.

**56.** In the oblique triangle $ABC$ (Fig. 22), given $A = 54°$, $B = 105°$ and $CB = 134$; find the other parts of the triangle and the area.

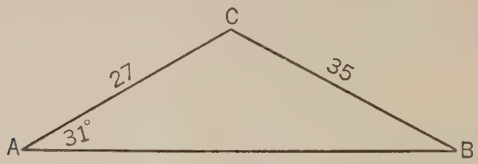

FIG. 23

**57.** In the oblique triangle $ABC$ (Fig. 23), given $AC = 27$ feet, $CB = 35$ feet, and $A = 31°$. Find the other parts of the triangle and the area.

**58.** In the oblique triangle $ABC$ (Fig. 24), given $AC = 38$ feet, $CB = 34$ feet, and $A = 39°$. Find the other parts of the triangle and the area. Is there more than one solution?

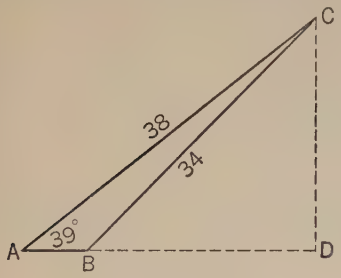


FIG. 24

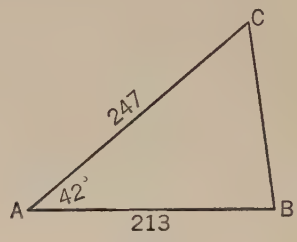


FIG. 25

**59.** In the oblique triangle $ABC$ (Fig. 25), given $AC = 247$ feet, $AB = 213$ feet and $A = 42°$. Find the other parts of the triangle and the area.

**60.** In the oblique triangle $ABC$ (Fig. 26), given $AC = 31$ inches, $CB = 38$ inches, and $AB = 51$ inches. Find the angles of the triangle and the area.

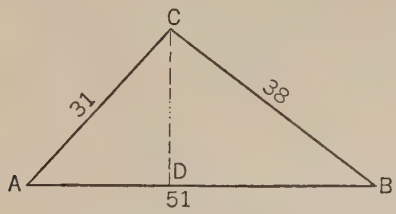


FIG. 26

HINT: Draw $CD$. From the right triangle $ACD$ we have, using the Pythagorean theorem,

$$CD^2 = AC^2 - AD^2 = 31^2 - AD^2. \tag{1}$$

From the right triangle $CDB$, we have

$$CD^2 = CB^2 - DB^2 = 38^2 - DB^2.$$

But

$$DB = AB - AD = 51 - AD,$$

hence

$$CD^2 = 38^2 - (51 - AD)^2. \tag{2}$$

Equating (1) and (2),

$$31^2 - AD^2 = 38^2 - (51 - AD)^2.$$

Solving for $AD$, we find

$$AD = \frac{31^2 - 38^2 + 51^2}{102} = 20.8.$$

**8. Identities.** In any right triangle, Fig. 27, we have the fundamental relation

$$a^2 + b^2 = c^2. \tag{1}$$

Dividing each term by $c^2$, we have

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1,$$

or

$$\sin^2 A + \cos^2 A = 1, \tag{A}$$

which is one of the most useful formulas in mathematics. Similarly, by dividing each term of (1) by $b^2$, we have

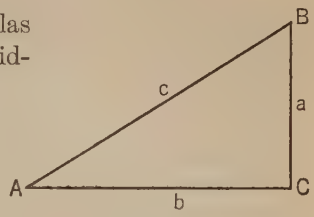

FIG. 27

$$\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2,$$

or

$$\tan^2 A + 1 = \sec^2 A. \tag{B}$$

Again dividing each term of (1) by $a^2$, we have

$$1 + \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2,$$

or

$$1 + \cot^2 A = \csc^2 A. \tag{C}$$

From the definition of the tangent of an angle, we have $\tan A = \frac{a}{b}$. Dividing both numerator and denominator by $c$, we have

$$\tan A = \frac{\frac{a}{c}}{\frac{b}{c}},$$

or

$$\tan A = \frac{\sin A}{\cos A}. \tag{D}$$

We have already noticed (Art. 1) that

$$\csc A = \frac{1}{\sin A}, \tag{E}$$

$$\sec A = \frac{1}{\cos A}, \tag{F}$$

and

$$\cot A = \frac{1}{\tan A}. \tag{G}$$

The equalities A, B, C, D, E, F, G are fundamental relations between the trigonometric functions. Since these equalities are true for all values of the angle $A$ they are called **identities.**

By means of these fundamental identities, many other identities may be proven; in particular, any trigonometric function may be written in terms of any other.

EXAMPLE 1. Write an expression for $\tan A$ in terms of $\cos A$.

From identity (D), we have

$$\tan A = \frac{\sin A}{\cos A}.$$

From identity (A) we find $\sin A = \sqrt{1 - \cos^2 A}$.

Hence,

$$\tan A = \frac{\sqrt{1 - \cos^2 A}}{\cos A}.$$

EXAMPLE 2. Prove the identity $\dfrac{1 + \cot^2 x}{\cot x} = \sec x \cdot \csc x$.

Substituting $\dfrac{\cos x}{\sin x}$ for $\cot x$, we have

$$\frac{1 + \cot^2 x}{\cot x} = \frac{1 + \dfrac{\cos^2 x}{\sin^2 x}}{\dfrac{\cos x}{\sin x}} = \frac{\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}}{\dfrac{\cos x}{\sin x}}$$

$$= \frac{\dfrac{1}{\sin^2 x}}{\dfrac{\cos x}{\sin x}} = \frac{1}{\sin x \cos x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \cdot \csc x.$$

EXAMPLE 3. Prove $\dfrac{\sec x - \csc x}{\sec x + \csc x} = \dfrac{\tan x - 1}{\tan x + 1}$.

Taking first the left-hand member, we have

$$\frac{\sec x - \csc x}{\sec x + \csc x} = \frac{\dfrac{1}{\cos x} - \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}} = \frac{\dfrac{\sin x - \cos x}{\cos x \sin x}}{\dfrac{\sin x + \cos x}{\cos x \sin x}} = \frac{\sin x - \cos x}{\sin x + \cos x}.$$

From the right-hand member, we have

$$\frac{\tan x - 1}{\tan x + 1} = \frac{\dfrac{\sin x}{\cos x} - 1}{\dfrac{\sin x}{\cos x} + 1} = \frac{\dfrac{\sin x - \cos x}{\cos x}}{\dfrac{\sin x + \cos x}{\cos x}} = \frac{\sin x - \cos x}{\sin x + \cos x}.$$

In Example 1, the new identity is derived directly from one of the fundamental identities; in Example 2, one member of the required identity is reduced to the form of the other by the use of known identities and algebraic reductions; in Example 3, each member was reduced to the same form. In most cases the method of Example 3, which includes that of Example 2 as a special case, can be used.

The student should be warned against the method of changing the proposed identity into one known to be true, by means of algebraic and trigonometric manipulations. For example:

Prove

$$\tan x \sqrt{1 - \cos^2 x} = \cos x - \sec x,$$

assuming the radical to be positive.

Squaring both sides, we have

$$\tan^2 x \cdot (1 - \cos^2 x) = \cos^2 x - 2 \cos x \sec x + \sec^2 x.$$

Substituting,

$$\tan x = \frac{\sin x}{\cos x} \quad \text{and} \quad \sec x = \frac{1}{\cos x},$$

there results

$$\frac{\sin^2 x}{\cos^2 x}(1 - \cos^2 x) = \cos^2 x - 2 + \frac{1}{\cos^2 x}.$$

Multiply both members by $\cos^2 x$, we find

$$\sin^2 x\,(1 - \cos^2 x) = \cos^4 x - 2 \cos^2 x + 1.$$

Dividing both sides by $1 - \cos^2 x$, we have

$$\sin^2 x = 1 - \cos^2 x,$$

or

$$\sin^2 x + \cos^2 x = 1,$$

which is the fundamental identity A. But this is no proof of the original identity, in fact it is easy to show for acute angles

that $\tan x \sqrt{1 - \cos^2 x}$ is not equal to $\cos x - \sec x$. The first expression is positive since for acute angles $\tan x$ is positive and the radical is positive. The second expression is negative because for an acute angle the secant is greater than the cosine.

A simpler example of such illogical argument is as follows: To prove $2 = 0$, assume

$$2 = 0.$$

Subtract 1 from each side, finding

$$2 - 1 = 0 - 1,$$

or

$$1 = -1.$$

Square both sides, obtaining

$$1 = 1.$$

Thus, by treating a false statement as if it were true it is possible to arrive at a result which is true.

## EXERCISES

Prove the following identities:

1. $\cos A \tan A = \sin A.$
2. $\sec x - \tan x \sin x = \cos x.$
3. $\tan \theta \sin \theta + \cos \theta = \sec \theta.$
4. $\sin A \cot A = \cos A.$
5. $\csc x = \dfrac{\sec x}{\sqrt{\sec^2 x - 1}}.$
6. $\cot B = \dfrac{1}{\sqrt{\sec^2 B - 1}}.$
7. $\dfrac{\sin z}{\csc z} = 1 - \dfrac{\cos z}{\sec z}.$
8. $\cos^2 A - \sin^2 A = 2 \cos^2 A - 1.$
9. $\cos \theta \sqrt{\sec^2 \theta - 1} = \sin \theta.$
10. $\cos^2 \theta (\tan^2 \theta + 1) = 1.$
11. $\tan^2 z \cos^2 z = \sin^2 z \csc^2 z - \cos^2 z.$
12. $(\sec A - 1)^2 - (1 - \cos A)^2 = (\sin A - \tan A)^2.$
13. $\cot^2 x \cos^2 x = \cot^2 x - \cos^2 x.$
14. $\cos^4 x - \sin^4 x = 2 \cos^2 x - 1.$

15. $\dfrac{1 + \sin x}{1 - \sin x} = (\tan x + \sec x)^2.$

16. $\sin z \cos z (\sec z + \csc z) = \sin z + \cos z.$

17. $\csc A \tan A \cos A = \sin A \cot A \sec A.$

18. $\tan B - \sin B = \dfrac{\sec B \sin^3 B}{1 + \cos B}.$

19. $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A.$

20. $\sin x \sec x = \csc x (\sec x + 1)(1 - \cos x).$

21. Express $\tan A$ in terms of $\sin A$.

22. Express $\sin \theta$ in terms of $\sec \theta$.

23. Express $\cos \theta$ in terms of $\cot \theta$.

24. Express $\sec \theta$ in terms of $\csc \theta$.

25. Express $\sin x$ in terms of each of the other trigonometric functions.

26. Express $\csc x$ in terms of each of the other trigonometric functions.

27. Prove $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x.$

28. $\sin x \sec x \cot x - 1 = \tan x \cos x - \sin x.$

29. $\dfrac{\sin A}{\csc A} + \dfrac{\cos A}{\sec A} - 1 = \tan B + \cot B - \sec B \csc B.$

30. $\dfrac{\cot x \cos x}{\cot x + \cos x} - \dfrac{\cot x - \cos x}{\cot x \cos x} + \dfrac{\cos^2 y}{1 - \sin y} = 1 + \sin y.$

If $A$ and $B$ are the acute angles of a right-angled triangle prove the following identities:

31. $(\sin A + \sin B)^2 + (\sin A - \sin B)^2 = 2.$

32. $\sin^2 B - \sin^2 A = 1 - 2 \sin^2 A.$

33. $1 + \cot^2 A = \dfrac{1}{1 - \sin^2 B}.$

34. $\dfrac{\sin A \cot B - \cos A \tan B}{\sec A - \sec B} = 1 + \cos A \cos B.$

35. $\dfrac{1 - \cos A}{1 + \sin B} = (\csc A - \tan B)^2.$

36. $\csc^4 B - \sec^2 A = \cot^4 B + \tan^2 A.$

37. $\cos^3 A + \cos^3 B = (\sin B + \cos B)(1 - \cos A \cos B).$

38. $(a \sin A - b \sin B)^2 + (a \cos A + b \cos B)^2 = c^2$

39. $\dfrac{1}{(\cos^2 A - \cos^2 B)^2} = 1 + \dfrac{4 \tan^2 B}{(1 - \cot^2 A)^2}.$

40. $\dfrac{\tan A - \tan B}{\tan A + \tan B} + 1 = \dfrac{2}{\sec B \csc A}.$

Test the following identities by substituting the values of the trigonometric functions of 30°, 45° and 60°.

**41.** $\sec x (\sin x - \cos x) + \csc x (\sin x + \cos x) = \sec x \csc x.$

**42.** $\sec^2 A + \csc^2 A = \sec^2 A \csc^2 A.$

**43.** $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x.$

**44.** $\dfrac{1 - \tan z}{1 + \tan z} = \dfrac{1 - \cot z}{1 + \cot z}.$

**45.** $(1 + \sec A + \tan A)(1 + \cot A - \csc A) = 2.$

**46.** $\dfrac{1 - \cot^2 x}{1 + \cot^2 x} = \sin^2 x - \cos^2 x.$

## CHAPTER II

## TRIGONOMETRIC FUNCTIONS OF ANY ANGLE.

**9. Angular magnitudes.** Up to this point, we have considered only angles less than a right angle. However, in plane geometry we often meet obtuse angles which are greater than 90°, and in rotation problems we have large angles even greater than 360°; so we need to extend our conception of angular magnitude.

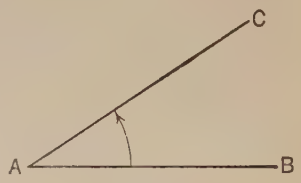


FIG. 28

Let us think of any angle, such as $\angle BAC$, as being formed, or generated, by rotating the ray $AC$ about the point $A$ from an initial coincidence with the ray $AB$. From this point of view, $AB$ is called the **initial side** and $AC$ the **terminal side** of the angle. If the direction of rotation is counter-clockwise, the angle is called **positive,** and if the rotation is clockwise the angle is called **negative.** Angles which have the same initial side and the same terminal side are called **co-terminal angles;** the angles $+45°$, $+405°$, $-315°$, $+45 + n \cdot 360°$ $[n = 1, 2, 3 \cdots]$ are all co-terminal angles. It is to be noted that an angle, such as $\angle BAC$ (Fig. 28), may be represented numerically in an infinite number of ways. Angular magnitudes may be added and subtracted.

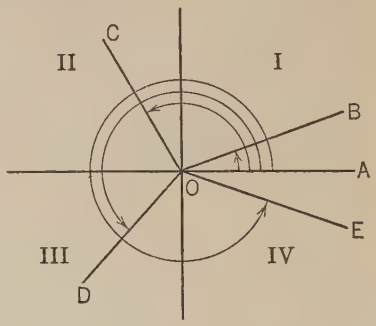


FIG. 29

Two mutually perpendicular lines divide the plane into four parts or quadrants, which are conventionally numbered I, II, III, IV as in Fig. 29. If $OA$ is taken in each case to be the initial side, then $\angle AOB$ is an angle in the first quadrant, $\angle AOC$ is in the second quadrant, $\angle AOD$ is in the third quadrant, and $\angle AOE$ is in the fourth quadrant. The angles 0°, 90°, 180°, 270°, and 360°, and their co-terminal angles, are called **quadrantal angles.**

In discussing properties of angles we say that an angle is in **standard position** when its initial side is horizontal and extends to the right from its vertex.

## EXERCISES

**1.** In which quadrant are each of the following angles: 150°, 400°, 190°, 500°, 310°, − 95°, − 6°, − 585°, − 340°?

**2.** In which quadrant are each of the following angles: − 190°, − 68°, − 280°, − 145°, 292°, 130°, 8°, 223°?

**3.** Give three positive and three negative angles each co-terminal with 108°.

**4.** Give three positive and three negative angles each co-terminal with 75°.

**5.** Which of the following angles are co-terminal: 60°, − 320°, 870°, 420°, 330°, − 300°, 1840°, − 30°, 690°, 150°?

**6.** Which of the following angles are co-terminal: − 56°, 562°, − 230°, 130°, − 304°, 776°, − 964°, − 1684°, 416°, 850°, − 590°, − 173°, 1210°, − 1310°?

**7.** What angle does the minute hand of a clock generate in 3 hrs.? In 4 hrs. 20 mins.?

**8.** What angle does the hour hand of a clock generate in 20 min. 15 sec.? In 2 hrs. 30 min.?

**9.** A flywheel makes 320 revolutions per minute. What angle does a spoke generate in 5 sec.? In 1.5 sec.?

**10.** If a wheel rotates at the rate of 150° per sec., how many revolutions does it make per minute?

**11.** In what quadrant is the exterior angle of an equilateral triangle?

**12.** In what quadrant is the interior angle of a regular polygon of five sides? Of six sides?

**13.** In what quadrant is the exterior angle of a regular polygon of eight sides? Of ten sides?

**14.** In what quadrant is the supplement to 65°?

**15.** In what quadrant is the supplement to 80°?

**16.** In what quadrant is the central angle of a regular hexagon?

**17.** In what quadrant is the central angle of a regular octagon?

**18.** Give an original illustration of the use of an angle greater than 90° and less than 360°?

**19.** Give an original illustration of the use of an angle greater than 360°.

**20.** What is the angular velocity per day of the moon about the earth? (The lunar month is 29 days, 12 hours, 44 minutes, 2.7 seconds.)

**10. Rectangular coördinates.** The student has probably used rectangular coördinates before in plotting curves or equations. The position of a point in a plane may be described by means of its distances from two mutually perpendicular lines. The horizontal line $XX'$ (Fig. 30) is the **axis of abscissas,** or the $X$-axis; the vertical line is the **axis of ordinates,** or the $Y$-axis; distances to the right and left of the $Y$-axis are **abscissas,** and distances above and below the $X$-axis are **ordinates.** By convention, distances measured up or to the right are positive while distances down or to the left are negative. Thus, the point $P$ (Fig. 30) has an abscissa $+ 4$ and an ordinate $+ 3$; the point $Q$ has $x = - 5$ and $y = + 2$. In symbols like $P(4, 3)$ and $Q(- 5,2)$, the abscissa is always written first. Abscissas are positive for points in the first and fourth quadrants, negative for points in the second and third quadrants; ordinates are positive for points in the first and second quadrants, and negative in the third and fourth quadrants. The point of

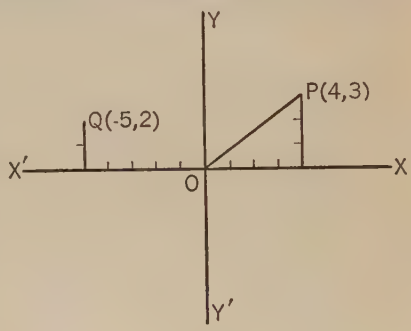

FIG. 30

intersection of the axes, as $O$, is called the **origin**; the distance of a point from the origin, as $OP$, is called the **radius vector,** or **distance,** of the point $P$. The vector of a point is always taken as positive in the definitions given in the next section. The abscissa and the ordinate of a point are the **rectangular coördinates** of that point. The position of a point in a plane relative to the axes of reference is known when its coördinates are known.

### EXERCISES

**1.** Locate the points (3, 5), (+ 5, − 2), (− 6, − 3), (− 4, + 3).

**2.** Arrange the following points in the order of their distances from the origin: (7, 2), (6, 7), (− 2, 8), (− 5, − 6), (2, − 3), (5, − 10) and (− 7, 7).

**3.** Make drawings showing the angles at origin between the positive $X$-axis and the lines joining the origin with the points: (3, 5), (2, − 7), (− 5, 8) and (− 9, − 2).

**4.** In what quadrants are the angles of Exercise 3?

**11. General definition of the trigonometric functions.** Up to this point, we have considered only the trigonometric functions of acute angles. By the use of rectangular coördinates, we can now define the trigonometric functions for any angle. Suppose that any angle $A$ (Fig. 31) be placed in standard position with its vertex at the origin and its initial side coinciding with the positive $X$-axis. In Fig. 31 two cases are shown, one where $A$ is in the first quadrant, one where $A$ is in the second quadrant. Let $P(x, y)$ be any point on the terminal side and drop a perpendicular from $P$ to the $X$-axis thus forming the **triangle of reference** with sides $x$, $y$ and $r$. We then define the six functions as follows:

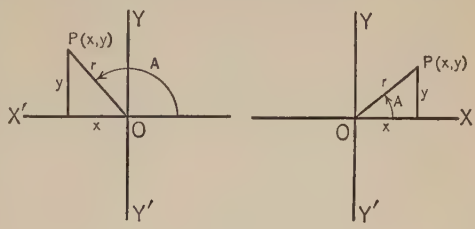


FIG. 31

$$\sin A = \frac{y}{r} = \frac{\text{ordinate}}{\text{distance}}, \qquad \csc A = \frac{r}{y} = \frac{\text{distance}}{\text{ordinate}},$$

$$\cos A = \frac{x}{r} = \frac{\text{abscissa}}{\text{distance}}, \qquad \sec A = \frac{r}{x} = \frac{\text{distance}}{\text{abscissa}},$$

$$\tan A = \frac{y}{x} = \frac{\text{ordinate}}{\text{abscissa}}, \qquad \cot A = \frac{x}{y} = \frac{\text{abscissa}}{\text{ordinate}},$$

in which $r$ is always positive but $x$ and $y$ may be positive or negative according to the position of the point $P$. These definitions should be memorized. Except when the denominators are zero, these definitions give definite numerical values for the trigonometric functions. These values being ratios are

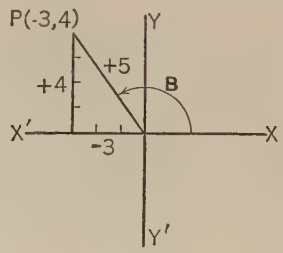


FIG. 32

independent of the unit used in measuring $x$, $y$ and $r$. The values of these functions depend only upon the angle and not at all upon which point $P$ on the terminal side is used. It is to be noted that the definitions previously given for acute angles are special cases of these general definitions. The exceptional cases where the ratios have zero denominators will be discussed in the next section.

As an illustration, consider the angle $B$ (Fig. 32) where $P$ has the coördinates $x = -3$, $y = 4$, and, where $r = 5$. Then, we have

$$\sin B = 4/5, \qquad \csc B = 5/4,$$
$$\cos B = -3/5, \qquad \sec B = -5/3,$$
$$\tan B = -4/3, \qquad \cot B = -3/4.$$

If the angle is acute, all the functions are positive, but for angles in the second, third and fourth quadrants some functions

are positive and some negative. For example, in the second quadrant the abscissa of any point on the terminal side is negative and its ordinate is positive; hence, in the second quadrant the sine and the cosecant are positive while the cosine, secant, tangent and cotangent are negative. The student should verify the signs in the following table:

| Function | First Quadrant | Second Quadrant | Third Quadrant | Fourth Quadrant |
|---|---|---|---|---|
| sin | + | + | − | − |
| cos | + | − | − | + |
| tan | + | − | + | − |
| csc | + | + | − | − |
| sec | + | − | − | + |
| cot | + | − | + | − |

## EXERCISES

**1.** Find the values of the six functions of 135°.

SOLUTION: Place 135° in the standard position (Fig. 33). Since the triangle of reference is a 45° triangle, we may select a point $P$ such that $x = -1$, $y = 1$, $r = \sqrt{1^2 + 1^2}$. Then by the definitions we get

$$\sin 135° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}, \qquad \csc 135° = \sqrt{2},$$

$$\cos 135° = \frac{-1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}, \qquad \sec 135° = -\sqrt{2},$$

$$\tan 135° = \frac{-1}{1} = -1, \qquad \cot 135° = -1.$$

Decimal values are obtained by approximating the radical $\sqrt{2}$. If the six functions are always tabulated in this way with reciprocals opposite, time will be saved in tabulating and in checking results with another student.

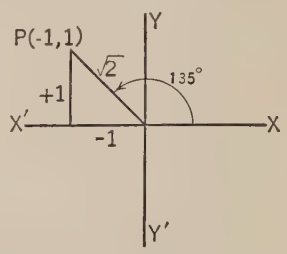

FIG. 33

**2.** Find the values of the six functions of 225°.

**3.** Find the values of the six functions of 315°.

**4.** Find the values of the six functions of 150°.

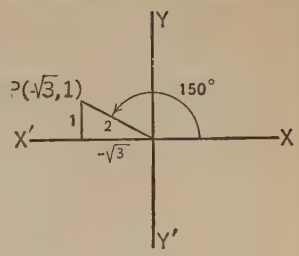

FIG. 34

SOLUTION: Since the triangle of reference is a 30°, 60°, 90° triangle (Fig. 34) and in such a triangle the hypotenuse is always twice the shorter leg, if we take $P$ so that $y = 1$, then $r = 2$ and

$$x = -\sqrt{2^2 - 1^2} = -\sqrt{3}.$$

Now, by the foregoing definitions, we get

$$\sin 150° = \frac{1}{2}, \qquad \csc 150° = 2,$$

$$\cos 150° = \frac{-\sqrt{3}}{2}, \qquad \sec 150° = \frac{-2}{\sqrt{3}} = -\frac{2}{3}\sqrt{3},$$

$$\tan 150° = \frac{1}{-\sqrt{3}} = -\frac{1}{3}\sqrt{3}, \qquad \cot 150° = -\sqrt{3}.$$

**5.** Find the values of the six functions of 120°, 210°, 240°, 300°, and 330° each of whose triangles of reference are 30°, 60°, 90° triangles but in different positions.

**6.** Make a table of the six functions of 30°, 45°, 60°, 120°, 135°, 150°, 210°, 225°, 240°, 300°, 315° and 330°. Paste this table, when carefully checked, in your text for quick reference. Note that these are the angles whose functions can be found easily and exactly without any tables because their triangles of reference are either 45°, 45°, 90° or 30°, 60°, 90° triangles whose properties are known.

**7.** Find the numerical value of

$$\sin 135° + \tan 210° + \cos 225° + \sec 300° + \tan 330.$$

**8.** Given $\sin A = 3/5$ and $\tan A > 0$, find the values of the other five functions of $A$.

**9.** Given $\cos A = -1/3$ and $\sin A < 0$, find the values of the other five functions of $A$.

**10.** Find the value of

$$\sin 225° - \tan 210° + \cos 315° + \sec 150°.$$

In what quadrants may the angle $A$ be if:

**11.** $\sin A = -2/3$.

**12.** $\cos A = 5/8$.

**13.** $\tan A = 3/2$.

**14.** $\sec A = -10/7$.

In what quadrant is the angle $A$ if:

**15.** $\sin A = 1/2$, $\cos A = -\sqrt{3}/2$.

**16.** $\tan A = 4/3$, $\cos A = -3/5$.

**17.** $\cot A = 5/12$, $\sin A = 12/13$.

**18.** $\sec A = 13/5$, $\sin A = -12/13$.

Make a drawing showing all possible angles and find the values of all the functions when:

**19.** $\cos B = -7/25$.

**20.** $\sec B = 17/8$.

**12. Functions of quadrantal angles.** Since we cannot divide by zero, the ten functions csc 0°, cot 0°, tan 90°, sec 90°, csc 180°, cot 180°, tan 270°, sec 270°, csc 360°, and cot 360° are not defined by the general definitions of the last section. As an illustration, let us consider tan 90°. Let $P$ be a point distant $b$ on the terminal side of a 90° angle in standard position. Then, for $P$ we have $x = 0$, $y = b$, $r = b$ and by the general definition $\tan 90° = \frac{b}{0}$, which is not a number since division by zero is an impossible operation. Thus, there is no tangent for 90°. However, each angle in the neighborhood of 90° has a tangent and we may study the variation of tan $A$ as $A$ approaches 90°. In Fig. 36, we see that $\tan A = \frac{y}{x}$. Now if angle $A$ approaches 90° while $x$ remains constant, point $P$ moves up on the tangent to the circle at $Q$, $y$ increases beyond all bounds and $\tan A = \frac{y}{x}$ increases without limit, or as we sometimes say, tan $A$ becomes infinite as $A$ approaches 90°. This fact is often expressed symbolically by the equality $\tan 90° = \infty$, which must never be understood as giving tan 90° a definite numerical value. Since the tangent is always negative in the second quadrant, if

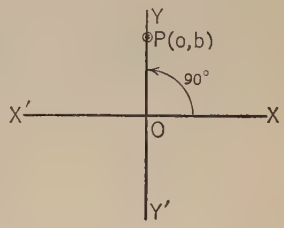


FIG. 35

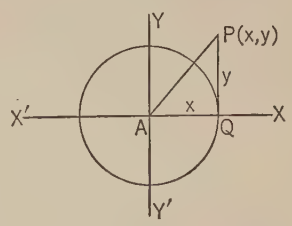


FIG. 36

$A$ approaches 90° through decreasing angles in the second quadrant we see that tan $A$ becomes negatively infinite.

Thus the six functions of 90° are as follows:

$$\sin 90° = \frac{b}{b} = 1, \qquad \csc 90° = \frac{b}{b} = 1,$$

$$\cos 90° = \frac{0}{b} = 0, \qquad \sec 90° = \frac{b}{0} = \infty,$$

$$\tan 90° = \frac{b}{0} = \infty, \qquad \cot 90° = \frac{0}{b} = 0.$$

Similar discussions will develop the values of the functions of the other quadrantal angles. The student should verify the values in the following table:

| | sine | cosine | tangent | cosecant | secant | cotangent |
|---|---|---|---|---|---|---|
| 0° | 0 | +1 | 0 | ∞ | +1 | ∞ |
| 90° | +1 | 0 | ∞ | +1 | ∞ | 0 |
| 180° | 0 | −1 | 0 | ∞ | −1 | ∞ |
| 270° | −1 | 0 | ∞ | −1 | ∞ | 0 |
| 360° | 0 | +1 | 0 | ∞ | +1 | ∞ |

The fact that certain functions of each of the quadrantal angles are not defined will have more meaning later when the graphs of the functions are considered.

## EXERCISES

Find the value of each of the following:

1. $\sin 0° + 2 \cos 90° + 3 \tan 180°$.
2. $4 \sin 90° - 5 \cos 180° - \sin 270°$.
3. $\csc 270° + \sec 180° - \cot 90°$.
4. $3 \cos 180° + 2 \sin 90° + 4 \cot 270° - \sec 180°$.
5. $\tan 360° - \tan 180° - \csc 90° + \cot 270°$.
6. $\tan 180° + \sin 120° + \cos 150° - \cos 90°$.

**13. Line representation of the trigonometric functions.** It is desirable to follow the changes in value of each function as the angle changes. Since the functions have been defined as ratios, their changes in value are not easy to follow if both numerator and denominator are changing at the same time. By a device called the **line function,** we can keep the denominator fixed and always equal to the radius of a unit circle. Then the change of value of the numerator will give the change of the function. Line functions help the student to visualize and to remember the variations of the trigonometric functions.

Let the angle $A$ be in the standard position with its vertex at the center of a unit circle (Figs. 37, 38, 39, 40). If the point $P$ is always taken on the unit circle, then the line segment $MP$, that

FIG. 37

FIG. 38

FIG. 39

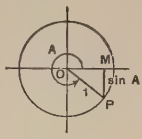


FIG. 40

is the ordinate of $P$, will always represent, in both magnitude and sign (up + and down −), the sine of $A$; and $OM$, that is the abscissa of $P$, will represent the cosine of $A$. Hence, if $r = 1$, then we have

$$\sin A = MP \quad \text{and} \quad \cos A = OM.$$

By means of these line functions, let us now trace the variation of sin $A$ through the four quadrants. If $A$ is zero, the ordinate $MP$ of $P$ is zero and so $\sin 0° = 0$. As $A$ increases to 90° the ordinate $MP$ increases through positive values to + 1 for $A = 90°$; thus, sin $A$ increases from 0 to + 1 as $A$ increases from 0° to 90°. In the second quadrant as $A$ increases from 90° to 180°, the ordinate $MP$ decreases through positive values from 1 to 0. In the third quadrant, the ordinate $MP$ is negative and sin $A$ decreases through negative values from 0 for $A = 180°$ to − 1 for $A = 270°$. In the fourth quadrant, sin $A$ increases from − 1 when $A = 270°$ to 0 for $A = 360°$.

Similarly, to trace the variation of cos $A$, by watching the changes in the abscissa $OM$ of point $P$ we see that:

*As $A$ increases from* $0°$ *to* $90°$, *cos $A$ decreases from* $+1$ *to* $0$.
*As $A$ increases from* $90°$ *to* $180°$, *cos $A$ decreases from* $0$ *to* $-1$.
*As $A$ increases from* $180°$ *to* $270°$, *cos $A$ increases from* $-1$ *to* $0$.
*As $A$ increases from* $270°$ *to* $360°$, *cos $A$ increases from* $0$ *to* $+1$.

In order to get a line representation for the tangent function which will give both the magnitude and the sign, we represent tan $A$ by the line $ST$ in Figs. 41 to 44. The line $ST$ is the geometric tangent to the unit circle perpendicular to the initial

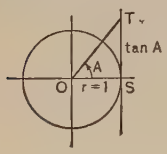


FIG. 41

FIG. 42

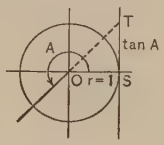


FIG. 43

FIG. 44

side of angle $A$. The terminal side of $A$, is prolonged backwards for angles in the second and third quadrants. In each quadrant (Figs. 41, 42, 43, 44), we have

$$\tan A = \frac{ST}{OS} = \frac{ST}{1} = ST = \text{ordinate of } T.$$

By watching the changes in $ST$ as the angle $A$ changes, we can trace the variation of tan $A$ and see that:

*As $A$ increases from* $0°$ *to* $90°$, *tan $A$ increases from* $0$ *without bound.*

*As $A$ decreases from* $180°$ *to* $90°$, *tan $A$ decreases from* $0$ *without bound.*

*As $A$ increases from* $180°$ *to* $270°$, *tan $A$ increases from* $0$ *without bound.*

*As $A$ decreases from* $360°$ *to* $270°$, *tan $A$ decreases from* $0$ *without bound.*

Since for $A = 90°$ and $270°$ the terminal side of $A$ and the geometric tangent to the unit circle are parallel and give no intersecting point $T$, we again see why tan 90° and tan 270° have no definite values and are said to become infinite.

The student should drill himself in giving orally the variations of the sine, cosine and tangent with their values for the quadrantal angles. After a little practice, he can easily read off the values and variations from just a mental picture of the line function figures and thus find an easy way to remember these facts.

If the variations of the sine, cosine and tangent be thus mastered, then the values of the cosecant, secant and cotangent may be obtained by tracing the changes in the reciprocals of these primary functions. It is possible, however, to give in a similar way line representations for the cosecant, secant and cotangent but this is left for the student who may be interested.

## EXERCISES

**1.** What angle in the first four quadrants has the largest sine? The smallest sine? The largest cosine? The smallest cosine?

**2.** For what angles is $\sin A = \cos A$? $\sin A = -\cos A$? $\sin A = -\sin A$? $\tan A = -\tan A$?

**3.** For how many angles between 0° and 360° may a trigonometric function have equal values? For how many angles may a function have equal numerical (absolute) values?

**4.** What range of angles in the first four quadrants gives all possible values of the sine function? The cosine? The tangent?

By means of line functions verify the following relations:

**5.** $\cos 240° = -\sin 30°$.

**6.** $\tan 60° = \tan 240°$.

**7.** $\sec 120° = \csc 330°$.

**8.** $\cos 360° = \tan 45°$.

**9.** $\sec 135° = -\sec 315°$.

**10.** $\cos 150° = \sin 300°$.

By means of line functions prove that:

**11.** $\sin (180° - C) = \sin C$.

**12.** $\tan (270° - C) = \cot C$.

**14. Radian measure of angles.** The protractor commonly used in measuring angles is scaled in degrees; but, for certain purposes, the radian is a more convenient unit than the degree. Circular or **radian** measure of angles will be useful later in plotting graphs of the functions.

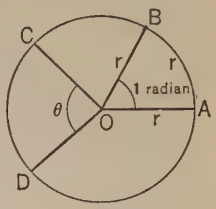


FIG. 45

*The radian is defined as an angle at the center of a circle which intersects an arc equal to the radius of the circle.*

Let $\angle AOB$ (Fig. 45) intersect an arc $AB$ whose length is $r$; then, by the foregoing definition, $\angle AOB$ is one radian. Let $\angle COD$ be any other central angle; then, since in the same circle central angles are proportional to their intercepted arcs, we have

$$\frac{\angle COD}{\angle AOB} = \frac{\angle COD}{1} = \frac{\text{arc } CD}{\text{arc } AB} = \frac{\text{arc } CD}{r};$$

that is,

$$\textbf{Radian measure of an angle} = \frac{\textbf{length of its arc}}{\textbf{length of the radius}}. \quad \text{(A)}$$

By clearing fractions in (A), we get the important relation

$$\text{arc } DC = r \cdot \theta, \quad \text{(B)}$$

which translated says:

*The length of an arc of a circle is equal to the radius multiplied by the central angle expressed in radians.*

The radian measure of an angle clearly is independent of the size of the circle used. Further it should be noted that this radian system of measuring angles, like the definitions of the trigonometric functions, uses ratios and therefore the numerical values obtained by such definitions are independent of the unit used in the parts of the ratio. We sometimes say that such measures are merely numbers and it is just this property which makes radian measure so useful in mathematical theory. Radian measures may be used as coefficients, exponents, etc. without difficulties due to a confusion of units. The relation

(B) makes possible many simplifications of form. For these reasons, radian measure is almost universally used in advanced mathematics.

Since the arc of a semi-circle is $\pi r$, by (A) we get

$$\textbf{180 degrees} = \pi \textbf{ radians}, \qquad \text{(C)}$$

an important relation, which should be memorized and used in changing radians to degrees or degrees to radians. For example, one radian is equal to $\frac{180}{\pi}$ degrees, or $57.2957° \cdots$; and further $30°$, $45°$, $60°$, $90°$, $135°$ are respectively equal to $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{4}$ radians. Radian measures are usually given in terms of $\pi$. To reduce degrees to radians multiply by $\frac{\pi}{180}$, and to reduce radians to degrees multiply by $\frac{180}{\pi}$. The student may get these multipliers confused unless he gets them each time from the memorized relation (C).

Another unit used in measuring angles is the "grade," which may be defined as the one hundredth part of a right angle. The grade is used in France and has the advantage of fitting into the system of decimal notation.

The United States Field Artillery uses units called *mils* in aiming guns. The mil is defined as an angle subtended by an arc of $\frac{1}{6400}$ of the circumference, which is approximately $\frac{1}{1000}$ of a radian, whence its name.

## EXERCISES

Reduce the following sexagesimal angles to radians:

**1.** $75°$. **3.** $165°$ **5.** $315°$. **7.** $32.4°$. **9.** $-10° 30' 30''$.
**2.** $15°$. **4.** $200°$. **6.** $-80°$. **8.** $85° 40' 2''$. **10.** $-210° 16'$.

Reduce the following angles to degrees:

**11.** $\frac{2}{3}\pi$ radians. **13.** $\frac{5}{4}\pi$ radians. **15.** 6 radians.
**12.** $\frac{5}{6}\pi$ radians. **14.** 2.5 radians. **16.** $-1.32$ radians.

**17.** How many radians does the minute hand of the clock pass over in 10 minutes? How many degrees?

**18.** How many radians does the minute hand of the clock pass over in $\frac{1}{2}$ hour? How many degrees?

**19.** How many radians at the center of a circle of radius 4 inches if the intercepted arc is 14 inches? How many degrees?

**20.** How many radians in the angle between the clock hands at 3 o'clock? How many degrees?

**21.** Find the value for $x = 5(\theta - \sin\theta)$, if $\theta = \frac{\pi}{4}$ radians. If $\theta = \frac{\pi}{3}$ radians. If $\theta = 2\pi$ radians.

**22.** Find the value for $y = 2\cos\theta + 3\theta$, if $\theta = \frac{\pi}{6}$ radians. If $\theta = \frac{3\pi}{4}$ radians. If $\theta = \frac{3\pi}{2}$ radians.

**15. Reduction to acute angles.** Trigonometric tables often give the values of the functions only for acute angles; hence, it is desirable to be able to express any function of any angle in terms of some acute angle. Since the triangle of reference for any angle in standard position contains two acute angles, it is easy to express functions in terms of acute angles by means of a simple figure. For example, consider the sin 200°. Placing 200° in standard position (Fig. 46), let $a$, $b$, and $c$ be positive lengths of the sides of the triangle of reference $OPQ$ which has acute angles 20° and 70° (these acute angles are not in standard position). By using the definitions of the functions we get,

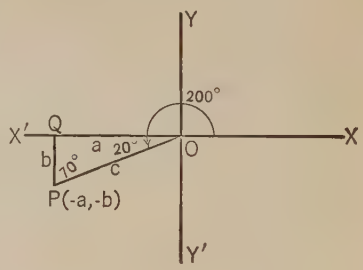

FIG. 46

$$\sin 200° = \frac{-b}{c} = -\frac{b}{c} = -\cos 70° = -\sin 20°,$$

$$\cos 200° = \frac{-a}{c} = -\frac{a}{c} = -\sin 70° = -\cos 20°,$$

$$\tan 200° = \frac{-b}{-a} = \frac{b}{a} = \cot 70° = \tan 20°,$$

$$\csc 200° = -\csc 20°,$$
$$\sec 200° = -\sec 20°,$$
$$\cot 200° = \cot 20°.$$

This reduction to acute angles may also be done by reduction formulas. If we note that by subtracting 90° some finite number of times from any positive angle greater than 90°, we al-

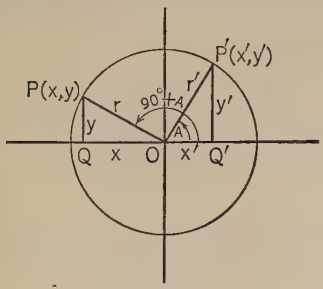


FIG. 47

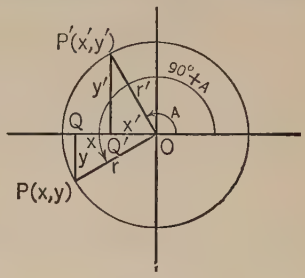


FIG. 48

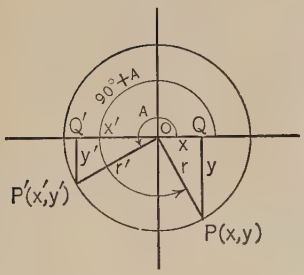


FIG. 49

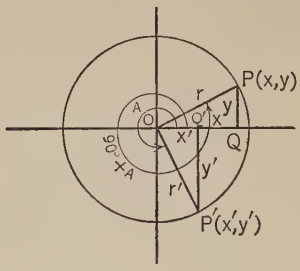


FIG. 50

ways get an acute angle, we see that this reduction may be done by applying some one of the following formulas a finite number of times:

$$\sin(90° + A) = \cos A, \qquad \csc(90° + A) = \sec A,$$
$$\cos(90° + A) = -\sin A, \qquad \sec(90° + A) = -\csc A, \quad \text{(A)}$$
$$\tan(90° + A) = -\cot A, \qquad \cot(90° + A) = -\tan A.$$

To prove these formulas, let angles $90° + A$ and $A$ each be placed in standard position. Let $OP = OP'$; (Figs. 47, 48, 49, 50); then $\triangle QOP = \triangle Q'OP'$, since they have the hypotenuse

and the acute angle of one triangle equal respectively to the hypotenuse and the acute angle of the other. We note in comparing the coördinates of $P(x, y)$ and $P'(x', y')$ that, in each of the Figures 47–50,

$$x = -y', \quad y = x', \quad r = r',$$

and by the definitions of the functions we get from these figures:

$$\sin(90° + A) = \frac{y}{r} = \frac{x'}{r'} = \cos A,$$

$$\cos(90° + A) = \frac{x}{r} = -\frac{y'}{r'} = -\sin A,$$

$$\tan(90° + A) = \frac{x}{y} = -\frac{y'}{x'} = -\cot A,$$

and the student should obtain the last three formulas of (A) in the same way.

To illustrate how these formulas may be used, consider $\cos 256°$.

$$\cos 256° = \cos(90° + 166°) = -\sin 166°,$$
$$\sin 166° = \sin(90° + 76°) = \cos 76°,$$

and hence

$$\cos 256° = -\sin 166° = -\cos 76°;$$

that is, we can get the value of $\cos 256°$ by taking the negative of $\cos 76°$, which is found in the tables.

By generalizing, we can simplify the use of these formulas, if we note that any positive angle $B$ may be expressed in the form $B = n \cdot 90° + A$, where $n$ is a positive integer and $A$ is an acute angle. By the repeated application of these formulas, it is clear that:

*Any function of* $(n \cdot 90 + A) = \pm$ *the co-function of* $A$, *if* $n$ *is odd,*

$= \pm$ *the same function of* $A$ *if* $n$ *is even.*

The ambiguous sign ($\pm$) is determined in a special case by our knowledge of the sign of the function in that quadrant.

To illustrate reductions by this generalization, consider the following:

1. $\tan 240° = \tan(2 \cdot 90° + 60°) = +\tan 60°.$

We have the tangent function, because $n$ is even, and it is positive because 240° is in the third quadrant, where the tangent is always positive.

2. $\sin 305° = \sin(3 \cdot 90° + 35°) = -\cos 35°.$

We have here the cosine, because $n$ is odd, and it is negative because 305° is in the fourth quadrant.

3. $\cos 522° = \cos(5 \cdot 90° + 72°) = -\sin 72°.$

We have the sine, because $n$ is odd, and it is negative because 522° is in the second quadrant.

## EXERCISES

By means of figures giving the acute angles of the triangles of reference, express each of the following functions in terms of some function of a positive acute angle:

**1.** sin 165°. **3.** cos 286°. **5.** cot 755°.
**2.** tan 147°. **4.** sec 193°. **6.** sec 1323°.

By means of formulas or the generalized rule, express each of the following functions in terms of an acute angle:

**7.** tan 145°. **9.** cos 305°. **11.** csc 460°.
**8.** sec 252°. **10.** sin 375°. **12.** cot 710°.

Express the following functions in terms of some function of an angle less than 45°:

**13.** sin 140°. **14.** cos 325°. **15.** tan 245°.

From the tables, find the numerical values of:

**16.** cos 175°. **17.** sin 203°. **18.** tan 100°.

Prove these relations for any $A$:

**19.** $\sin(180° + A) = -\sin A.$
**20.** $\cos(180° - A) = -\cos A.$
**21.** $\tan(360° - A) = -\tan A.$

**16. Functions of negative angles.** Trigonometric tables do not give directly the values of functions of negative angles, so it is sometimes desirable to express functions of negative angles in terms of positive angles. This may be done by the following formulas:

$$\begin{aligned} \sin(-A) &= -\sin A, & \csc(-A) &= -\csc A, \\ \cos(-A) &= \cos A, & \sec(-A) &= \sec A, \\ \tan(-A) &= -\tan A, & \cot(-A) &= -\cot A, \end{aligned} \qquad \text{(B)}$$

in which $A$ is any angle.

To prove these formulas, place $A$ and $-A$ in standard position, as in Figs. 51 and 52.

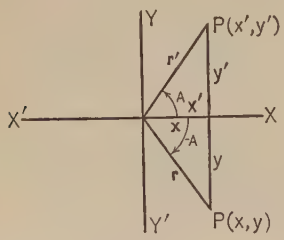


FIG. 51

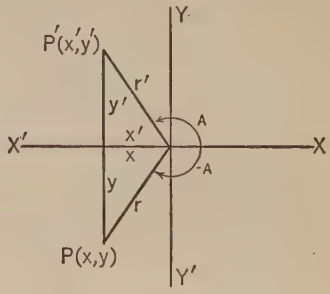


FIG. 52

From the figures we note that the coördinates of $P(x, y)$ and $P'(x', y')$ always have the relations

$$x = x', \quad y = -y', \quad \text{and} \quad r = r',$$

and by using the definitions of the trigonometric functions, we obtain

$$\sin(-A) = \frac{y}{r} = \frac{-y'}{r'} = -\frac{y'}{r'} = -\sin A,$$

$$\cos(-A) = \frac{x}{r} = \frac{x'}{r'} = \cos A,$$

$$\tan(-A) = \frac{y}{x} = \frac{-y'}{x'} = -\frac{y'}{x'} = -\tan A.$$

The student should work out the other three formulas of (B) by the same method.

## EXERCISES

Express each of the following in terms of some positive acute angle:

1. sin (− 45°).
2. cos (− 60°).
3. tan (− 30°).
4. csc (− 150°).
5. sec (− 315°).
6. cot (− 135°).
7. cot (− 375°).
8. sec (− 222°).
9. csc (− 145°).
10. tan (− 89°).
11. cos (− 235°).
12. sin (− 330°).

**17. Graphs of the trigonometric functions.** Like line representations, the graphs of the trigonometric functions are helpful to the student in learning and remembering the variations in numerical value of these functions as the angle changes.

Consider the following table. For each $x$ selected, there is

| Point | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | Degrees | −90 | −75 | −60 | −45 | −30 | −15 | 0 | 15 | 30 | 45 | 60 |
| | Radians | $-\frac{\pi}{2}$ | $-\frac{5\pi}{12}$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | $-\frac{\pi}{12}$ | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
| $y$ | | −1 | −.965 | −.866 | −.707 | −.500 | −.258 | 0 | .258 | .500 | .707 | .866 |

| Point | | $P_{12}$ | $P_{13}$ | $P_{14}$ | $P_{15}$ | $P_{16}$ | $P_{17}$ | $P_{18}$ | $P_{19}$ | $P_{20}$ | $P_{21}$ | $P_{22}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | Degrees | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 |
| | Radians | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | $\pi$ | $\frac{13\pi}{12}$ | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ |
| $y$ | | .965 | 1. | .965 | .866 | .707 | .500 | .258 | 0 | .258 | .500 | .707 |

| Point | | $P_{23}$ | $P_{24}$ | $P_{25}$ | $P_{26}$ | $P_{27}$ | $P_{28}$ | $P_{29}$ | $P_{30}$ | $P_{31}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | Degrees | 240 | 255 | 270 | 285 | 300 | 315 | 330 | 345 | 360 | | |
| | Radians | $\frac{4\pi}{3}$ | $\frac{17\pi}{12}$ | $\frac{3\pi}{2}$ | $\frac{19\pi}{12}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | $\frac{23\pi}{12}$ | $2\pi$ | | |
| $y$ | | .866 | .965 | 1 | .965 | .866 | .707 | .500 | .258 | 0 | | |

a corresponding value of $y = \sin x$ obtained from a table of sines. Let these pairs of corresponding values be the rectangular coördinates of points in a plane. Plot these points just as you did in plotting equations in your algebra and join successive points to form a smooth curve which is the graph of the sine function. If we select values for $x$ every 15°, or for every $\frac{\pi}{12}$ radians, from $-90$ to $+360$, we get the table of corresponding values given on page 49. When plotted, these values give the

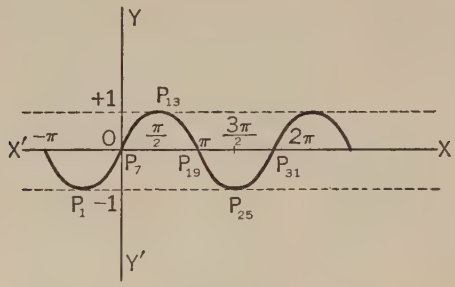


FIG. 53

curve in Fig. 53. It is clear that this curve continues on indefinitely in each direction, repeating itself every 360° or $(2\pi)$ radians.

For some purposes the curve of Fig. 53 should take the same unit on the two axes. This is not necessary; in fact, when using squared paper it is sometimes well to select different units on the two axes such that each scale may be easily read off from the squares. However, when the units are different on the two axes the slope of the curve is not a true representation of the rate of change.

Instead of taking the values of the sine from a table, the sine curve may be quickly constructed by means of line representations on a unit circle. Divide the unit circle into 24 equal parts, or any other number of parts that may be convenient. Drop perpendiculars from each division point to the horizontal diameter, and then with compasses or dividers transfer these perpendicu-

lars as the proper ordinates of the sine curve, taking the radius of the circle as the unit on the scales of $x$ and $y$.

In a similar way, the graphs of $y = \cos x$ (Fig. 54) and of $y = \tan x$ (Fig. 55) may be constructed.

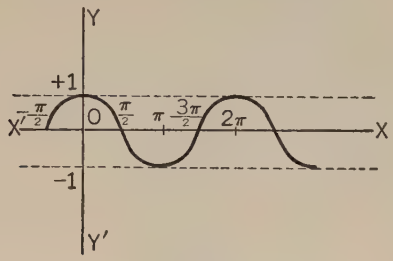


FIG. 54

We note that the sine and the cosine curves are continuous for all values of $x$, but the tangent curve is discontinuous for $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \cdots$; that is, the tangent curve would run off any paper near these points however large it be, and we again

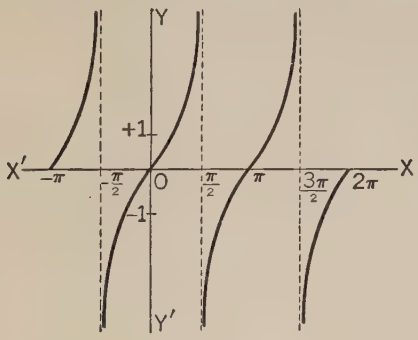


FIG. 55

see why there is no tangent of 90°. By remembering these curves, the variations of the functions may be read off from these mental pictures. These curves may also serve to recall or check many relations between functional values.

## EXERCISES

1. Construct the graph for $y = \sin x$, without the use of tables but by means of the unit circle.
2. Construct the graph for $y = \cos x$ in like manner.
3. Construct the graph for $y = \tan x$.
4. Construct the graph for $y = \csc x$.
5. Construct the graph for $y = \sec x$.
6. Construct the graph of $y = \cot x$.
7. Construct the graph of $y = \sin x + \cos x$.
8. Construct the graph for $y = 2 \sin x$.
9. Construct the graph for $y = \sin 2x$.
10. Construct the graph for $y = \sin \frac{x}{2}$.
11. Construct the graph for $y = \sin \frac{1}{x}$.
12. Construct the graph for $y = \cos 2x$.
13. Construct the graph for $y = \cos 3x$.
14. Construct graphs for $y = \sin (x - 90°)$, $y = \sin (x + 90°)$.
15. Construct the graph for $y = \cos (x + 180°)$.
16. Construct the graphs for $y = \sin (-x)$ and $y = -\sin x$.
17. Construct the graph for $y = \cos (-x)$ and $y = -\cos x$.
18. By means of graphs show that

$$\sin (180° - x) = \sin x,$$
$$\cos (180° - x) = -\cos x.$$

**18. Periodicity.** When studying the variations of the trigonometric functions by line representations or by graphs, it was no doubt noticed that the values of each function repeat themselves at regular intervals as the angle increases; that is, the trigonometric functions are **periodic.**

Since

$$\sin (x + 2\pi) = \sin x,$$

the sine is a periodic function with a period of $2\pi$. Similarly the cosine, secant and cosecant functions are periodic each with a period of $2\pi$. But since

$$\tan (x + \pi) = \tan x,$$

the tangent and cotangent have periods equal to $\pi$.

It is this property of periodicity which gives the trigonometric functions certain important applications. For example, the sine may be used in the study of wave or oscillatory motions which are common natural phenomenon.

**19. Fundamental identities for any angle.** The fundamental identities proven in Chapter I for acute angles are also true for angles of any magnitude. Consider the relations:

$$\sin^2\theta + \cos^2\theta = 1,$$
$$1 + \tan^2\theta = \sec^2\theta,$$
$$1 + \cot^2\theta = \csc^2\theta,$$

all of which may be derived from the relation

$$x^2 + y^2 = r^2,$$

which latter relation involves only the squares of the coördinates $x$ and $y$ and, therefore, is not dependent upon the sign of either $x$ or $y$.

Since the radius vector $r$ is always taken positive in the defining relations, it is clear that

$$\tan\theta = \frac{y}{x} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin\theta}{\cos\theta}$$

and

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

are true for any $\theta$.

However, in certain derived relations involving radicals, such as

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta},$$

it is necessary to be careful to use the proper sign of the radical. Thus if $\theta$ is in either the third or fourth quadrant, we have

$$\sin\theta = -\sqrt{1 - \cos^2\theta},$$

but if $\theta$ is in the first or second quadrant, we have

$$\sin\theta = +\sqrt{1 - \cos^2\theta}.$$

Our knowledge of the sign of each function in the various quadrants will enable us to select the proper sign for the radical.

In case the quadrant for the angle is not known, carry the double sign until some new fact determines the sign.

## EXERCISES

Verify the following identities by substituting the angles 45°, 120°, 240° and 315°:

1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.
2. $\sin^2\theta + \cos^2\theta = 1$.
3. $1 + \tan^2\theta = \sec^2\theta$.
4. $1 + \cot^2\theta = \csc^2\theta$.

Verify the following identities by substituting the angles $\pi/3$, $3\pi/4$, $7\pi/6$, and $11\pi/6$:

5. $\dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$.
6. $\cos\theta\csc\theta\tan\theta = 1$.
7. $\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$.
8. $\dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta} = 2\tan\theta$.

Prove the following identities:

9. $(\tan\theta + \cot\theta)\sin\theta\cos\theta = 1$.
10. $\dfrac{1 - \sin\theta}{1 + \sin\theta} = (\sec\theta - \tan\theta)^2$.
11. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$.
12. $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$.
13. $\dfrac{1 + \sin\theta}{1 - \sin\theta} - \dfrac{1 - \sin\theta}{1 + \sin\theta} = 4\tan\theta\sec\theta$.
14. $\dfrac{\csc\theta}{(\cot\theta + \tan\theta)} = \cos\theta$.
15. $\dfrac{\cos^2\theta}{(1 - \sin\theta)} = 1 + \sin\theta$.
16. $\dfrac{(1 - 2\cos^2\theta)}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$.
17. $\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$.
18. $\dfrac{(1 + \cos\theta)}{(1 - \cos\theta)} = (\csc\theta + \cot\theta)^2$.
19. $\dfrac{(1 - \cot^2\theta)}{(1 + \cot^2\theta)} = \sin^2\theta - \cos^2\theta$.
20. $\dfrac{\cot\theta + \sin\theta}{(1 + \cos\theta)} = \csc\theta$.

## CHAPTER III

## FUNCTIONS OF THE SUM OF TWO ANGLES AND RELATED FORMULAS

**20. Directed line segments.** It is often helpful in the analysis of a geometric figure to think of line segments as having quality, or sign, as well as magnitude. We are familiar with directed lines in a system of rectangular coördinates. By convention, horizontal lines are considered positive when running from left to right, and negative when running from right to left. Vertical lines are positive when running up and negative when running down. Oblique lines are considered either as always positive, or directed as marked on the figure.

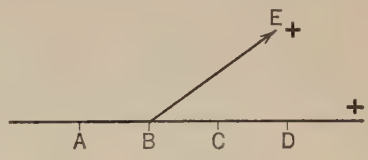

Fig. 56

If $A$, $B$, $C$ and $D$ are four points on a directed line, then the following obvious equalities illustrate the use of directed line segments:

$$AB = -BA,$$
$$AB + BC + CD = AD,$$
$$AB = AC + CB = AC - BC.$$

The use of directed line segments gives greater generality since a set of equalities in line segments often applies to many figures similarly drawn but varying greatly in appearance. For example, the line segment equalities of pages 58 and 59, Art. 22, apply equally well to Figs. 59, 60 and 61.

**21. Projections.** We have already considered some simple properties of projections in connection with triangle problems

(p. 19). The idea of projection is very useful in proving formulas for the sine and cosine of the sum of two angles.

The **projection of a point** upon a line is the foot of the perpendicular let fall from the point to the line. For example, $A'$ is the projection of $A$ on the line $MN$ (Fig. 57). Let $AB$ be any line segment; drop perpendiculars to the line $MN$ from the points $A$ and $B$. Then *the segment $A'B'$ is called the orthographic projection of segment $AB$ upon the line $MN$.*

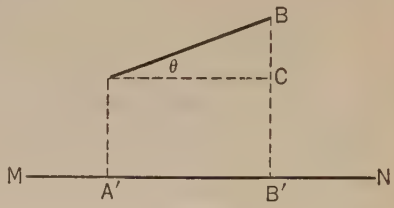

FIG. 57

Through $A$ draw a line parallel to $MN$ intersecting the perpendicular let fall from $B$ at $C$. Then $\angle CAB = \theta$ is equal to the angle between $AB$ and $MN$. Since

$$\frac{AC}{AB} = \cos\theta \quad \text{and} \quad AC = A'B',$$

we have the important relation

$$A'B' = AB\cos\theta, \tag{1}$$

or in word form:

*The projection of any line segment upon any other line is equal to the length of the segment multiplied by the cosine of the angle between the two lines.*

This relation has many applications, such as that of finding the resultant of forces acting at a point or of finding the component of a velocity in a given direction. In applications, it is usually practically possible and simpler to consider both the segment and its projection as positive and the angle $\theta$ acute. However, this relation is true for any directions for the segment $AB$ and the line $MN$ and for any value of $\theta$. The proof of such a general relation is left to the student.

Let $ABC$ be any triangle and $MN$ any line in the same plane (Fig. 58). Then $A'B'$, $B'C'$ and $C'A'$ are respectively

the projections of $AB$, $BC$ and $CA$, and we have the relations

$$A'B' = A'C' + C'B', \tag{2}$$

$$A'C' = A'B' + B'C' = A'B' - C'B', \tag{3}$$

which in words says that:

*The projection upon any line of one side of a triangle is equal to the sum of the projections upon the same line of the other two sides.*

As a test, the student may wish to prove the following more general theorem:

*The projection of a line joining two points is always equal to the sum of the projections, upon the same line, of the parts of any broken line of any finite number of parts joining the same two points.*

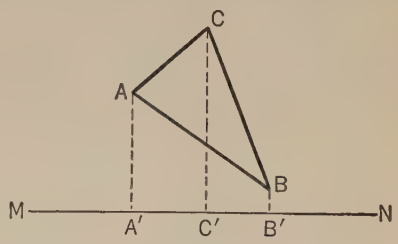


FIG. 58

## EXERCISES

**1.** Give the coördinates of the projections upon the $X$-axis of the following points: $(1, 3)$, $(3, 1)$, $(-1, 4)$, $(-4, 2)$, $(-5, -2)$, and $(+2, -6)$; also give their projections upon the $Y$-axis.

**2.** Give the projections upon the $X$-axis of the line segments joining the following pairs of points:

$A = (1, 1)$, $B = (2, 5)$; $C = (2, 3)$, $D = (-3, 1)$; $M = (-4, -2)$, $N = (2, 4)$; $L = (-5, 2)$, $K = (3, 2)$; $O = (3, 4)$, $P = (3, -5)$; also give their projections upon the $Y$-axis.

**3.** When are projections of line segments equal to zero? When equal to the line segment projected? When less than the line segment projected? Can a projection ever be greater than the line segment projected?

**4.** Given the triangle $A = (1, 3)$, $B = (7, 1)$, $C = (5, 8)$. Find the numerical value of Proj. $AB$ + Proj. $BC$ + Proj. $CA$ on the $X$-axis; on the $Y$-axis. (Proj. $AB$ means "projection of $AB$.")

**5.** Find the projection of the side of a square upon its diagonal.

**6.** Given a rectangle 10 inches long and 6 inches wide. Find the projection of each side upon the diagonal.

**7.** Given the parallelogram (0, 0), (6, 0), (9, 3), (3, 3). Find the projection of the long side on the long diagonal.

**8.** Given the rhombus (0, 0), (4, 3), (7, 7), (3, 4). Find the projection of a side on the long diagonal.

**22. The sine and the cosine of sums.** It probably has been observed that sin 90° is not equal to the sum of sin 30° and sin 60°; that is, in general a trigonometric function of a sum is not equal to the sum of the same functions of the parts of the sum. In fact, the trigonometric symbols are not distributive, but obey certain special addition formulas of their own. We shall give two proofs of the following important formulas:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y, \tag{4}$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y, \tag{5}$$

These relations should be memorized in word form; thus, for (4):

*The sine of a sum of two angles is equal to the sine of the first angle times the cosine of the second angle plus the cosine of the first angle times the sine of the second angle.*

Write out a similar word statement for (5).

### Projection Proof

Let $\angle AOB$ be $x$, $\angle BOC$ be $y$, then $\angle AOC$ is $x + y$, (in Fig. 59, $x$, $y$ and $x + y$ are each taken to be less than 90°).

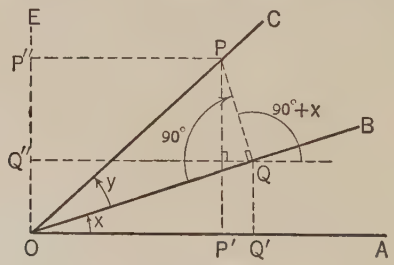


FIG. 59

From any point $P$ on $OC$ drop a perpendicular to $OB$ intersecting at $Q$; draw $OE$ perpendicular to $OA$. Now projecting the sides of $\triangle OQP$ upon $OA$, we get by Art. 21,

$$OP' = OQ' + Q'P',$$

or
$$\begin{aligned} OP\cos(x+y) &= OQ\cos x + QP\cos(90^\circ + x) \\ &= OQ\cos x - QP\sin x. \quad \text{(Art. 15)} \end{aligned}$$

Dividing by $OP$, we have

$$\cos(x+y) = \frac{OQ}{OP}\cos x - \frac{QP}{OP}\sin x,$$

and hence

$$\cos(x+y) = \cos y\cos x - \sin y\sin x,$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y. \tag{5}$$

Again, if we project the sides of $\triangle OQP$ upon $OE$, we get

$$OP'' = OQ'' + Q''P'',$$

or

$$OP\cos[90^\circ - (x+y)] = OQ\cos(90^\circ - x) + QP\cos x$$

and by Art. 4, this relation becomes

$$OP\sin(x+y) = OQ\sin x + QP\cos x.$$

Dividing by $OP$, this relation becomes

$$\sin(x+y) = \frac{OQ}{OP}\sin x + \frac{QP}{OP}\cos x,$$

whence

$$\sin(x+y) = \cos y\sin x + \sin y\cos x$$
$$\sin(x+y) = \sin x\cos y + \cos x\sin y. \tag{4}$$

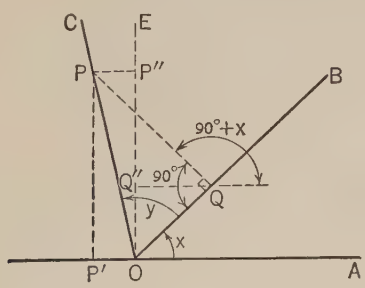


FIG. 60

The reader may verify the fact that this analysis holds for Fig. 60 where $x < 90^\circ$, $y < 90^\circ$, but $x + y > 90^\circ$.

Formulas (4) and (5) have been proved here for angles $x$ and $y$

each less than 90°. They are true, however, for any $x$ and any $y$, as will be shown later in Art. 23. It will be good practice for the student to derive these formulas for other values of $x$ and $y$, such as those given in Fig. 61.

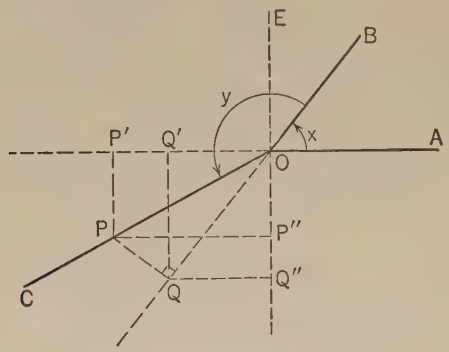


FIG. 61

## Second Proof

In Fig. 62 let $\angle AOB$ be $x$ and $\angle BOC$ be $y$, then $\angle AOC$ is $x + y$. From any point $P$ on $OC$ drop perpendiculars to $OB$ intersecting at $Q$ and to $OA$ intersecting in $S$. From $Q$ draw $QR$

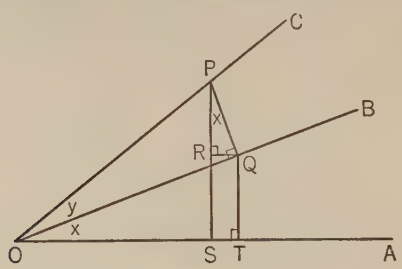


FIG. 62

parallel to $OA$ and $QT$ parallel to $PS$. Then $\angle QPR$ and $\angle AOQ$ are equal, since their sides are respectively perpendicular. We have

$$\sin(x + y) = \frac{SP}{OP} = \frac{SR + RP}{OP} = \frac{TQ + RP}{OP} = \frac{TQ}{OP} + \frac{RP}{OP}.$$

Multiplying the numerator and the denominator of the first fraction by $OQ$ and of the second fraction by $QP$, we get

$$\sin(x+y) = \frac{TQ}{OQ}\cdot\frac{OQ}{OP} + \frac{RP}{QP}\cdot\frac{QP}{OP},$$

whence

$$\sin(x+y) = \sin x\cos y + \cos x\sin y. \tag{4}$$

Again, from the same figure, we have

$$\cos(x+y) = \frac{OS}{OP} = \frac{OT-ST}{OP} = \frac{OT-RQ}{OP} = \frac{OT}{OP} - \frac{RQ}{OP},$$

and multiplying numerator and denominator of the first fraction by $OQ$ and of the second fraction by $PQ$, we get

$$\cos(x+y) = \frac{OT}{OQ}\cdot\frac{OQ}{OP} - \frac{RQ}{PQ}\cdot\frac{PQ}{OP},$$

whence

$$\cos(x+y) = \cos x\cos y - \sin x\sin y. \tag{5}$$

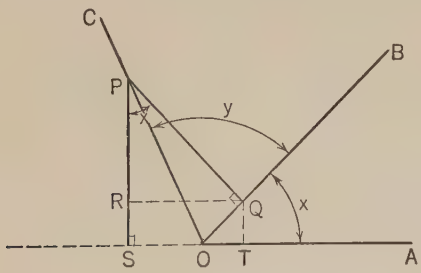


FIG. 63

In Fig. 62 $x$, $y$, and $x+y$ are each acute angles. The student can show that this analysis will work for other figures, such as Fig. 63, where $x < 90°$, $y < 90°$ but $x + y > 90°$.

**23. Generalization.** Let us assume that the formulas

$$\sin(x+y) = \sin x\cos y + \cos x\sin y, \tag{4}$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y, \tag{5}$$

are true for some values of $x$ and $y$.

By formulas of Art. 15, we have

$$\sin(x+y) = -\cos(x+y+90^\circ) \qquad \text{(a)}$$
$$\cos(x+y) = \sin(x+y+90^\circ), \qquad \text{(b)}$$
$$\sin y = -\cos(y+90^\circ), \qquad \text{(c)}$$
$$\cos y = \sin(y+90^\circ). \qquad \text{(d)}$$

Now by substituting (a), (c) and (d) in (4), we get

$$-\cos[x+(y+90^\circ)]$$
$$= \sin x \sin(y+90^\circ) - \cos x \cos(y+90^\circ),$$

which becomes, after changing signs and rearranging terms in the second member,

$$\cos[x+(y+90^\circ)]$$
$$= \cos x \cos(y+90^\circ) - \sin x \sin(y+90^\circ). \qquad \text{(e)}$$

In a similar way, by substituting (b), (c) and (d) in (5) we get

$$\sin[x+(y+90^\circ)]$$
$$= \cos x \sin(y+90^\circ) + \sin x \cos(y+90^\circ),$$

$$\sin[x+(y+90^\circ)]$$
$$= \sin x \cos(y+90^\circ) + \cos x \sin(y+90^\circ). \qquad \text{(f)}$$

We note that (4) and (5) are symmetric in $x$ and $y$; so in deriving (e) and (f) we have proven that (4) and (5) are true for either $x$ or $y$ increased by $90^\circ$. But in Art. 22 we proved (4) and (5) for $x$ and $y$ any acute angles. Hence, (4) and (5) are true for any $x$ and any $y$, since (e) and (f) may be applied any finite number of times and any negative angle can be considered as co-terminal with some positive angle.

To illustrate, suppose we wish to prove that

$$\sin(210^\circ + 140^\circ) = \sin 210^\circ \cos 140^\circ + \cos 210^\circ \sin 140^\circ.$$

We may write $210^\circ$ and $140^\circ$ as $(2\cdot 90^\circ + 30^\circ)$ and $(90^\circ + 50^\circ)$ respectively.

By the last section, since $30^\circ$ and $50^\circ$ are acute, we have

$$\sin(30^\circ + 50^\circ) = \sin 30^\circ \cos 50^\circ + \cos 30^\circ \sin 50^\circ,$$

and applying (f) to this three times we get,

$$\sin(30° + 140°) = \sin 30° \cos 140° + \cos 30° \sin 140°,$$
$$\sin(120° + 140°) = \sin 120° \cos 140° + \cos 120° \sin 140°,$$
$$\sin(210° + 140°) = \sin 210° \cos 140° + \cos 210° \sin 140°.$$

**24. Tangent and cotangent of sums.** We can derive formulas for $\tan(x + y)$ and $\cot(x + y)$ algebraically by using (4) and (5) of Art. 23 and other known formulas, as follows:

$$\tan(x + y) = \frac{\sin(x + y)}{\cos(x + y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}.$$

Dividing numerator and denominator by $\cos x \cos y$, we have

$$\tan(x + y) = \frac{\dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} - \dfrac{\sin x \sin y}{\cos x \cos y}},$$

whence

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}. \tag{6}$$

And again

$$\cot(x + y) = \frac{\cos(x + y)}{\sin(x + y)} = \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y},$$

and dividing numerator and denominator by $\sin x \sin y$ and reducing, this relation becomes

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}. \tag{7}$$

We can get similar formulas for $\sec(x + y)$ and for $\csc(x + y)$ by taking reciprocals of $\cos(x + y)$ and of $\sin(x + y)$ and reducing; but these formulas are seldom used.

**25. Functions of differences.** By using one or more of the formulas (4), (5), (6), (7), and the negative angle formulas of Art. 16, we get the following difference formulas:

$$\sin(x - y) = \sin[x + (-y)] = \sin x \cos(-y) + \cos x \sin(-y),$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y. \tag{8}$$

By similar methods, the student should prove that

$$\cos(x - y) = \cos x \cos y + \sin x \sin y. \tag{9}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}. \tag{10}$$

$$\cot(x - y) = \frac{\cot x \cot y + 1}{\cot x - \cot y}. \tag{11}$$

## EXERCISES

**1.** Show that $\sin(45° + 30°) \neq \sin 45° + \sin 30°$.

NOTE: The sign $\neq$ means "is not equal to."

**2.** Show that $\cos(60° + 30°) \neq \cos 60° + \cos 30°$.

**3.** Calculate, in terms of radicals, $\sin 75°$, $\cos 75°$, $\tan 75°$, noting that $75° = 45° + 30°$.

**4.** Calculate, in terms of radicals, $\sin 135°$, $\cos 135°$, noting that $135° = 90° + 45°$.

**5.** Find $\cos(60° + x)$ in terms of functions of $x$.

**6.** Find $\tan(x + 30°)$ in terms of functions of $x$.

**7.** Find, in terms of radicals, the six functions of $15°$ noting that $15° = 60° - 45°$.

**8.** Find in terms of radicals, the six functions of $15°$, using the fact that $15° = 45° - 30°$.

**9.** If $\sin x = \frac{5}{13}$ and $\sin y = \frac{3}{5}$ where $x$ and $y$ are acute angles, find $\sin(x + y)$, $\cos(x + y)$ and $\tan(x + y)$.

**10.** If $\tan x = \frac{4}{3}$ and $\tan y = \frac{3}{4}$ where $x$ and $y$ are acute angles, find $\sin(x - y)$, $\cos(x - y)$ and $\tan(x - y)$.

Prove the following identities:

**11.** $\cos(60° + x) + \sin(30° + x) = \cos x$.

**12.** $\tan(y \pm 45°) + \cot(y \mp 45°) = 0$.

**13.** $\cos\left(\frac{\pi}{6} + x\right) - \cos\left(\frac{\pi}{6} - x\right) = -\sin x$.

**14.** $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$.

**15.** $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$.

**16.** $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$.

**17.** Show that $2\sin(45° + A) = \sqrt{2}(\cos A + \sin A)$.

**18.** Prove $\sin(60° + A) - \cos(30° + A) = \sin A$.

**19.** By means of the formulas for the sine and cosine of the sum of two angles express $\sin(90° + A)$ and $\cos(90° + A)$ in terms of $\sin A$ and $\cos A$.

**20.** Prove $\tan 15° = 2 - \sqrt{3}$.

HINT: $15° = 45° - 30°$.

**21.** Find $\sin 15°$ and $\cos 15°$ without referring to tables.

**22.** Show that $\cos(30° + A) + \cos(30° - A) = \sqrt{3}\cos A$.

**23.** Given $\sin x = 0.191$, $\cos y = 0.946$ where $x$ and $y$ are acute angles. Find $\sin(x + y)$ and $\cos(x + y)$ and verify the results from the table on page 13.

**24.** Given $\tan x = 0.306$ and $\tan y = 0.554$ where $x$ and $y$ are acute angles. Find $\tan(x + y)$ and verify the result from the table on page 13.

**25.** Work Exercise 23 when $x$ is an obtuse angle.

**26.** Work Exercise 24 when $y$ is an angle in the third quadrant.

**27.** Given $\tan A = \frac{2}{3}$, $\tan(A + B) = 1$; find $\tan B$.

**28.** Given $\tan A = n$, $A + B = 45°$. Show that $\tan B = \dfrac{1 - n}{1 + n}$.

**26. Functions of multiple angles.** If we let $y$ equal $x$ in formulas (4), (5), (6) and (7) of Art. 23 and Art. 24, we get the double angle formulas. From (4) we find

$$\sin(x + x) = \sin x \cos x + \cos x \sin x,$$

and hence

$$\sin 2x = 2 \sin x \cos x. \tag{12}$$

Again, we have

$$\cos(x + x) = \cos x \cos x - \sin x \sin x,$$

and consequently

$$\cos 2x = \cos^2 x - \sin^2 x, \tag{13}$$

$$= 1 - 2\sin^2 x, \tag{14}$$

$$= 2\cos^2 x - 1. \tag{15}$$

In a similar way, we get

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}, \tag{16}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}. \tag{17}$$

By a succession of such steps, we could derive formulas for any integral multiple of an angle. To illustrate, by (4) we have

$$\sin 3x = \sin(x + 2x) = \sin x \cos 2x + \cos x \sin 2x,$$

which by aid of (12) and (14) becomes

$$\begin{aligned}\sin 3x &= \sin x\,(1 - 2\sin^2 x) + \cos x\,(2\sin x\cos x)\\ &= \sin x - 2\sin^3 x + 2\sin x\cos^2 x\\ &= \sin x - 2\sin^3 x + 2\sin x\,(1 - \sin^2 x)\\ &= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x,\\ \sin 3x &= 3\sin x - 4\sin^3 x. \qquad (18)\end{aligned}$$

NOTE. Formula (18) is often used to prove that it is not possible to trisect any angle by ruler and compasses construction. See Ball's "Mathematical Recreations," p. 285.

The student should prove that

$$\cos 3x = 4\cos^3 x - 3\cos x, \qquad (19)$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}. \qquad (20)$$

**27. Functions of half angles.** By formula (A) of Art. 8 and (13) of Art. 26 we have

$$\cos^2 A + \sin^2 A = 1, \qquad (a)$$

$$\cos^2 A - \sin^2 A = \cos 2A, \qquad (b)$$

and by subtracting (b) from (a), we get

$$2\sin^2 A = 1 - \cos 2A,$$

$$\sin^2 A = \frac{1 - \cos 2A}{2},$$

$$\sin A = \sqrt{\frac{1 - \cos 2A}{2}},$$

or, if $x = 2A$, this relation becomes

$$\sin\frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}. \qquad (21)$$

Again by adding (a) and (b) we get

$$2\cos^2 A = 1 + \cos 2A,$$

$$\cos^2 A = \frac{1 + \cos 2A}{2},$$

$$\cos A = \sqrt{\frac{1 + \cos 2A}{2}},$$

and letting $x = 2A$, this relation becomes

$$\cos\frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}. \tag{22}$$

Dividing (21) by (22), we get

$$\tan\frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}, \tag{23'}$$

and by multiplying the numerator and the denominator by $\sqrt{1 - \cos x}$, we get

$$\tan\frac{x}{2} = \frac{1 - \cos x}{\sqrt{1 - \cos^2 x}} = \frac{1 - \cos x}{\sin x}.$$

By multiplying the numerator and the denominator of (23′) by $\sqrt{1 + \cos x}$, we get

$$\tan\frac{x}{2} = \frac{\sqrt{1 - \cos^2 x}}{1 + \cos x} = \frac{\sin x}{1 + \cos x};$$

that is,

$$\tan\frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}. \tag{23}$$

His knowledge of the signs of the functions in each quadrant will enable the student to select the proper sign for the radicals in these formulas.

## EXERCISES

1. From $\sin 30°$, find $\sin 60°$.
2. From $\cos 45°$, find $\cos 90°$.
3. From $\cos 60°$, find $\cos 30°$.
4. From $\sin 90°$, find $\sin 45°$.
5. From $\tan 60°$, find $\tan 120°$.
6. From $\cos 45°$, find $\cos 135°$.
7. Develop a formula for $\sin 4x$ in terms of $\sin x$.
8. Develop a formula for $\cos 4x$ in terms of $\cos x$.

Prove the following identities:

9. $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$.
10. $\csc 2x = \dfrac{\sec x \csc x}{2}$.
11. $\dfrac{1 + \cos x}{\sin x} = \cot\dfrac{x}{2}$.
12. $\left(\sin\dfrac{x}{2} - \cos\dfrac{x}{2}\right)^2 = 1 - \sin x$.

**13.** $\cot x = \dfrac{\sin 2x}{1 - \cos 2x}$.

**14.** $\dfrac{\sin 2x}{2} = \dfrac{\tan x}{1 + \tan^2 x}$.

**15.** $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$.

**16.** $\sin 2x (\tan x + \cot x) = 2$.

**17.** $\cot \dfrac{x}{2} - \tan \dfrac{x}{2} = 2 \cot x$.

**18.** $\sin x - \cos x = \pm \sqrt{1 - \sin 2x}$. Explain the use of the $\pm$ sign.

**19.** $\sin x + \cos x = \pm \sqrt{1 + \sin 2x}$. Explain the use of the $\pm$ sign.

**20.** Show that $\tan (45° + x) - \tan (45° - x) = 2 \tan 2x$.

**21.** Given $\sin \dfrac{x}{2} = \dfrac{3}{5}$, find $\tan x$.

**22.** Given $\cos \dfrac{x}{2} = \dfrac{4}{5}$, find $\cot x$.

**23.** Given $\sin x = \dfrac{5}{13}$, and let $x$ be an angle in the second quadrant; find the values of the six trigonometric functions of $2x$.

**24.** Given $\cos x = -\dfrac{3}{5}$, and let $x$ be an angle in the third quadrant; find the values of the six trigonometric functions of $2x$.

**25.** Given $\tan x = \dfrac{3}{4}$, and let $x$ be an angle in the first quadrant; find the values of $\sin \dfrac{x}{2}$, $\cos \dfrac{x}{2}$, $\tan \dfrac{x}{2}$.

**26.** Given $\cos 2x = -\dfrac{1}{2}$ and let $x$ be an angle in the second quadrant; find the values of $\sin \dfrac{x}{2}$, $\cos \dfrac{x}{2}$, $\tan \dfrac{x}{2}$.

**27.** Prove $\tan 2x \tan x + 1 = \dfrac{\tan 2x}{\tan x} - 1$.

**28.** Prove $\tan \dfrac{x}{2} \tan x + 1 = \tan x \cot \dfrac{x}{2} - 1$.

**29.** Prove $\tan \left(45° + \dfrac{x}{2}\right) = \cot \left(45° - \dfrac{x}{2}\right)$.

**30.** Prove $\tan (45° + x) = \sec 2x + \tan 2x$.

**28. Product formulas.** It is sometimes desirable to transform sums and differences into products to make them more useful in logarithmic computation. The formulas of this section are useful for such purposes; for example see Art. 46.

We have previously derived the following formulas:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y, \tag{4}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y, \tag{8}$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y, \tag{5}$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y. \tag{9}$$

If we let $x + y = A$ and $x - y = B$ and solve these equations for $x$ and $y$ we get

$$x = \tfrac{1}{2}(A+B), \qquad y = \tfrac{1}{2}(A-B).$$

By substituting these values of $x$ and $y$ in the above four formulas, we get

$$\sin A = \sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) + \cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B), \tag{a}$$

$$\sin B = \sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) - \cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B), \tag{b}$$

$$\cos A = \cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) - \sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B), \tag{c}$$

$$\cos B = \cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) + \sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B). \tag{d}$$

Now by adding (a) and (b), we get

$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B). \tag{24}$$

Subtracting (b) from (a) gives

$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B). \tag{25}$$

Adding (c) and (d) gives

$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B). \tag{26}$$

Subtracting (d) from (c) gives

$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B). \tag{27}$$

Instead of memorizing these sum and difference formulas, it is usually better to memorize the method of deriving them.

## EXERCISES

Express as a product:

1. $\sin 50° + \sin 10°$.
2. $\cos 75° + \cos 15°$.
3. $\cos 80° - \cos 20°$.
4. $\sin 60° - \sin 20°$.
5. $\cos 4x + \cos 2x$.
6. $\sin 6x + \sin 2x$.
7. Show that $\sin 20° + \sin 40° = \cos 10°$.
8. Show that $\cos 50° + \cos 70° = \cos 10°$.

Prove the following identities:

9. $\dfrac{\sin x + \sin y}{\sin x - \sin y} = \dfrac{\tan \frac{1}{2}(x + y)}{\tan \frac{1}{2}(x - y)}$.

10. $\dfrac{\cos x - \cos y}{\cos x + \cos y} = -\dfrac{\tan \frac{1}{2}(x + y)}{\cancel{\tan} \cot \frac{1}{2}(x - y)}$.

11. $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$.

12. $\dfrac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} = \tan x$.

13. $\sin\left(x + \dfrac{\pi}{3}\right) + \sin\left(x - \dfrac{\pi}{3}\right) = \sin x$.

14. Prove $\cot A + \tan B = \dfrac{\cos (A - B)}{\sin A \cos B}$.

15. Prove $\cot A - \tan B = \dfrac{\cos (A + B)}{\sin A \cos B}$.

16. Prove $\tan A + \tan B = \dfrac{\sin (A + B)}{\cos A \cos B}$.

17. Prove $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$.

18. Prove $\dfrac{\sin 19° - \sin 41°}{\cos 19° - \cos 41°} = \sqrt{3}$.

19. Prove $\dfrac{\sin 47° + \sin 73°}{\cos 47° + \cos 73°} = \sqrt{3}$.

20. Prove $\dfrac{\sin (2A - 3B) + \sin 3B}{\cos (2A - 3B) + \cos 3B} = \tan A$.

## CHAPTER IV

## COMPUTATION AND LOGARITHMS

**29. Approximate numbers.** Most of the numbers used in trigonometry are approximations. For example, when we read from the table on page 13 that $\sin 35° = .574$, it does not mean that the sine of 35° is exactly .574 but that .574 gives the value of $\sin 35°$ as nearly as can be done with three figures. The approximate value .574 in this table really means any number between the exact numbers .5735 and .5745 and is "correct to 3 significant figures." From a four-place table of sines we find $\sin 35° = .5736$ meaning some number between .57355 and .57365.

Most of the numbers which come from measurements are approximate values. If we are measuring to the tenth of an inch and say that the length of a table is 27.3 inches, we mean that the table is longer than 27.25 inches and shorter than 27.35 inches. The difference between the exact value and the approximate value is called the **absolute error.** Thus the exact number $\frac{1}{3}$ may be represented by the three-place approximation .333. The absolute error is then $\frac{1}{3} - .333$. The quotient of the absolute error by the exact value is called the **relative or percentage error.**

**30. Significant figures.** When we say that a number is correct to a certain number of significant figures we usually refer to the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, but under certain circumstances 0 may be significant. It is always so if it occurs between two other significant figures. For example, in the following numbers taken from a "five-place" table the zeros are significant, 2.5013, 3.0015. On the other hand, the zeros in 0.00136 are not significant but are used simply to locate the

decimal point. Zeros at the end of a number may or may not be significant. If we say that the distance to the sun is 93,000,000 miles, the last zeros are not significant, for we do not know just what numbers should be there since we cannot measure the distance correct to a mile. If the distance to the sun measured to the nearest mile turned out to be 93,000,000 then all the zeros would be significant.

The position of the decimal point has no influence on the number of significant figures. Thus, let 57.1428, 0.571428 and 0.00571428 represent $\frac{400}{7}$, $\frac{4}{7}$ and $\frac{0.04}{7}$ approximately to 6 significant figures. The relative error of each is found to be 0.000001. The relative error is unchanged as the decimal point is moved. Thus the number of significant figures is a measure of relative or percentage error.

**31. Rejecting figures.** It is often necessary to reject figures at the end of an approximate number. For example, if we are working with four-place data the last two figures in the number .214631 are unnecessary and we write simply .2146. However, if the first rejected figure is greater than 5 or 5 followed by figures not all zero the last unrejected figure should be increased by 1. Thus, it is clear that .3176 is a closer approximation to .317592 than is .3175. If the rejected figure is 5 or 5 followed by nothing but zeros, it is often customary among computers to increase the last unrejected number by 1 if it is an odd number, but to leave it unchanged if it is an even number. For example, .42765 becomes .4276 but .317350 becomes .3174 when these numbers are cut down to 4 significant figures. If this rule is followed in a long piece of computation, the errors tend to compensate one another. This rule has been followed in working the problems in this book.

**32. Computation with approximate numbers.** The results of calculations based upon approximate numbers are approximate numbers, and we cannot expect these results to be any more exact than the least accurate number used in the calcula-

tion. If we add together the approximate numbers 2.176, 30.1752 and 5.12 in the usual way, we have

$$\begin{array}{r} 2.176 \\ 30.1752 \\ 5.12 \\ \hline 37.4712, \end{array}$$

where the last two digits in the result are worthless and the result should be written 37.47. It should be noticed that in addition we are concerned with the number of decimal places rather than with the number of significant figures. In the sum of a number of approximate numbers, we should not retain more decimal places than are found in the number with the smallest number of decimal places.

If we multiply together the approximate numbers 30.1752 and 2.176 in the usual way, we have

$$\begin{array}{r} 30.1752 \\ 2.176 \\ \hline 1810512 \\ 2112264 \\ 301752 \\ 603504 \\ \hline 65.6612352 \end{array}$$

We know however that 30.1752 means some number between 30.17515 and 30.17525 and that 2.176 means some number between 2.1755 and 2.1765. All that we can say then about the product is that it is some number lying between

$$2.1755 \cdot 30.17515 = 65.646038825$$

and

$$2.1765 \cdot 30.17525 = 65.676431625.$$

In the product of 30.1752 and 2.176, it is useless to keep more than 4 significant figures; that is,

$$30.1762 \cdot 2.176 = 65.66,$$

where the last figure is somewhat in doubt. In general, a product cannot be more accurate than its least accurate factor, and we should not keep more significant figures than are in the least accurate factor.

Again, in division we have

$$\frac{30.1752}{2.176} = 13.867279 + \cdots.$$

All we can know about the quotient of these two approximate numbers is that it is a number lying between

$$\frac{30.17515}{2.1765} = 13.864070 + \cdots$$

and

$$\frac{30\ 17525}{2.1755} = 13.870489 + \cdots.$$

Hence, it is useless to write the quotient to more than 4 significant figures. In general the quotient of two approximate numbers should not contain more significant figures than are in the less accurate of the two numbers which are the dividend and the divisor.

## EXERCISES AND PROBLEMS

**1.** Explain the difference between the approximate numbers 16.3, 16.30, 16.300 where the zeros are to be considered as significant.

**2.** Distinguish between the numbers 3, 3.0, 3.00, 3.000.

**3.** The numbers 0.2727, 2.727, 2727 represent $\frac{3}{11}$, $\frac{30}{11}$, $\frac{30000}{11}$ approximately to four significant figures. Find the absolute and relative errors of each.

**4.** The numbers 0.0286, 2.86, 28600 represent $\frac{0.2}{7}$, $\frac{20}{7}$, $\frac{200000}{7}$ approximately to three significant figures. Find the absolute and relative errors of each.

**5.** To ten significant figures, $\pi = 3.141592654$. Write $\pi$ to nine significant figures; also to eight, to seven, to six, to five, to four, to three, to two significant figures.

**6.** Add the following numbers which are to be considered as the results of measurements: 12.5, 1.132, 5.2163, 0.126.

**7.** The product of the approximate numbers 17.3 and .181 may take on any value between what two numbers?

**8.** The product of the approximate numbers .0316, 1.203 and 13.05 may take on any value between what two numbers?

**9.** Show that the quotient of 20.71 by 43.4 is some number between 0.476653+ and 0.477854+.

**10.** The fraction $\frac{20}{.376}$ may take on any value lying between what two numbers? The numerator is an exact number while the denominator is an approximate number.

**11.** The fraction $\frac{3.161}{1.83}$ may take on any value lying between what two numbers? Both numerator and denominator are to be considered as approximate numbers.

**12.** The numbers in the fraction $\frac{(.211)(7.18)}{.310}$ are the results of measurement. Between what two four-figure numbers does the fraction lie?

**13.** There are 2.539978 centimeters in 1 inch. A board measures $16\frac{1}{2}$ feet in length measured to the nearest tenth of an inch. Express the length in centimeters.

**14.** What is the maximum absolute error for the approximate number 2.017? What is the maximum relative error?

**15.** The numbers 1.52, 306.11, 91.43, 0.12 are all true to two decimal places. Find the maximum absolute error and the maximum relative error for each.

**16.** The numbers 603, 1.26, 76200, .00147 are all true to three significant figures. Find the maximum absolute error and the maximum relative error for each.

**33. Logarithms.** The solution of triangle problems in trigonometry involves much numerical calculation. One of the most efficient aids in reducing this numerical work to a minimum is found in the use of logarithms. If $N$ and $a$ are two positive numbers ($a \neq 1$), the exponent $x$ applied to $a$ to give $N$ is called the **logarithm** of $N$ to the base $a$. In symbols we write

$$a^x = N, \tag{1}$$

where $x$ is the logarithm of $N$ to the base $a$, written

$$x = \log_a N. \tag{2}$$

Equations (1) and (2) are the inverse, one of the other, in the same way that each of the equations $x^2 = y$, $x = \sqrt{y}$ is an inverse of the other. The number $a$ is called the **base** of the system of logarithms.

EXAMPLES: In $3^2 = 9$, 2 is the logarithm of 9 to the base 3, or $\log_3 9 = 2$; in $10^3 = 1000$, 3 is the logarithm of 1000 to the base 10, or $\log_{10} 1000 = 3$; in $10^{-1} = 0.1$, $-1$ is the logarithm of 0.1 to the base 10, or $\log_{10} 0.1 = -1$.

The number 10 is the base most commonly used and many tables of logarithms of numbers to the base 10 have been computed. In general, logarithms are approximate numbers.

## EXERCISES

**1.** If 2 is the base of a system of logarithms, what is the logarithm of 4? of 8? of 128? of 1/4? of 1?

**2.** If 7 is the base of a system of logarithms, what is the logarithm of 7? of 343? of 1/7? of 1/49? of 1?

**3.** Write the following equations in the inverse form:

$$\log_2 16 = 4,\ \log_{10} 100 = 2,\ \log_3 9 = 2,\ \log_9 (1/3) = -1/2,\ \log_{10} 1 = 0.$$

**4.** Write the inverse of each of the following equations:

$$2^5 = 32,\ 10^1 = 10,\ 7^3 = 343,\ 16^{-\frac{1}{2}} = 1/4,\ a^0 = 1.$$

**5.** In the definition of a logarithm on page 75, why is an exception made of the base $a = 1$?

**6.** Solve the following equations for $x$:

$$\log_7 x = 2;\quad \log_{10} 10000 = x;\quad \log_x 25 = 2;$$
$$\log_x (\tfrac{1}{2}) = -1/3;\quad \log_8 x = 2/3.$$

**7.** Name some exceptions to the statement that in general logarithms are approximate numbers.

**8.** Give at least one objection to the use of a number less than 1 for the base of a system of logarithms.

**34. Fundamental Laws.** There are four fundamental laws which will be used over and over again in the application of logarithms to the solution of triangles and to other problems. These laws are easily derived from the laws of exponents.

I. *The logarithm of a product equals the sum of the logarithms of its factors.*

Taking first the case where there are only two factors, $N$ and $M$ we write

$$x = \log_a N,$$
$$y = \log_a M.$$

From the definition of a logarithm, we may write

$$a^x = N,$$
$$a^y = M.$$

Hence, we have

$$a^x \cdot y^y = a^{x+y} = MN,$$

or

$$\log_a MN = x + y,$$

and consequently

$$\log_a MN = \log_a M + \log_a N.$$

For three or more factors the proof is obvious.

II. *The logarithm of a quotient equals the logarithm of the dividend minus the logarithm of the divisor.*

Let the quotient be $\frac{N}{M}$. As before, write

$$x = \log_a N,$$
$$y = \log_a M,$$
$$a^x = N, \quad a^y = M,$$

and

$$\frac{N}{M} = \frac{a^x}{a^y} = a^{x-y}.$$

We may then write

$$\log_a \frac{N}{M} = x - y,$$

or

$$\log_a \frac{N}{M} = \log_a N - \log_a M.$$

III. *The logarithm of the power of a number equals the logarithm of the number multiplied by the index of the power.*

Let

$$x = \log_a N.$$

Then

$$N = a^x,$$

and

$$N^p = a^{px},$$

whence

$$\log_a N^p = px,$$

or

$$\log_a N^p = p \cdot \log N.$$

IV. *The logarithm of the root of a number equals the logarithm of the number divided by the index of the root.*

Let

$$x = \log_a N, \quad a^x = N.$$

Then

$$\sqrt[q]{N} = N^{\frac{1}{q}} = a^{\frac{x}{q}},$$

and

$$\log_a \sqrt[q]{N} = \frac{x}{q},$$

whence

$$\log_a \sqrt[q]{N} = \frac{\log_a N}{q}.$$

## EXERCISES

**1.** Given to five significant figures, $\log_{10} 2 = .30103$, $\log_{10} 3 = .47712$, $\log_{10} 7 \doteq .84510$; find $\log_{10} 4$, $\log_{10} 9$, $\log_{10} \frac{3}{4}$, $\log_{10} 28$, $\log_{10} 32$.

**2.** Using the values in Exercise 1; find $\log_{10} \frac{2}{3}$, $\log_{10} \frac{3}{2}$, $\log_{10} 343$, $\log_{10} \sqrt{2}$, $\log_{10} \sqrt[3]{7}$, $\log_{10} 5$.

**3.** Prove $\log_a \dfrac{M \cdot N}{P} = \log_a M + \log_a N - \log_a P$.

**4.** Prove $\log_a \dfrac{1}{P} = -\log_a P$.

**35. Common logarithms.** Logarithms with the base 10 are called **common logarithms.** In this text, we shall use common logarithms only. Since there can be no confusion, the base will not be indicated. For example, when we write log 17, we shall mean $\log_{10} 17$. In this system, the logarithm of any power of 10 is an integer. We may tabulate as follows

$$
\begin{array}{llllll}
10^3 & = 1000. & \text{or} & \log 1000. & = & 3, \\
10^2 & = 100. & \text{or} & \log 100. & = & 2, \\
10^1 & = 10. & \text{or} & \log 10. & = & 1, \\
10^0 & = 1. & \text{or} & \log 1. & = & 0, \\
10^{-1} & = .1 & \text{or} & \log .1 & = & -1, \\
10^{-2} & = .01 & \text{or} & \log .01 & = & -2, \\
10^{-3} & = .001 & \text{or} & \log .001 & = & -3,
\end{array}
$$

or in general

$$\log 10^p = p.$$

Since moving the decimal point in a number to the right is equivalent to multiplication by some power of 10, it follows that if the decimal point in a number be moved one place to the right, the logarithm of the number is increased by 1, and the decimal part of the logarithm is not changed. For example, we find from a table of logarithms that

$$\log 3.146 = 0.49776.$$

Since $31.46 = 3.146 \times 10$, then we have

$$\log 31.46 = \log 3.146 + \log 10 = 1.49776.$$

Again $\quad \log 314.6 = \log 31.46 + \log 10 = 2.49776.$

Since moving a decimal point to the left is the same as dividing by some power of 10, it follows that the logarithm is decreased only by an integer. For a number greater than 10 the decimal part of the logarithm remains unchanged if we move the decimal point one place to the left. For example, we have

$$\log 31.46 = 1.49776.$$

Moving the decimal point in 31.46 one place to the left, we have

$$3.146 = \frac{31.46}{10},$$

and

$$\begin{aligned}\log 3.146 = \log 31.46 - \log 10 &= 1.49776 - 1 \\ &= 0.49776.\end{aligned}$$

Now, if we move the decimal point one place more to the left getting .3146, we have

$$\log 3.146 = \log 3.146 - \log 10 = 0.49776 - 1$$
$$= -0.50224.$$

Again, we have
$$\log .03146 = 0.49776 - 2$$
$$= -1.50224.$$

Here, we have two ways of writing the logarithm of a number less than 1. In the first, we have a positive decimal part and a negative integral part; in the second, we have a single negative number. The first method, for reasons to be seen, is preferable, although the latter method is useful upon occasions. If we use the first method we can say that every logarithm is made up of two parts, a positive decimal part and a part which is either an integer or zero. This integral or zero part is called the **characteristic** of the logarithm, while the decimal part is called the **mantissa.** Moving a decimal point of a number either to the right or to the left changes the characteristic but leaves the mantissa unchanged.

**36. Rules for characteristics.** Since $\log 1 = 0$ and $\log 10 = 1$, the common logarithm of any number between 1 and 10 is some number between 0 and 1; that is, the characteristic is 0. For example, $\log 3.146 = 0.49776$. The common logarithm of any number between 10 and 100 is some number between 1 and 2, that is the characteristic is 1. For example,

$$\log 31.46 = 1.49776.$$

If the number lies between 100 and 1000 the characteristic is 2. For example $\log 314.6 = 2.49776$. It follows that the logarithm of a number with one digit to the left of the decimal is zero plus the mantissa; with two digits to the left of the decimal, it is one plus the mantissa; and so on leading up to the formal rule:

*The characteristic of the common logarithm of a number greater than* 1 *is one less than the number of digits in the integral part of the number.*

The logarithm of .1 is $-1$, hence the logarithm of any number between .1 and 1 is some number between $-1$ and 0; that is, minus one plus the mantissa. For example,

$$\log .3146 = -1 + .49776.$$

For a number between .01 and .1 the logarithm is minus two plus the mantissa. For example,

$$\log .03146 = -2 + .49776.$$

This leads easily to the general rule for negative characteristics:

*The characteristic of the logarithm of any number less than* 1 *is negative and numerically one greater than the number of zeros immediately following the decimal point.*

For example,

$$\log .0003146 = -4 + .49776.$$

It is customary to write the minus sign over the characteristic and to omit the plus sign. For example, we may write

$$\begin{aligned} \log .03146 &= \bar{2}.49776, \\ \log .0003146 &= \bar{4}.49776. \end{aligned}$$

Another form for writing logarithms with negative characteristics, which has many advantages, is illustrated in the following examples:

$$\begin{aligned} \log .3146 &= \bar{1}.49776 = 9.49776 - 10. \\ \log .03146 &= \bar{2}.49776 = 8.49776 - 10. \\ \log .00003146 &= \bar{5}.49776 = 5.49776 - 10. \end{aligned}$$

Here, if $-n$ is the characteristic, it is replaced by $(10 - n) - 10$. The positive part, $10 - n$, is placed before and $-10$ after the mantissa. If $n$ is so large that $10 - n$ is not positive, take $n = (20 - n) - 20$. If logarithms are written in this form, the rule for characteristics becomes:

*To obtain the characteristic of the logarithm of a number less than* 1, *subtract from* 9 *the number of zeros immediately following*

*the decimal point, write the result before the mantissa and* $-10$ *after it.*

In this text we shall use this method exclusively.

We have seen that moving the decimal point changes only the characteristic of the logarithm of a number. That is, the mantissa is independent of the decimal point. While the characteristic is a whole number and can be found by a rule the mantissa is an approximate number and is found from tables.

**37. Tables of logarithms.** Tables of logarithms are really tables of mantissas. The characteristic must be obtained by the rules given in the previous article. These tables are called "four-place," "five-place," according to the number of significant figures given. A "three-place" table can be used for some calculations. Its accuracy is approximately equal to that of the ordinary slide rule in the hands of a skillful computer. A four-place table is accurate enough for use with ordinary data obtained by the use of field instruments where angles are measured to the nearest minute. A five-place table is accurate enough for all data except such as may be obtained by the use of very delicate instruments. It should be used when angles are measured to the nearest tenth of a minute. Where angles are measured to the nearest second, a six-place table is necessary. Tables of six, seven, eight and more places are useful and indispensable in certain problems of astronomy, geodesy and the mathematics of finance. Usually a table giving one place more than the data should be used and the final result cut down to the same number of significant figures as the data. Four-figure accuracy means accuracy within one hundredth of one per cent. This degree of accuracy is sufficient for most work. The use of a five-place table enables us to be reasonably sure of the final result to four significant figures if the data warrants such accuracy. In the answers to problems in this text, five-place tables are used and the result cut to four significant figures.

**38. To find the logarithm of a number from the table.** In an ordinary five-place table, the mantissas for all numbers

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **300** | 47 712 | 727 | 741 | 756 | 770 | 784 | 799 | 813 | 828 | 842 |
| 301 | 857 | 871 | 885 | 900 | 914 | 929 | 943 | 958 | 972 | 986 |
| 302 | 48 001 | 015 | 029 | 044 | 058 | 073 | 087 | 101 | 116 | 130 |
| 303 | 144 | 159 | 173 | 187 | 202 | 216 | 230 | 244 | 259 | 273 |
| 304 | 287 | 302 | 316 | 330 | 344 | 359 | 373 | 387 | 401 | 416 |
| 305 | 430 | 444 | 458 | 473 | 487 | 501 | 515 | 530 | 544 | 558 |
| 306 | 572 | 586 | 601 | 615 | 629 | 643 | 657 | 671 | 686 | 700 |
| 307 | 714 | 728 | 742 | 756 | 770 | 785 | 799 | 813 | 827 | 841 |
| 308 | 855 | 869 | 883 | 897 | 911 | 926 | 940 | 954 | 968 | 982 |
| 309 | 996 | *010 | *024 | *038 | *052 | *066 | *080 | *094 | *108 | *122 |
| **310** | 49 136 | 150 | 164 | 178 | 192 | 206 | 220 | 234 | 248 | 262 |
| 311 | 276 | 290 | 304 | 318 | 332 | 346 | 360 | 374 | 388 | 402 |
| 312 | 415 | 429 | 443 | 457 | 471 | 485 | 499 | 513 | 527 | 541 |
| 313 | 554 | 568 | 582 | 596 | 610 | 624 | 638 | 651 | 665 | 679 |
| 314 | 693 | 707 | 721 | 734 | 748 | 762 | 776 | 790 | 803 | 817 |
| 315 | 831 | 845 | 859 | 872 | 886 | 900 | 914 | 927 | 941 | 955 |
| 316 | 969 | 982 | 996 | *010 | *024 | *037 | *051 | *065 | *079 | *092 |
| 317 | 50 106 | 120 | 133 | 147 | 161 | 174 | 188 | 202 | 215 | 229 |
| 318 | 243 | 256 | 270 | 284 | 297 | 311 | 325 | 338 | 352 | 365 |
| 319 | 379 | 393 | 406 | 420 | 433 | 447 | 461 | 474 | 488 | 501 |
| **320** | 515 | 529 | 542 | 556 | 569 | 583 | 596 | 610 | 623 | 637 |
| 321 | 651 | 664 | 678 | 691 | 705 | 718 | 732 | 745 | 759 | 772 |
| 322 | 786 | 799 | 813 | 826 | 840 | 853 | 866 | 880 | 893 | 907 |
| 323 | 920 | 934 | 947 | 961 | 974 | 987 | *001 | *014 | *028 | *041 |
| 324 | 51 055 | 068 | 081 | 095 | 108 | 121 | 135 | 148 | 162 | 175 |
| 325 | 188 | 202 | 215 | 228 | 242 | 255 | 268 | 282 | 295 | 308 |
| 326 | 322 | 335 | 348 | 362 | 375 | 388 | 402 | 415 | 428 | 441 |
| 327 | 455 | 468 | 481 | 495 | 508 | 521 | 534 | 548 | 561 | 574 |
| 328 | 587 | 601 | 614 | 627 | 640 | 654 | 667 | 680 | 693 | 706 |
| 329 | 720 | 733 | 746 | 759 | 772 | 786 | 799 | 812 | 825 | 838 |
| **330** | 851 | 865 | 878 | 891 | 904 | 917 | 930 | 943 | 957 | 970 |
| 331 | 983 | 996 | *009 | *022 | *035 | *048 | *061 | *075 | *088 | *101 |
| 332 | 52 114 | 127 | 140 | 153 | 166 | 179 | 192 | 205 | 218 | 231 |
| 333 | 244 | 257 | 270 | 284 | 297 | 310 | 323 | 336 | 349 | 362 |
| 334 | 375 | 388 | 401 | 414 | 427 | 440 | 453 | 466 | 479 | 492 |
| 335 | 504 | 517 | 530 | 543 | 556 | 569 | 582 | 595 | 608 | 621 |
| 336 | 634 | 647 | 660 | 673 | 686 | 699 | 711 | 724 | 737 | 750 |
| 337 | 763 | 776 | 789 | 802 | 815 | 827 | 840 | 853 | 866 | 879 |
| 338 | 892 | 905 | 917 | 930 | 943 | 956 | 969 | 982 | 994 | *007 |
| 339 | 53 020 | 033 | 046 | 058 | 071 | 084 | 097 | 110 | 122 | 135 |
| **340** | 148 | 161 | 173 | 186 | 199 | 212 | 224 | 237 | 250 | 263 |
| 341 | 275 | 288 | 301 | 314 | 326 | 339 | 352 | 364 | 377 | 390 |
| 342 | 403 | 415 | 428 | 441 | 453 | 466 | 479 | 491 | 504 | 517 |
| 343 | 529 | 542 | 555 | 567 | 580 | 593 | 605 | 618 | 631 | 643 |
| 344 | 656 | 668 | 681 | 694 | 706 | 719 | 732 | 744 | 757 | 769 |
| 345 | 782 | 794 | 807 | 820 | 832 | 845 | 857 | 870 | 882 | 895 |
| 346 | 908 | 920 | 933 | 945 | 958 | 970 | 983 | 995 | *008 | *020 |
| 347 | 54 033 | 045 | 058 | 070 | 083 | 095 | 108 | 120 | 133 | 145 |
| 348 | 158 | 170 | 183 | 195 | 208 | 220 | 233 | 245 | 258 | 270 |
| 349 | 283 | 295 | 307 | 320 | 332 | 345 | 357 | 370 | 382 | 394 |
| **350** | 407 | 419 | 432 | 444 | 456 | 469 | 481 | 494 | 506 | 518 |

| Prop. Pts. | | | | |
|---|---|---|---|---|
| | | | | **15** |
| 1 | 1.5 | | | |
| 2 | 3.0 | | | |
| 3 | 4.5 | | | |
| 4 | 6.0 | | | |
| 5 | 7.5 | | | |
| 6 | 9.0 | | | |
| 7 | 10.5 | | | |
| 8 | 12.0 | | | |
| 9 | 13.5 | | | |
| | | | | **14** |
| 1 | 1.4 | | | |
| 2 | 2.8 | | | |
| 3 | 4.2 | | | |
| 4 | 5.6 | | | |
| 5 | 7.0 | | | |
| 6 | 8.4 | | | |
| 7 | 9.8 | | | |
| 8 | 11.2 | | | |
| 9 | 12.6 | | | |
| | | | | **13** |
| 1 | 1.3 | | | |
| 2 | 2.6 | | | |
| 3 | 3.9 | | | |
| 4 | 5.2 | | | |
| 5 | 6.5 | | | |
| 6 | 7.8 | | | |
| 7 | 9.1 | | | |
| 8 | 10.4 | | | |
| 9 | 11.7 | | | |
| | | | | **12** |
| 1 | 1.2 | | | |
| 2 | 2.4 | | | |
| 3 | 3.6 | | | |
| 4 | 4.8 | | | |
| 5 | 6.0 | | | |
| 6 | 7.2 | | | |
| 7 | 8.4 | | | |
| 8 | 9.6 | | | |
| 9 | 10.8 | | | |

from 100 to 9999 are given. On page 83 is a sample page of such a table. On the left margin under $N$ are found the first three digits of the number whose logarithm is to be found. The fourth digit is found in black type in the top row of the table indicating the column in which the logarithm is to be found. For example, to find the logarithm of 3.146, find 314 in the left margin. Move along the row indicated by 314 until we come to the column marked 6 at the top. We find here 776. These are the last three digits of the logarithm. The first two are obtained from the first column of the table, giving 49776 as the mantissa. Applying the rule for characteristics, we find

$$\log 3.146 = 0.49776.$$

EXERCISES

1. Find log 3246.
2. Find log 30.10.
3. Find log 3.406.
4. Find log 321.7.
5. Find log 3509.
6. Find log 33330.
7. Find log 308500.
8. Find log .03111.
9. Find log .3378.
10. Find log .003175.

**39. Interpolation.** From the table, one can obtain directly the logarithm of a four-figure numbers. To find the logarithm of a number with five or more significant figures, interpolation is necessary. For example, to find log 34615, we assume that it lies half way between log 34610 and log 34620. We find from the table

$$\log 34610 = 4.53920,$$
$$\log 34620 = 4.53933.$$

The difference between these logarithms is 13, not counting decimals. Taking half of 13 or 6.5 and adding it to 4.53920 we get 4.539265 as the number half way between 4.53910 and 4.53920. Since we keep all numbers to five places of decimals, we write

$$\log 34615 = 4.53926.$$

Again to find log 34613, we assume that it is the number lying .3 of the way from log 34610 to log 34620; that is, .3 of the way

from 4.53920 to 4.53933. Now the difference is 13 not counting decimals, and $.3 \cdot 13 = 3.9$ or 4 approximately. Hence,

$$\log 34613 = 4.53924.$$

This process is called interpolation by proportional parts. In this process we have to multiply the difference between two consecutive numbers in the table by certain decimals. This multiplication is found already performed in the little multiplication tables in the column headed "Prop. Pts."

### EXERCISES

1. Find log 333.33.
2. Find log 3.1121.
3. Find log 32177.
4. Find log .35079.
5. Find log .034567.
6. Find log 33.218.
7. Find log 3213.2.
8. Find log .00034114.
9. Find log 3324500.
10. Find log 343210.

**40. To find the number corresponding to a given logarithm.** The inverse process of finding the number corresponding to a given logarithm is shown in the following example. Given $\log A = 3.49592$, find $A$. For the time being, we disregard the characteristic and notice only the figures in the mantissa, 49592. This number is not found in the body of the table, but we find 49582 and 49596, one a little less the other a little greater than 49592. Since, we have

$$49596 - 49582 = 14$$

and

$$49592 - 49582 = 10,$$

the number 49592 lies $\frac{10}{14}$ of the way from 49582 to 49596. From the table we find 3132 corresponding to 49582 and 3133 to 49596. Now, putting in the characteristic 3, we write

$$\begin{aligned} \log 3132 &= 3.49582, \\ \log A &= 3.49592, \\ \log 3133 &= 3.49596, \end{aligned}$$

where we assume that $A$ lies $\frac{10}{14}$ of the way from 3132 to 3133.

Now $\frac{10}{14} = .7$ to one significant figure, so $A = 3132.7$.

### EXERCISES

Find numbers corresponding to the following logarithms:

**1.** 1.47885.
**2.** 0.50010.
**3.** 2.51370.
**4.** 3.53445.
**5.** 9.52840 − 10.
**6.** 9.54039 − 10.
**7.** 8.53290 − 10.
**8.** 7.51460 − 10.
**9.** 6.48010 − 10.
**10.** 4.52130.

**41. Logarithms in computation.** The use of logarithms in computation is shown in the following examples.

EXAMPLE 1. Find the value of $1.817 \times 3.139 \times 42.96 \times 818.4$.

From Art. 34 the logarithm of the product is the sum of the logarithms of the factors. From the tables, we obtain

$$\begin{aligned} \log 1.817 &= 0.25935 \\ \log 3.139 &= 0.49679 \\ \log 42.96 &= 1.63306 \\ \log 818.4 &= 2.91297 \\ \hline \log \text{product} &= 5.30217. \end{aligned}$$

In the tables we find 20053 corresponding to the mantissa 30217. This gives the product as 200530, but as the data was given to four significant figures only we should not write the product to more than four significant figures, and we have then for the product 200500.

If we had used a four-place table of logarithms, we should have found

$$\begin{aligned} \log 1.817 &= 0.2594 \\ \log 3.139 &= 0.4968 \\ \log 42.96 &= 1.6331 \\ \log 818.4 &= 2.9130 \\ \hline \log \text{product} &= 5.3023 \\ \text{product} &= 200400. \end{aligned}$$

By actual multiplication, we find by the method of Art. 32 that the product will be some number between 200405.8+ and 200651.3+ so that probably 200500 is more nearly correct than 200400.

EXAMPLE 2. Find $\left(\frac{2.714}{1.927}\right)^5$.

From Art. 34 the logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor. From the tables, we have

$$\begin{aligned} \log 2.714 &= 0.43361 \\ \log 1.927 &= 0.28488 \\ \hline \log \text{quotient} &= 0.14873. \end{aligned}$$

From Art. 34 the logarithm of the fifth power of a number is five times the logarithm of the number; hence, it follows that

$$\log\left(\frac{2.714}{1.927}\right)^5 = 0.74365,$$

$$\left(\frac{2.714}{1.927}\right)^5 = 5.542.$$

EXAMPLE 3. Find $\sqrt[3]{\frac{(981.7)(3.176)}{2.614}}$.

$$\begin{aligned} \log 981.7 &= 2.99198 \\ \log 3.176 &= 0.50188 \\ & \phantom{=} \overline{3.49386} \\ \log 2.614 &= 0.41731 \\ & \phantom{=} \overline{3.07655}. \end{aligned}$$

Since (Art. 34) the logarithm of the cube root of a number is one-third of the logarithm of the number, we have

$$\log \sqrt[3]{\frac{(981.7)(3.176)}{2.614}} = 1.02552,$$

$$\sqrt[3]{\frac{(981.7)(3.176)}{2.614}} = 10.61.$$

## EXERCISES AND PROBLEMS

Find by use of logarithms the results of the following exercises to four significant figures.

1. $(3.176)^2$.
2. $(51.83)^3$.
3. $\sqrt{8.216}$.
4. $\sqrt[3]{2950}$.
5. $\sqrt[4]{2950}$.
6. $\sqrt[7]{2950}$.
7. $3649 \times 2847$.
8. $25300 \times 92.31 \times 6.473$.
9. $\frac{7527}{83.21}$.
10. $\frac{6.758}{33.26}$.
11. $\frac{3.126 \times 52.71}{8.143 \times 0.5162}$.
12. $\frac{\sqrt{5}\,\sqrt[3]{7}}{\sqrt[3]{6}\,\sqrt{8}}$.
13. $\frac{\sqrt{\pi}}{\sqrt[3]{\pi}}$.
14. $\frac{(3.142)^2\,\sqrt{642.1}}{\sqrt{5.411}}$.

Much time can be saved in such problems as this by making a form before beginning the computation. For this exercise, the form is

$$\begin{aligned}
\log 3.142 &= \text{———} \\
\log (3.142)^2 &= \text{———} \\
\log 642.1 &= \text{———} \\
\log \sqrt{642.1} &= \text{———} \\
\log (3.142)^2 \sqrt{642.1} &= \text{———} \\
\log 5.411 &= \text{———} \\
\log \sqrt{5.411} &= \text{———} \\
\hline
\log \text{final result} &= \text{———} \\
\text{final result} &= \text{———}
\end{aligned}$$

**15.** $\dfrac{\sqrt{621.6}}{2.714\sqrt{31.45}}$.

**16.** $\dfrac{(21.43)^{\frac{2}{5}}\,(7.146)^{\frac{3}{5}}}{\sqrt{7.132}}$.

**17.** $\sqrt{2}\,\sqrt[3]{3}\,\sqrt[4]{4}$.

**18.** $\dfrac{1}{(2.716)^{\frac{1}{2}}\,(3.818)^{\frac{1}{3}}}$.

**19.** $\sqrt[3]{0.02163}$.

SOLUTION:

$$\begin{aligned}
\log 0.02163 &= 8.33506 - 10 \\
&= 28.33506 - 30 \\
\log \sqrt[3]{0.02163} &= 9.44502 - 10 \\
\sqrt[3]{0.02163} &= 0.2786
\end{aligned}$$

**20.** $\sqrt[5]{0.9321}$.

# CHAPTER V

## SOLUTION OF TRIANGLES

**42. Introductory.** In general, if three of the six parts of a triangle are given, provided the given parts are not all angles, the remaining parts can be found. The finding of these unknown parts is called the solution of the triangle. It is the object of this chapter to develop formulas and exhibit plans of calculation for such solutions. If a triangle is represented in the usual way with sides $a$, $b$ and $c$ and angles $A$, $B$ and $C$, we may have three parts not all angles given in many different ways, but all these ways are included in four cases.

CASE I. *Given a side and two angles.*

CASE II. *Given two sides and an angle opposite one of them.*

CASE III. *Given two sides and the included angle.*

CASE IV. *Given the three sides.*

It is of course assumed that the data given is such that a triangle is possible. For example, in Case I the two given angles must not total 180° or more, and in Case IV one of the given sides must not be greater than the sum of the other two.

**43. Law of sines.** A theorem called the law of sines enables us to solve Cases I and II. Consider the triangle $ABC$ with

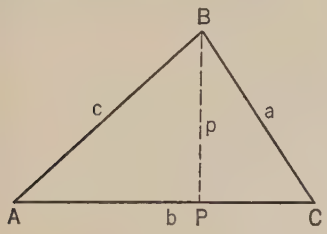


FIG. 64

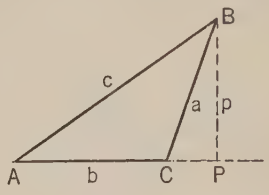


FIG. 65

sides $a$, $b$ and $c$. Let $P$ be the foot of the perpendicular dropped from the vertex at $B$ upon the opposite side $b$ (Fig. 64), or upon

the opposite side produced (Fig. 65). In either figure, the triangles $APB$ and $BPC$ are right triangles, and from the definition of the sine of an angle we may write

$$\sin A = \frac{p}{c}. \tag{1}$$

Recalling that $\sin(180 - C) = \sin C$, we find from either figure

$$\sin C = \frac{p}{a}. \tag{2}$$

Solving both (1) and (2) for $p$, and equating the two values, we find

$$c \sin A = a \sin C, \tag{3}$$

or

$$\frac{a}{\sin A} = \frac{c}{\sin C}.$$

In a similar way, by dropping perpendiculars from $A$ upon the opposite side, or the opposite side produced, we get

$$\frac{b}{\sin B} = \frac{c}{\sin C},$$

and we may write

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}. \tag{4}$$

This is the **Law of Sines.** Put into the form of a theorem, the law of sines is:

*In any triangle the sides are proportional to the sines of the opposite angles.*

### 44. Case I. Given a side and two angles.

EXAMPLE. Given $b = 176.2$, $A = 31°54'$, $B = 75°15'$. Find $a$, $c$ and $C$.

SOLUTION: Since

$$A + B + C = 180°$$
$$C = 180° - (A + B) = 72°51'.$$

To find $a$, choose a formula from the law of sines containing $a$ and three knowns; namely,

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad \text{or} \quad a = \frac{b \sin A}{\sin B}.$$

In a similar way, we have

$$c = \frac{b \sin C}{\sin B}.$$

Solving by logarithms, we obtain

$$\log a = \log b + \log \sin A - \log \sin B,$$
$$\log c = \log b + \log \sin C - \log \sin B.$$

| | |
|---|---|
| $\log b = 2.24601$ | $\log b = 2.24601$ |
| $\log \sin A = 9.72299 - 10$ | $\log \sin C = 9.98025 - 10$ |
| sum $= 1.96900$ | sum $= 2.22626$ |
| $\log \sin B = 9.98545 - 10$ | $\log \sin B = 9.98545 - 10$ |
| $\log a = 1.98355$ | $\log c = 2.24081$ |
| $a = 96.28$ | $c = 174.1.$ |

NOTE. In this example we have written down the $-10$ which comes with the logarithms of sines and cosines. This is usually unnecessary and hereafter we shall omit it.

## EXERCISES AND PROBLEMS

Solve the following triangles, having given

**1.** $b = 63.18$, $A = 25° 19'$, $C = 66° 30$.

**2.** $a = 7.436$, $A = 11° 36'$, $B = 23° 12'$.

**3.** $c = 21.45$, $B = 107° 10'$, $C = 29° 55'$.

**4.** $a = 1046$, $B = 76° 13'$, $C = 76° 13'$.

**5.** $c = 6476$, $A = 19° 37'$, $B = 98° 42'$.

**6.** $b = 104.1$, $B = 77° 9'$, $C = 81° 29'$.

**7.** In Fig. 66, $B$ is an inaccessible point. To find the distance $AB$, the angles $A$ and $C$ and the distance $AC$ are measured. $A = 48° 15'$, $C = 55° 33'$ and $AC = 667.4$ feet.

**8.** The angle of elevation of the top of a church steeple is observed to be $12° 46'$. At a point 150 feet nearer to the church the angle of elevation is $18° 4'$. Find the height of steeple.

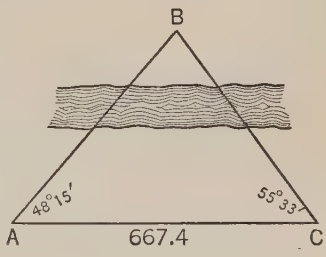


FIG. 66

**45. Case II. Given two sides and the angle opposite one of them.** Let the given parts be $a$, $c$ and $A$. In Case II, often called the ambiguous case, there may be no solution, one solution or two solutions as shown below.

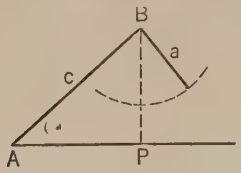


FIG. 67

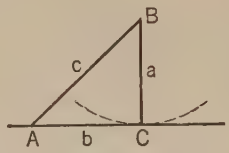


FIG. 68

1. The side $a$ may be too short to complete the triangle as shown in Fig. 67. The length of the perpendicular from $C$ to the side $b$ is $c \sin A$. Hence, if $a$ is less than $c \sin A$ there is no solution.

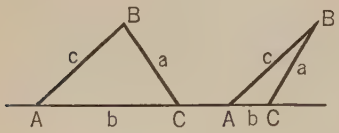


FIG. 69

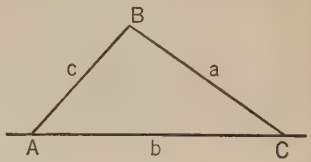


FIG. 70

2. If $a = c \sin A$, the triangle is a right-angled triangle, and there is one solution. (Fig. 68.)

3. If $a$ is greater than $c \sin A$ but less than $c$ there are two solutions. (Fig. 69.)

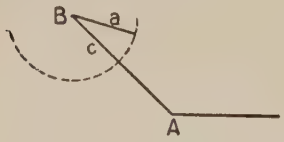


FIG. 71

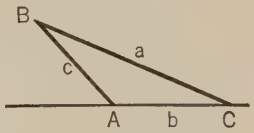


FIG. 72

4. If $a$ is greater than or equal to $c$ there is one solution. (Fig. 70.)

If the angle $A$ is greater than 90°, there are two more cases.

5. If $a$ is equal to or less than $c$, there is no solution. (Fig. 71.)

6. If $a$ is greater than $c$, there is one solution. (Fig. 72.)

A sketch made to scale will generally tell the number of solutions to be expected.

EXAMPLE 1. Given $b = 22.83$, $c = 28.71$, $B = 53° 33'$. Find $a$, $A$ and $c$.

SOLUTION: Making a sketch to scale, we have Fig. 73, showing no solution. If still in doubt as to whether the side $b$ will reach the line of side $a$, we find the perpendicular $c \sin B = 23.09$ which is greater than $b$. If we had gone ahead with the solution without making a preliminary sketch, the work would have been as follows:

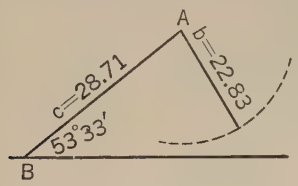


FIG. 73

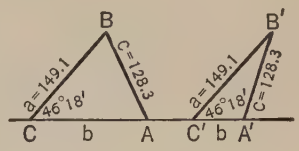


FIG. 74

From the law of sines, we have

$$\sin C = \frac{c \sin B}{b}$$

$$\begin{aligned} \log c &= 1.45803 \\ \log \sin B &= \underline{9.90546} \\ \text{sum} &= 1.36349 \\ \log b &= \underline{1.35851} \\ \log \sin C &= 0.00498 \end{aligned}$$

or

$$\sin C = 1.015.$$

We know that the sine of any angle is never greater than unity; hence, there is no angle $C$ and the triangle is impossible.

EXAMPLE 2. Given $a = 149.1$, $c = 128.3$, $C = 46° 18'$. Find $b$, $A$, $B$.

SOLUTION: A preliminary sketch drawn to scale shows two solutions.

From the law of sines, we have

$$\sin A = \frac{a \sin C}{c}$$

$$\begin{aligned} \log a &= 2.17348 \\ \log \sin C &= \underline{9.85912} \\ \text{sum} &= 2.03260 \\ \log c &= \underline{2.10823} \\ \log \sin A &= 9.92437 \\ A &= 57° 9.5'. \end{aligned}$$

This gives the angle $A$ of the triangle $CAB$. (Fig. 74.) The angle at $A'$ is $180° - A = 122° 50.5'$.

Completing first the solution of the triangle $CAB$, we have

$$B = 180° - (A + C) = 76° 32.5'.$$

$$\begin{aligned} b &= \frac{a \sin B}{\sin A} \\ \log a &= 2.17348 \\ \log \sin B &= 9.98790 \\ \text{sum} &= \overline{2.16138} \\ \log \sin A &= 9.92437 \\ \log b &= \overline{2.23701} \\ b &= 172.6. \end{aligned}$$

For the second triangle (Fig. 74), we have

$$B' = 180 - (A' + C) = 10° 51.5',$$

$$\begin{aligned} b' &= \frac{a \sin B'}{\sin A'} \\ \log a &= 2.17348 \\ \log \sin B' &= 9.27504 \\ \text{sum} &= \overline{1.44852} \\ \log \sin A' &= 9.92437 \\ \log b' &= \overline{1.52415} \\ b' &= 33.43. \end{aligned}$$

## EXERCISES AND PROBLEMS

Solve the following triangles, given

**1.** $a = 577.7$, $c = 1212$, $A = 28° 28'$.

**2.** $a = 36.00$, $b = 19.20$, $B = 32° 14'$.

**3.** $c = 234.5$, $b = 247.3$, $C = 72° 39'$.

**4.** $a = 6471$, $b = 5792$, $B = 63° 42'$.

**5.** $b = 102.3$, $c = 39.21$, $B = 16° 55'$.

**6.** $b = 117.6$, $c = 243.8$, $C = 38° 29'$.

**7.** $a = 81.18$, $c = 71.18$, $C = 51° 9'$.

**8.** $a = 684.2$, $b = 730.5$, $A = 24° 40.'$

**9.** $c = 1086$, $b = 841.5$, $C = 100°$.

**10.** $b = 25.31$, $c = 16.28$, $B = 131° 31'$.

**11.** $b = 21.43$, $c = 21.43$, $B = 125° 45'$.

**12.** $a = 343$, $c = 340$, $C = 109° 15'$.

**13.** It is desired to measure the distance $AB$ between two points on the opposite sides of a lake. A point $C$ easily accessible to $A$ and to $B$

is chosen. It is found that $AC = 8461$ and $BC = 10246$. At $A$ the angle $BAC$ is found to be $26° 33'$. Find $AB$.

**14.** A flagpole 76 feet high standing on a slope casts a shadow 129 feet long straight down the slope. At the end of the shadow the angle subtended by the pole is $29° 16'$. What is the elevation of the sun?

**46. Law of tangents.** From the law of sines, we have

$$\frac{a}{b} = \frac{\sin A}{\sin B}. \tag{5}$$

Adding unity to each member of this equation, it becomes

$$\frac{a}{b} + 1 = \frac{\sin A}{\sin B} + 1,$$

or

$$\frac{a + b}{b} = \frac{\sin A + \sin B}{\sin B}. \tag{6}$$

In a similar way, the subtraction of unity from each member of (5) gives

$$\frac{a - b}{b} = \frac{\sin A - \sin B}{\sin B}. \tag{7}$$

By division, we obtain from (6) and (7)

$$\frac{a - b}{a + b} = \frac{\sin A - \sin B}{\sin A + \sin B}. \tag{8}$$

In Art. 28, we have proved that

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B),$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$$

By substitution in (8), we find

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}. \tag{9}$$

This is the **Law of Tangents.** Put into words, the law of tangents is:

*In any triangle, the difference of two sides is to their sum as the tangent of half the difference of the opposite angles is to the tangent of half their sum.*

The law of tangents is convenient in the logarithmic solution of triangles, if two sides and the included angle are given.

**47. Case III. Given two sides and included angle.**

EXAMPLE. Given $a = 21.47$, $c = 128.4$, $B = 43° 37'$. Find $b$, $A$ and $C$.

SOLUTION: From the law of tangents, we have

$$\tan \tfrac{1}{2}(C - A) = \frac{c - a}{c + a} \tan \tfrac{1}{2}(C + A).$$

We choose the formula for $\tan \frac{1}{2}(C - A)$ instead of $\tan \frac{1}{2}(A - C)$ to avoid negative numbers.

$$\begin{aligned}
c - a &= 106.93 \\
c + a &= 149.87 \\
\tfrac{1}{2}(C + A) &= 68°11.5' \\
\log (c - a) &= 2.02910 \\
\log \tan \tfrac{1}{2}(C + A) &= \underline{0.39778} \\
\text{sum} &= 2.42688 \\
\log (c + a) &= \underline{2.17571} \\
\log \tan \tfrac{1}{2}(C - A) &= 0.25117 \\
\tfrac{1}{2}(C - A) &= 60°42.9' \\
\tfrac{1}{2}(C + A) &= \underline{68°11.5'} \\
\text{Adding, we find} \qquad C &= 128°54.4'. \\
\text{Subtracting, we have} \qquad A &= 7°28.6'.
\end{aligned}$$

To find the side $b$, we use the law of sines, and obtain

$$b = \frac{a \sin B}{\sin A}$$

$$\begin{aligned}
\log a &= 1.33183 \\
\log \sin B &= \underline{9.83874} \\
\text{sum} &= 1.17057 \\
\log \sin A &= \underline{9.11435} \\
\log b &= 2.05622 \\
b &= 113.8.
\end{aligned}$$

## EXERCISES AND PROBLEMS

Solve the following triangles, having given

**1.** $b = 10.36$, $c = 24.39$, $A = 38° 7'$.

**2.** $b = 9164$, $c = 6097$, $A = 68° 56'$.

**3.** $a = 1043$, $b = 9347$, $C = 107° 22'$.

**4.** $a = 834.9$, $b = 646.3$, $C = 121° 17'$.

**5.** $a = 123.9$, $c = 444.4$, $B = 75°$.

**6.** $a = 43.70$, $c = 30.10$, $B = 81° 30'$.

**7.** In order to find the distance $AB$ (Fig. 75) between two points on the opposite side of a dense wood, a point $C$ visible from both $A$ and $B$ is selected and the distances $AC$ and $BC$ and the angle $ACB$ are measured. It is found that $AC = 8316$ feet, $BC = 9841$ feet, and angle $ACB = 38° 16'$. Find the distance $AB$.

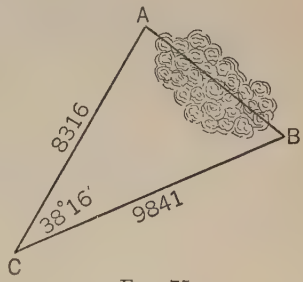

FIG. 75

**8.** The diagonals of a parallelogram are 18.3 inches and 22.7 inches long, respectively. The smaller angle between them is $46° 17'$. What are the lengths of the sides of the parallelogram?

**48. Law of cosines.** Problems coming under Case III may also be solved by using a formula called the law of cosines. This method is not well adapted to logarithmic computation, but is useful if the sides of the triangle are given to only one or two significant figures, or if only one side is to be found. The solu-

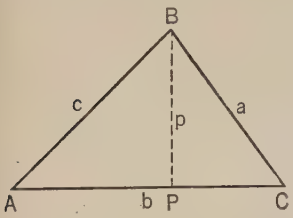

FIG. 76

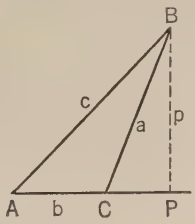

FIG. 77

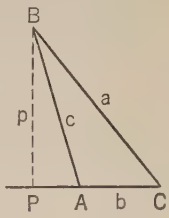

FIG. 78

tion of triangles is only a small part of the usefulness of the law of cosines. It is frequently met in more advanced mathematics.

From the Pythagorean theorem, we have for each of the triangles in Figs. 76–78

$$a^2 = p^2 + \overline{PC}^2, \tag{10}$$

$$c^2 = p^2 + \overline{AP}^2. \tag{11}$$

Solving the last equation for $p^2$ and substituting the result in the first equation, we have

$$a^2 = c^2 - \overline{AP}^2 + \overline{PC}^2. \tag{12}$$

Taking into consideration that $PC$ is negative in Fig. 77 and $AP$ is negative in Fig. 78, we may write

$$PC = b - AP,$$

and equation (12) becomes

$$\begin{aligned} a^2 &= c^2 - \overline{AP}^2 + (b - AP)^2 \\ &= c^2 - \overline{AP}^2 + b^2 - 2b\overline{AP} + \overline{AP}^2 \\ &= c^2 + b^2 - 2b\overline{AP}. \end{aligned} \tag{13}$$

But $AP = c \cos A$, and hence, we obtain

$$\boldsymbol{a^2 = c^2 + b^2 - 2bc \cos A.} \tag{14}$$

This is the **Law of Cosines.** Put into the form of a theorem, the law of cosines is:

*The square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice their product multiplied by the cosine of the included angle.*

EXAMPLE: Given $a = 9$, $c = 21$, $B = 32°$. Find $b$.

SOLUTION: From the law of cosines, we have

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ &= 81 + 441 - 2 \cdot 9 \cdot 21 \cdot .84805 \\ &= 201.44 \\ b &= 14.2. \end{aligned}$$

## EXERCISES AND PROBLEMS

**1.** Given $a = 8$, $b = 10$, $C = 51°$. Find the side $c$.

**2.** Given $b = 10$, $c = 20$, $A = 25° 23'$. Find $a$.

**3.** Given $a = 34$, $c = 42$, $B = 76° 30'$. Find $b$.

**4.** Given $a = 43$, $b = 43$, $C = 62°$. Find $c$.

**5.** Two roads start from the same point making an angle of 75° with each other. Two boys, one on a bicycle and the other on foot, leave the intersection at the same time and travel on the different roads. The

bicyclist travels at the rate of 12 miles per hour. The boy on foot walks 3 miles per hour. How far apart are the boys at the end of 40 minutes?

**6.** One signal station lies directly north and another northeast of an observation point. Guns fired simultaneously from the two stations are heard at the observation point in 3 seconds and 3.5 seconds, respectively. Find the distance between the two stations, assuming sound to travel at the rate of 1100 feet per second.

**49. Area of a triangle.** From elementary geometry, we know that the area of a triangle is equal to one-half the product of the base and altitude.

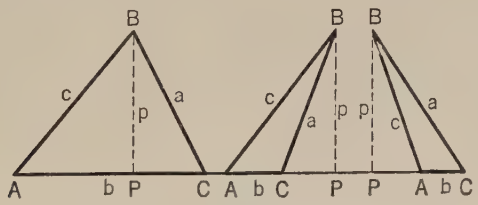


FIG. 79

For the triangles in Fig. 79, we have

$$\text{Area} = \tfrac{1}{2}pb.$$

But from Art. 1

$$p = c \sin A.$$

Hence, we have the formula

$$\textbf{Area} = \tfrac{1}{2}bc \sin A. \tag{15}$$

Since any side may be considered as the base, we get two more formulas similar to (15)

$$\textbf{Area} = \tfrac{1}{2}ac \sin B. \tag{16}$$

$$\textbf{Area} = \tfrac{1}{2}ab \sin C. \tag{17}$$

There are many formulas for the area of a triangle besides those given above. One of the most valuable is the formula for the area in terms of the sides only, often called **Heron's formula.** The formula may be derived as follows:

Squaring both members of (15), we have

$$\begin{aligned}(\text{Area})^2 &= \tfrac{1}{4}b^2c^2 \sin^2 A \\ &= \tfrac{1}{4}b^2c^2 (1 - \cos^2 A) \\ &= \tfrac{1}{4}b^2c^2 (1 + \cos A)(1 - \cos A).\end{aligned}$$

From (14) page 98 we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc};$$

hence,

$$\begin{aligned}(\text{Area})^2 &= \frac{bc}{2}\left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right) \cdot \frac{bc}{2}\left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right) \\ &= \frac{2bc + b^2 + c^2 - a^2}{4} \cdot \frac{2bc - b^2 - c^2 + a^2}{4} \\ &= \frac{(b + c)^2 - a^2}{4} \cdot \frac{a^2 - (b - c)^2}{4} \\ &= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \cdot \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}.\end{aligned}$$

Letting $2s = a + b + c$, we may write

$$\frac{b + c + a}{2} = s,$$

$$\frac{b + c - a}{2} = s - a,$$

$$\frac{a - b + c}{2} = s - b,$$

$$\frac{a + b - c}{2} = s - c.$$

Hence, we obtain

$$(\text{Area})^2 = s(s - a)(s - b)(s - c),$$

or

$$\textbf{Area} = \sqrt{s(s - a)(s - b)(s - c)}, \qquad (18)$$

which is Heron's formula.

The area may also be expressed in terms of the radius of the inscribed circle and $s$. Let $r$ (Fig. 80) be the radius of the

circle with $O$ as a center inscribed in the triangle. The area of the triangle $ABC$ is then made up of the areas of the three triangles $AOC$, $AOB$ and $BOC$, all having the same altitude $r$.

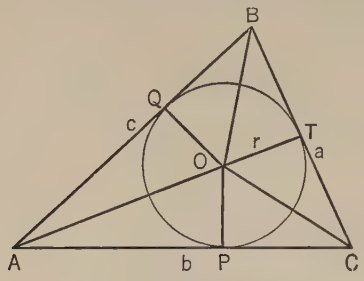


FIG. 80

We may write

$$\begin{aligned}\text{Area } ABC &= \text{Area } AOC + \text{Area } AOB + \text{Area } BOC \\ &= \tfrac{1}{2}br + \tfrac{1}{2}cr + \tfrac{1}{2}ar \\ &= r \cdot \frac{a+b+c}{2} \\ &= r \cdot s. \qquad (19)\end{aligned}$$

Comparing formulas (18) and (19), we can easily find an expression for the radius of the inscribed circle in terms of the sides. Thus, we have

$$r \cdot s = \sqrt{s(s-a)(s-b)(s-c)},$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}. \qquad (20)$$

## EXERCISES AND PROBLEMS

Find the areas of the following triangles:

| | | | |
|---|---|---|---|
| **1.** | $a = 162.8$, | $b = 89.6$, | $C = 72° 25'$. |
| **2.** | $b = 12.51$, | $c = 21.23$, | $A = 51° 13'$. |
| **3.** | $a = 31.4$, | $b = 40.8$, | $c = 49.7$. |
| **4.** | $a = 84{,}3$, | $b = 100.6$, | $c = 36.2$. |
| **5.** | $a = 825$, | $B = 63° 48'$, | $B = 49° 25'$. |
| **6.** | $b = 266$, | $B = 57° 18'$, | $C = 66° 8'$. |

**7.** Prove that the area of a triangle in terms of the side $a$ and the angles $A$, $B$ and $C$ is

$$\frac{\frac{1}{2}a^2 \sin B \sin C}{\sin A}.$$

**8.** Let $R$ be the radius of the circle circumscribing the triangle $ABC$ (Fig. 81).

$$\text{Prove that } 2R = \frac{b}{\sin B}.$$

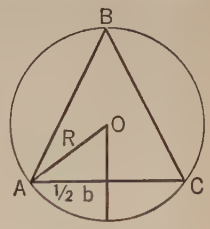


FIG. 81

**9.** Prove that the area of a triangle is equal to

$$\frac{abc}{4R},$$

where $R$ is the radius of the circumscribing circle.

**50. Case IV. Given the three sides.** We are now in a position to solve the triangle when the three sides are given.

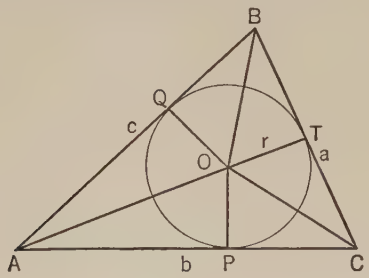


FIG. 82

From Fig. 82, we know from geometry that

$$AP = AQ,$$
$$BQ = BT,$$
$$CP = CT,$$

and that the lines $AO$, $BO$, $CO$ bisect the angles $A$, $B$ and $C$, respectively.

From Art. 49, we have

$$\begin{aligned} 2s &= a + b + c \\ &= BT + TC + AP + PC + AQ + QB \\ &= 2BT + 2TC + 2AP, \end{aligned}$$

or $$s = BT + TC + AP = a + AP.$$

From this relation, it follows that

$$AP = s - a.$$

Since $AO$ bisects the angle at $A$, we have from the right triangle $AOP$,

$$\tan \frac{A}{2} = \frac{r}{s - a}. \tag{21}$$

In a similar way, we find

$$\tan \frac{B}{2} = \frac{r}{s - b}, \tag{22}$$

$$\tan \frac{C}{2} = \frac{r}{s - c}. \tag{23}$$

These formulas are convenient for the logarithmic calculation of the angles when the sides are given.

EXAMPLE: Given $a = 10.26$, $b = 18.24$, $c = 15.98$. Find $A$, $B$ and $C$.

SOLUTION: Find first $\log r$, where $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$.

$$\begin{aligned} 2s &= a + b + c = 44.48 \\ s &= 22.24 \\ s - a &= 11.98 \\ s - b &= 4.00 \\ s - c &= 6.26 \end{aligned}$$

$$\begin{aligned} \log (s - a) &= 1.07846 \\ \log (s - b) &= 0.60206 \\ \log (s - c) &= 0.79657 \\ \text{sum} &= 2.47709 \\ \log s &= 1.34713 \\ \log r^2 &= 1.12996 \\ \log r &= 0.56498. \end{aligned}$$

Using the formulas $\tan \frac{A}{2} = \frac{r}{s-a}$, $\tan \frac{B}{2} = \frac{r}{s-b}$, $\tan \frac{C}{2} = \frac{r}{s-c}$, we have

$$\begin{aligned} \log r &= 0.56498 \\ \log (s - a) &= 1.07846 \\ \log \tan \tfrac{1}{2}A &= 9.48652 \\ \tfrac{1}{2}A &= 17^\circ\, 2.6' \\ A &= 34^\circ\, 5.2' \end{aligned} \qquad \begin{aligned} \log r &= 0.56498 \\ \log (s - b) &= 0.60206 \\ \log \tan \tfrac{1}{2}B &= 9.96292 \\ \tfrac{1}{2}B &= 42^\circ\, 33.4' \\ B &= 85^\circ\, 6.8' \end{aligned} \qquad \begin{aligned} \log r &= 0.56498 \\ \log (s - c) &= 0.79657 \\ \log \tan \tfrac{1}{2}C &= 9.76841 \\ \tfrac{1}{2}C &= 30^\circ\, 24' \\ C &= 60^\circ\, 48' \end{aligned}$$

CHECK: $A + B + C = 180^\circ$.

## EXERCISES AND PROBLEMS

Solve the following triangles:

**1.** $a = 2.236$, $b = 2.449$, $c = 2.646$.

**2.** $a = 191$, $b = 339$, $c = 490$.

**3.** $a = 14.49$, $b = 55.44$, $c = 66.91$.

4. $a = 113, \quad b = 147, \quad c = 41.$

5. The sides of a triangle are given as exact integers all different. Find to the nearest minute the angles of the triangle of the smallest possible perimeter. Also find the area.

6. A house is situated on a hillside. A ladder 32 feet long just reaches the eaves which are 25 feet from the ground. The base of the ladder is 13 feet from the house measured down the slope. What is the angle of inclination of the hill at this place?

**51. Checking solutions.** Numerical computations should be checked for accuracy. The solution of a triangle may be checked by substitution in some triangle formula not used in the solution.

For example, in the solution on page 91, we found $a = 96.28$, $c = 174.14$, $C = 72° 51'$. The formula,

$$\frac{c - a}{c + a} = \frac{\tan \frac{1}{2}(C - A)}{\tan \frac{1}{2}(C + A)},$$

contains all three of the parts of the triangle which were to be found. The formula was not used in the solution, hence if upon substitution we found both sides to be equal we could consider the solution to be correct. Substitution gives

$$\frac{77.82}{270.38} = \frac{\tan 20°28.5'}{\tan 52°22.5'}.$$

Using logarithms,

| | | | |
|---|---|---|---|
| log 77.82 | = 1.89104 | log tan 20°28.5′ | = 9.57216 |
| log 270.38 | = 2.43198 | log tan 52°22.5′ | = 0.11306 |
| Difference | = 9.45906. | Difference | = 9.45910. |

Since the logarithms check through to four decimal places, we may consider our solution checked.

However there are some formulas which are especially adapted to checking purposes. In particular it is desirable, though not at all necessary, to use a formula containing all six parts of the triangle. Two formulas often called **Mollweide's equations** are of this kind and are derived as follows:

Since $$\frac{a}{c} = \frac{\sin A}{\sin C} \quad \text{and} \quad \frac{b}{c} = \frac{\sin B}{\sin C},$$

we may write
$$\frac{a}{c} + \frac{b}{c} = \frac{\sin A}{\sin C} + \frac{\sin B}{\sin C}, \tag{24}$$
or
$$\frac{a+b}{c} = \frac{\sin A + \sin B}{\sin C}. \tag{25}$$

But from equations (24) page 69 and (12) page 65 this equation may be written
$$\frac{a+b}{c} = \frac{2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{2 \sin \frac{1}{2}C \cos \frac{1}{2}C}. \tag{26}$$

Since $C = 180 - (A + B)$,
$$\cos \tfrac{1}{2}C = \cos [90 - \tfrac{1}{2}(A+B)] = \sin \tfrac{1}{2}(A+B),$$
and (26) may be written
$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(A-B)}{\sin \frac{1}{2}C}. \tag{27}$$

This is one of Mollweide's equations. The other is derived in a similar way and is
$$\frac{a-b}{c} = \frac{\sin \frac{1}{2}(A-B)}{\cos \frac{1}{2}C}. \tag{28}$$

Checking Example 2, page 93, by the use of (27), we have for the first solution, $a + b = 321.7$, $c = 128.3$, $\frac{1}{2}(A - B) = -9°.41.5'$, $\frac{1}{2}C = 23°9'$, and substitution in (27) gives
$$\frac{321.7}{128.3} = \frac{\cos 9°41.5'}{\sin 23°9'}.$$

Using logarithms,

| | | | |
|---|---|---|---|
| log 321.7 | = 2.50745 | log cos 9°41.5′ | = 9.99376 |
| log 128.3 | = 2.10823 | log sin 23°9′ | = 9.59455 |
| Difference | = 0.39922. | Difference | = 0.39921. |

For the second solution, substitution in (27) gives
$$\frac{182.53}{128.3} = \frac{\cos 55°59.5'}{\sin 23°9'},$$

| | | | |
|---|---|---|---|
| log 182.53 | = 2.26133 | log cos 55°59.5′ | = 9.74766 |
| log 128.3 | = 2.10823 | log sin 23°9′ | = 9.59455 |
| Difference | = 0.15310. | Difference | = 0.15311. |

Both solutions check to the fourth decimal place.

## EXERCISES

**1.** Check the example worked under Case 1 by equation (27).

**2.** Check Example 2, page 93, by equation (28).

**3.** Check the solution of the example on page 96.

**4.** Check the answer to Exercise 3, page 97.

**5.** Write down two forms for each of Mollweide's equations besides those given.

**6.** Prove the formulas

$$b = c \cos A + a \cos C \text{ and } a = b \cos C + c \cos B,$$

using the figures on page 89.

**7.** Write formulas corresponding to the formula in Exercise 6 for $b$ and for $c$, and give the formula as a rule.

**8.** Discuss the use of the formulas of Exercise 6 as checking formulas for the solution of triangles.

## PROBLEMS

**1.** Two observers a mile apart on a horizontal road observe a balloon directly over the road. The angles of elevation of the balloon estimated by the two observers are 60° and 75°. What is the height of the balloon?

**2.** A diagonal of a parallelogram is 18.18 inches long. At one end it makes angles of 31° 27′ and 25° 38′ with the sides of the parallelogram. Find the lengths of the sides of the parallelogram.

**3.** It is desired to measure the inaccessible length $AC$ (Fig. 83). A line $AB$ 300 feet long is measured along the river. The angles $CAB$ and $ABC$ are measured and found to be 65° 16′ and 70° 3′, respectively.

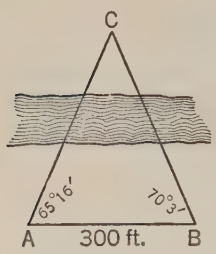


FIG. 83

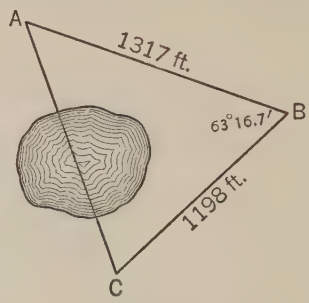


FIG. 84

**4.** $A$ and $C$ (Fig. 84) are points on opposite sides of a pond. The distances $AB$ and $BC$ are 1317 feet and 1198 feet, respectively. The angle at $B$ is measured and found to be 63° 16.7′. Find $AC$.

**5.** The two diagonals of a parallelogram make an angle of $26°26.7'$ with each other. One diagonal is 158.8 feet long and one side 100 feet long. Find the length of the other diagonal.

**6.** Two roads intersect with an angle of $53°$. At 10 o'clock an automobile passes the point of intersection at 30 miles per hour. Five minutes later an automobile passes the intersection on the other road at 50 miles per hour going in the same general direction as the first car. How far apart are the two cars at 10:10 o'clock?

**7.** To find the height of a mountain peak, a horizontal line $AB$, 750 feet long is laid off (Fig. 85). At $A$ the angle of elevation of the top is $54°8'$. Let $C$ represent the top of the mountain and $D$ a point directly under $C$ in the horizontal plane in which $AB$ lies. The angles $DAB$ and $ABD$ are $72°30'$ and $66°45'$ respectively. Find the height of the mountain above $A$.

**8.** If the second automobile in Problem 6 had been going in the opposite direction how far apart would the two automobiles be at 10:10 o'clock?

**9.** Prove that the area of any quadrilateral is equal to one half the product of the diagonals by the sine of the included angle.

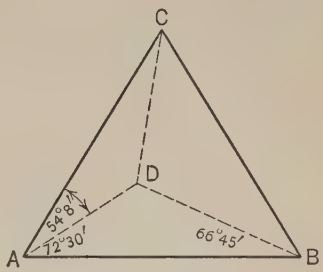

FIG. 85

**10.** Two roads intersect at an angle of $\theta°$. At 10 o'clock an automobile passes the intersection at a rate of $r_1$ miles per hour. After $m$ minutes a second automobile passes the intersection on the other road at a rate of $r_2$ miles per hour. Derive the formulas showing the distance apart at $t$ minutes past 10, ($a$) when both automobiles are going in the same general direction, ($b$) when going in opposite directions.

**11.** A right triangle rests on its hypotenuse, whose length is 132 feet; its plane is inclined $63.2°$ to the horizontal. One of its acute angles is $55°$. Find the height of the vertex of the right angle above the horizontal plane through the hypotenuse.

**12.** Two observers $A$ and $B$ are on different mountain peaks. A third person $C$ fires a gun. $A$ finds the time between the flash and the report to be 1.3 seconds. For $B$ the time is 2.2 seconds. The angle $ACB$ is measured and found to be $119°30'$. If the velocity of sound is 1140 feet per second what is the distance between observers?

**13.** Three circles mutually tangent externally have radii equal to 10, 15 and 20 inches. Find the angles between the lines joining their centers.

**14.** A building 28 feet high is surmounted by a flagpole 36 feet high. In measuring the angles of elevation it is found that the angle of elevation

of the top of the flagpole is just twice the angle of elevation of the top of the building. How far away was the observer?

**15.** A man standing on a cliff 780 feet high can see two rocks in the sea. One rock is two miles away, the other three miles away. With a sextant he finds the angle between the lines of sight to the rocks to be 110°. Find the distance between the rocks.

**16.** Observations from a point 51 feet above a river give 47° 53′ for the angle of depression of the near side of the river and 6° 32′ for the angle of depression of the far side. How wide is the river?

**17.** A house and a garage stand on level ground. From the top of the house the angles of depression of the top and bottom of the garage are 44° and 55°, respectively; the height of the house is 50 feet. What is the height of the garage?

**18.** The sides of a parallelogram are 16 and 32 feet. The angle between them is 100°. What is the area of the parallelogram?

**19.** In an equilateral triangle, with sides equal to 20 feet, draw a line from one vertex to the opposite side. If this line divides the angle into parts having a ratio of 1 to 2, find the length of the segments into which it divides the opposite side.

**20.** An equilateral triangle with sides 12 feet long and with one side on the level ground leans against a perpendicular wall. How high is the vertex of the triangle which touches the wall if the plane of the triangle is inclined 35° to the ground?

**21.** It is desired to prolong the line $AB$ beyond an obstacle. At $B$ an angle of 66° 14′ is made and at $C$ an angle of 94° 30′ (Fig. 86). $BC$ measures 226 feet. Compute $CD$ and the angle at $D$.

FIG. 86

**22.** A tree 15 feet high standing 4 feet from an embankment falls upon the embankment and reaches 13 feet up its face. Find the slope of the embankment.

**23.** Two points are in a horizontal line with the foot of a tower. The distance between the points is $d$. Let $A$ be the angle of elevation of the top of the tower from the more remote point and $B$ the angle from the nearer point. Show that the height of the tower is $\dfrac{d \sin A \sin B}{\sin (B - A)}$.

**24.** Two trains leave the same station at the same time on straight tracks which form an angle of 125°. The trains travel at the average speeds of 35 and 25 miles per hour. How far apart are the trains at the end of the first hour and a half?

**25.** A triangular lot has 97.3 feet frontage on one street, 161.7 feet on another. The rear line of the lot is 183.4 feet long. What is the area of the lot?

**26.** The parallel sides of a trapezoid are 80 feet and 133 feet. The angles at the end of the latter side are 61° 18′ and 46° 27′. Find the lengths of the non-parallel sides.

**27.** Two points $A$ and $B$ are separated by a swamp. To find the distance $AB$ a point $C$ is located which is easily accessible to both $A$ and $B$ (Fig. 87). It is found that $AC = 947$ feet and $BC = 1483$ feet and the angle $ACB$ is 114° 18′. Find $AB$.

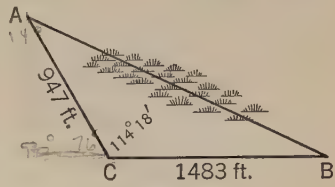

FIG. 87

**28.** A radio tower top $A$ is observed from two points $B$ and $C$, 123 feet apart. The angles $ACB$ and $ABC$ were found to be 29° 17′ and 135°, respectively. Find the distance of the tower from the nearer point.

**29.** In a right triangle $ABC$ where $C$ is the right angle, prove

$$\tan \tfrac{1}{2}A = \sqrt{\frac{c-b}{c+b}}.$$

**30.** In the triangle $ABC$, the angles $A$ and $B$ are 46° 7′ and 56° 19′, respectively. The side $c$ is 214 feet. Find the area of the triangle.

**31.** The sides of a parallelogram are 14.3 and 20.5 feet long. The angle between the shorter diagonal and the shorter side is 100°. Find the length of the longer diagonal.

**32.** Show that the area of a regular polygon of $n$ sides is $\frac{na^2}{4} \cot \frac{180°}{n}$ where $a$ is the length of one side.

**33.** What would have been the height of the balloon in Problem 1 if it had been found that the road sloped 2 feet per 100 horizontal feet, the observer who measured the angle 60° being higher than the other observer? The distance of one mile between observers is measured along the road.

**34.** A triangular lot has a frontage of 166 feet. The other sides are 88 and 156 feet. The lot is to be divided into two triangular lots of equal area and equal frontage. What are the dimensions of the two lots?

**35.** The area of a parallelogram is 1000 square feet. The sides are 50 and 30 feet. What are the angles?

**36.** A flagpole stands on a hill. At some distance down the hill is a level place on which a horizontal line $AB$ 100 feet long can be laid out. The line is so directed that its continuation would pass under the pole. From

$A$ the angles of elevation of the base and top of the flagpole are $13°3.1'$ and $40°58.3'$, respectively. From $B$ the angles of elevation of base and top are $7°15.6'$ and $23°43.7'$, respectively. What is the height of the flagpole?

**37.** The area of a triangle whose angles are 45°, 65° and 70° is 180 square feet. What is the length of the longest side?

**38.** Three circles of radii 40, 20, 10 are tangent to each other, the two smaller circles being within the largest (Fig. 88). What is the area of the smaller curvilinear triangle $ABC$ formed?

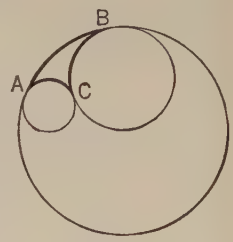

FIG. 88

**39.** Two sides and the included angle of a triangle are measured and found to be 26.3 inches, 47.9 inches and $9°42'$, respectively. What error would be made in computing the area if an error of one minute was made in reading the angle?

**40.** In Problem 36 let the length of the line $AB$ be $a$ feet. Let the angles of elevation of the base and top of the pole from $A$ be $\alpha$ and $\beta$, respectively, and the corresponding angles from $B$ be $\alpha'$ and $\beta'$. Let $h$ be the height of the flagpole. Show that,

$$h = a\left(\frac{\sin\beta\sin\beta'}{\sin(\beta-\beta')} - \frac{\sin\alpha\sin\alpha'}{\sin(\alpha-\alpha')}\right).$$

**41.** Three circles of radii 40 ft., 20 ft. and 10 ft. are tangent externally (Fig. 89). What is the area of the curvilinear triangle $ABC$?

**42.** Obtain a formula for the area of a parallelogram when two adjacent sides and the included angle are given.

**43.** An observer is so situated that he can see the reflection of the top of a tree in a pond. The angle of elevation of the top of the tree is $30°11.3'$. The angle of depression of its image in the pond is $51°57.2'$. Find the height of the tree top above the surface of the pond if the observer is 24.6 feet above the pond.

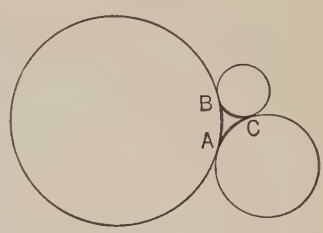

FIG. 89

**44.** The angle of elevation of the top of a tree standing in a level field is measured from three points in the field. The angles of elevation are all found to be equal. Find an expression for the height of the tree in terms of the angle of elevation and a side and angle opposite of the triangle formed by the three points of observation.

**45.** It is desired to measure the distance between two inaccessible rocks $A$ and $B$. A line $CD$ (Fig. 90) 1000 feet long is laid out on the shore and the following angles measured: $ACB = 17° 20.1'$, $ADB = 40° 52.8'$, $DCA = 43° 44.7'$, $CDA = 45° 31'$. Find the distance between the rocks.

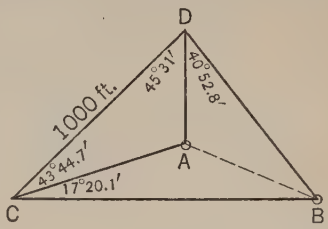

FIG. 90

**46.** A wall stands at the foot of a slope. From a point some distance up the slope the angle of elevation of the top of the wall and the angle of depression of the base are measured by an instrument whose eyepiece is 4.6 feet above the ground. The angle of elevation of the top is $5° 2.1'$ and the angle of depression of the base is $28° 37.3'$. The slope rises 4.3 inches per horizontal foot. What is the height of the wall?

## CHAPTER VI

## COMPLEX NUMBERS. HYPERBOLIC FUNCTIONS.

**52. Real Numbers.** In the study of algebra we found two classes of numbers, the so-called real and imaginary numbers. The real numbers include zero, the positive and the negative integers, fractions, and the irrational numbers like $\sqrt{2}$, $\pi$, etc. In this course in trigonometry, we have used thus far only the real numbers.

If we construct a scale extending in both directions from an arbitrary point 0, Fig. 91, any real number may be represented

Fig. 91

by the segment of the line extending from 0 to a point of the line. For example the number $+3$ is represented by the line segment from 0 to the point marked 3 on the scale. The number $-3$ is represented by the segment three units in length drawn in the opposite direction from 0. The fundamental operations of arithmetic, that is addition, subtraction, multiplication and division can be performed graphically on the scale of Fig. 91. For example, to add 3 to 2 we place the two segments representing 3 and 2 end to end on the scale as shown in Fig. 92.

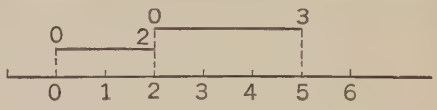


Fig. 92

To multiply 2 by 3, draw through 0 a line making any angle with the horizontal scale, Fig. 93. On this line construct a scale to the same

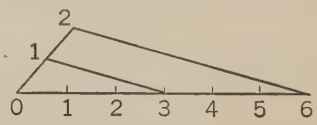


Fig. 93

unit. Connect 1 on this scale with 3 on the original scale by a straight line. The line parallel to this line through the point 2 on the new scale passes through the point $6 = 2 \cdot 3$ on the original scale.

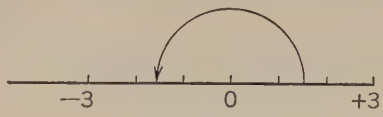


FIG. 94

To multiply graphically any real number by $-1$ we simply rotate the segment representing the number through an angle of 180° (Fig. 94).

## EXERCISES

**1.** Represent on a scale the following numbers:

$$2, \quad -4, \quad 3.5, \quad \pi, \quad -\pi, \quad -1.3, \quad \tfrac{2}{3}, \quad -\tfrac{3}{4}.$$

**2.** Remembering that the length of the diagonal of a square of side 1 is $\sqrt{2}$, construct graphically a segment on the scale representing $\sqrt{2}$.

**3.** Perform the following additions on a scale:

$$1+3, \quad 1+1+2, \quad 1+2+3, \quad -2+3+2+1, \quad -3+1.$$

**4.** How may subtraction be performed on the scale?

**5.** Perform the following additions and subtractions on the scale:

$$3-2, \quad 3-2+1, \quad 2+3-5+1, \quad -2+3+3-4,$$
$$4-(-2)+1, \quad 6-1-2-(-4).$$

**6.** Perform the following multiplications graphically:

$$2 \cdot 4, \quad 3 \cdot 3, \quad 2 \cdot (-3), \quad (-2) \cdot (-3).$$

**7.** Perform the multiplication $2 \cdot 3 \cdot 4$ graphically.

**8.** By a diagram show how to obtain graphically the product $x \cdot y$.

**9.** Divide 6 by 3 graphically. (HINT: Mark the points 0, 1 and 3 on a horizontal scale. Through 0 draw a line making any angle with the

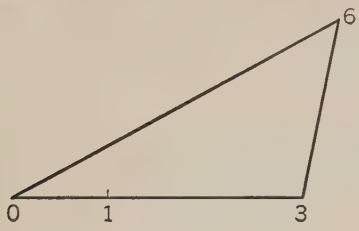


FIG. 95

horizontal scale. On this line mark the point 6. Connect 3 and 6 by a line.) (Fig. 95.)

**10.** Perform the following divisions graphically:

$$\frac{8}{2}, \frac{7}{3}, \frac{9}{4}, \frac{6}{5}, \frac{2}{3}.$$

**11.** By a diagram show how to obtain graphically the quotient $\frac{x}{y}$.

**12.** Perform the following divisions graphically:

$$\frac{-8}{2}, \frac{8}{-2}, \frac{-6}{-3}.$$

**53. Imaginary numbers.** Imaginary numbers are numbers of the form $\sqrt{-a}$, where $a$ is a positive real number. Any imaginary number may be written as the product of a real number and $\sqrt{-1}$. For example, we have

$$\sqrt{-9} = 3 \cdot \sqrt{-1},$$
$$\sqrt{-7} = \sqrt{7} \cdot \sqrt{-1}.$$

It is customary to use the symbol $i$ for $\sqrt{-1}$, hence we may say that any imaginary number may be put in the form $x \cdot i$, where $x$ is a real number.

Since $i^2 = -1$, the operation of multiplying a real number by $-1$ is that of multiplying by $i^2$ or $i \cdot i$. That is, the operation of multiplication by $i$ performed twice is equivalent to multiplication by $-1$. We may then consider the operation of multiplying a real number by $i$ as that operation which performed twice is equivalent to a rotation of 180°; in other words, multiplication by $i$ may be interpreted geometrically as a rotation through 90°. The number $xi$ is then represented by a segment of a vertical line through $O$, $x$ units in length. (Fig. 96.)

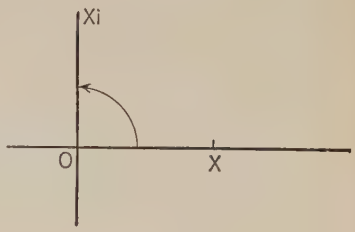

FIG. 96

If the real numbers are represented on a horizontal scale, the imaginary numbers may be represented on a vertical scale through $O$. This vertical line is often called the axis of imaginaries.

## EXERCISES

1. Perform the following operations on the vertical scale:
$$2i + i, \quad 2i - i, \quad 3i + 2i, \quad 3i - 2i, \quad 2 \cdot 3i, \quad (-2) \cdot 3i.$$
2. Find $i^3$ graphically. (HINT: $i^3 = i \cdot i \cdot i$.)
3. Find $i^4$ graphically.
4. Make a table giving the first ten powers of $i$.
5. Perform the following operations graphically:
$$2 \cdot i \cdot 3 \cdot i, \quad 2i \cdot 3i^2, \quad i \cdot i \cdot 2 \cdot 4 \cdot i \cdot i.$$
6. Show that multiplication by $\sqrt[3]{-1}$ may be considered as a rotation through 60°.
7. Show that $\frac{1}{i} = -i$.
8. Give a graphical representation of division by $i$.
9. Represent the following numbers graphically:
$$\frac{2}{i}, \frac{2}{i^2}, \frac{2}{i^3}, i + \frac{1}{i}.$$
10. Represent $\sqrt{i}$ graphically.

**54. Complex numbers.** Numbers of the form $a + bi$, where $a$ and $b$ are real are called **complex numbers.** If $b = 0$, the number is real, and when $a = 0$ it is imaginary, hence, complex numbers include both the real numbers and the imaginary

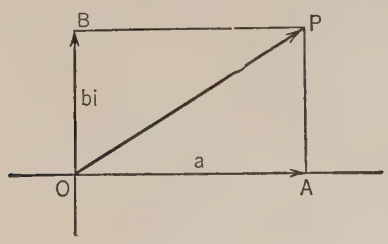


FIG. 97

numbers as special classes. Any complex number $a + bi$ may be represented by the directed segment of a line in the plane as follows: Let two scales, one horizontal, one vertical, intersect at $O$ (Fig. 97). On the horizontal scale represent $a$, the real

part of the complex number, by the segment $OA$, and on the vertical scale represent $bi$, the imaginary part by $OB$. Through $A$ and $B$, draw lines parallel to $OB$ and $OA$, respectively, intersecting in $P$. The directed segment $OP$ will then represent the complex number $a + bi$. Such a directed segment representing both a length and a direction is called a **vector** and we shall use this term hereafter.

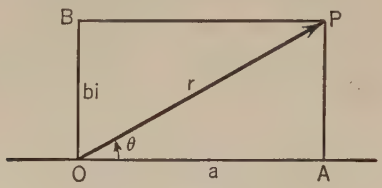


FIG. 98

There is another representation of the complex number called the **polar form.** Let $r$ represent the length of the vector $OP$, and $\theta$ the angle $AOP$ (Fig. 98). We have then

$$\begin{aligned} a &= r\cos\theta \\ b &= r\sin\theta \end{aligned} \tag{1}$$

and the complex number $a + bi$ may be written

$$a + bi = r(\cos\theta + i\sin\theta). \tag{2}$$

The angle $\theta$ is often called the **argument** of the complex number. The length $r$ is called the **absolute value** (sometimes the modulus) of the complex number.

If we think of all vectors as having one end at the intersection of the real and the imaginary axes, we may say that the complex number $a + bi$ is represented by the point $P$ which is determined by the numbers $a$ and $b$ as in Fig. 98.

**55. Equal complex numbers.** We shall define two complex numbers as equal, if and only if they are represented by the same point. From this it follows that the two complex numbers $a + bi$ and $x + yi$ are equal if and only if $a = x$ and $b = y$. Hence, when any two expressions containing real and imagi-

nary parts are equal, we may equate the real parts of one to the real parts of the other, and the imaginary parts of one to the imaginary parts of the other. From this it follows as a special case that the only way in which it is possible for a complex number $a + bi$ to be equal to zero is when $a = 0$ and $b = 0$.

**56. Conjugate complex numbers.** Complex numbers which differ only in the sign of the imaginary part are called **conjugate complex numbers.** That is, the complex numbers $a + bi$ and $a - bi$ are conjugate to each other.

## EXERCISES

Represent the following numbers graphically:

**1.** $3 + 2i,\ 1 - i,\ -2 + 4i,\ -1 + i,\ 0 - i.$

**2.** $\frac{1}{2} + 3i,\ 3 + \frac{1}{2}i,\ 3 - \frac{1}{2}i,\ -2 - 3i,\ 6i.$

**3.** $2(\cos\ 45° + i \sin 45°)$, $3(\cos\ 60° + i \sin 60°)$, $5(\cos 120° + i \sin 120°)$, $(\cos 225° + i \sin 225°)$, $\frac{1}{2}(\cos 300° + i \sin 300°)$.

**4.** $7(\cos\ 90° + i \sin 90°)$, $2(\cos 0° + i \sin 0°)$, $\frac{1}{2}(\cos\ 30° + i \sin 30°)$, $\frac{3}{4}(\cos 270° + i \sin 270°)$, $(\cos 135° + i \sin 135°)$.

Construct the angle and the absolute value (the modulus) of the following complex numbers:

**5.** $1 - 2i.$

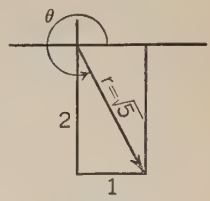

FIG. 99

SOLUTION: The absolute value $r$ is given by the hypotenuse of the right triangle with 1 and 2 for sides, or

$$r = \sqrt{1 + 2^2} = \sqrt{5}.$$

To find the angle $\theta$, we have

$$\tan\ \theta = \frac{-2}{1}, \qquad \sin\ \theta = \frac{-2}{\sqrt{5}}$$

Since $\sin\ \theta$ and $\tan\ \theta$ are both negative, $\theta$ lies in the fourth quadrant, see Fig. 99.

**6.** $1 + 2i.$

**7.** $-1 + 2i.$

**8.** $2 + 3i.$

**9.** $3 + 2i.$

**10.** $i.$

**11.** $1 - i.$

**12.** $1.$

**13.** $-2.$

**14.** $-2 - i.$

**15.** $3 - 4i.$

**16.** $-3 - 4i.$

Write the following complex numbers in the form $a + bi$:

**17.** $2(\cos 60° + i \sin 60°)$.
**18.** $\cos 30° + i \sin 30°$.
**19.** $\frac{1}{2}(\cos 45° + i \sin 45°)$.
**20.** $3(\cos 120° + i \sin 120°)$.
**21.** $4(\cos 90° + i \sin 90°)$.
**22.** $7(\cos 0° + i \sin 0°)$.
**23.** $5(\cos 270° + i \sin 270°)$.
**24.** $3(\cos 300° + i \sin 300°)$.

What two equations arise from each of the following equations?

**25.** $(x - y) + (x + y)i = 2 + 3i$.

SOLUTION: Equating real and imaginary parts, we have

$$x - y = 2, \qquad x + y = 3.$$

**26.** $2x + 3y + (x - 2y)i = x + iy$.
**27.** $x - 2y - (2x + 3y)i = 2 + i$.
**28.** $(x^2 + y^2) + (x + y + 1)i = 4$.
**29.** $(x + 3y - 2) + (x - 3y + 2)i = 1 + i$.
**30.** $x + 2i + xi + y + 4 - i = 0$.

**57. Addition and subtraction of complex numbers.** Assuming that the laws of algebra hold for complex numbers, we have

$$(a + bi) + (x + yi) = (a + x) + (b + y)i, \tag{3}$$

that is, the sum of two complex numbers is a complex number whose real part is the sum of the real parts and whose imaginary part is the sum of the imaginary parts. Similarly, the difference of two complex numbers is a complex number whose real part is the difference of the real parts and whose imaginary part is the difference of the imaginary parts. In symbols,

$$(a + bi) - (x + yi) = (a - x) + (b - y)i. \tag{4}$$

Complex numbers may be easily added or subtracted graphically. Let the complex numbers $a + bi$ and $x + yi$ be repre-

sented as vectors $OA$ and $OB$ (Fig. 100). Complete the parallelogram $OBPA$. The diagonal of the parallelogram $OP$ is

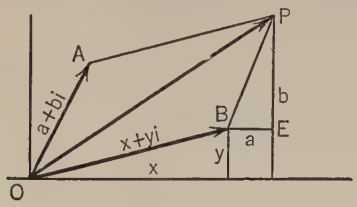


FIG. 100

the vector which represents the complex number

$$(x + a) + (y + b)i,$$

which is the sum of the two complex numbers represented by the vectors $OA$ and $OB$.

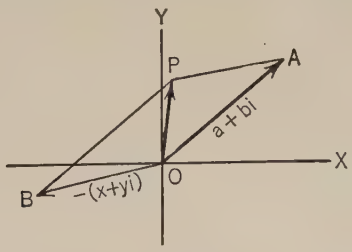


FIG. 101

To subtract $x + yi$ from $a + bi$, we simply add $-(x + yi)$ to $a + bi$ as shown in Fig. 101.

## EXERCISES

Perform the following additions and subtractions both algebraically and graphically:

**1.** $(2 + 3i) + (1 + 2i)$.

**2.** $(2 + 3i) + (2 + 3i)$.

**3.** $(4 + 3i) - (1 + 2i)$.

**4.** $(1 + i) - (2 + i)$.

**5.** $i + (3 + 2i)$.

**6.** $(3 + 4i) - (1 + 4i)$.

**7.** $2 - (1 - 3i)$.

**8.** $(1 + i) + (4 - 6i) + (3 + 2i)$.

**9.** $\left(\frac{1}{2} + \frac{3i}{2}\right) + \left(\frac{3}{2} - i\right) + \left(1 + \frac{5i}{2}\right)$.

**58. Multiplication of complex numbers.** Assuming the laws of algebra to hold for complex numbers, we have

$$\begin{aligned}(a + bi)(x + yi) &= ax + ayi + bxi + byi^2 \\ &= (ax - by) + i(ay + bx). \qquad (5)\end{aligned}$$

That is, the product of two complex numbers is a complex number. The geometric significance is easier seen when the complex numbers are put into the polar form.

Let $\qquad a + bi = r_1(\cos\theta_1 + i\sin\theta_1)$

and $\qquad x + yi = r_2(\cos\theta_2 + i\sin\theta_2).$

Multiplying, we have

$$\begin{aligned}(a + bi)(x + yi) &= r_1 r_2 [\cos\theta_1\cos\theta_2 + i(\sin\theta_1\cos\theta_2 \\ &\qquad + \cos\theta_1\sin\theta_2) + i^2\sin\theta_1\sin\theta_2] \\ &= r_1 r_2 [\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 \\ &\qquad + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)] \\ &= r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]. \qquad (6)\end{aligned}$$

This is a complex number whose absolute value is the product of the absolute values of the given numbers, and whose angle is the sum of their angles.

The graphical representation of the product of two complex

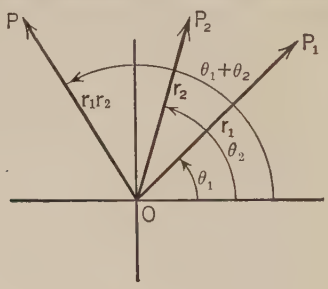


FIG. 102

numbers is shown in Fig. 102 where $OP_1$ and $OP_2$ represent the given numbers. The vector $OP$ represents their product. The length of $OP$ is equal to the product of the absolute values or $r_1 r_2$ and the angle is equal to the sum of $\theta_1$ and $\theta_2$.

## EXERCISES

Perform the following multiplications both algebraically and graphically:

**1.** $(1 + i)(2 + 2\sqrt{3}i)$.

SOLUTION:

$$(1 + i)(2 + 2\sqrt{3}i)$$
$$= 2 + 2i + 2\sqrt{3}i + 2\sqrt{3}i^2 = (2 - 2\sqrt{3}) + (2 + 2\sqrt{3})i.$$

In the polar form these numbers are

$$1 + i = \sqrt{2}(\cos 45° + i \sin 45°)$$
$$2 + 2\sqrt{3}\, i = 4(\cos 60° + i \sin 60°).$$

The absolute value of the product is then $4\sqrt{2}$ and the angle is

$$(45° + 60°) = 105°.$$

In Fig. 103, let $OP_1$ and $OP_2$ represent the two numbers. Draw a line through $O$ making an angle of 105° with the line $OX$. Measure off on this line a distance $OP = 4\sqrt{2}$. The vector $OP$ then represents the product of $(1 + i)$ and $(2 + 2\sqrt{3}i)$.

**2.** $(1 + i)(\sqrt{3} + i)$.

**3.** $(2\sqrt{3} + 2i)(\sqrt{3} + i)$.

**4.** $(1 + i)^2$.

**5.** $i(1 + i)$.

**6.** $(2 + 2i)(-\sqrt{3} + i)$.

**7.** $(1 + i)(-\sqrt{3} - 2i)$.

**8.** $(1 + i)(2 + 2i)(3 + 3i)$.

**9.** $(1 + i)^3$.

**10.** $(3 + \sqrt{3}i)(3 - \sqrt{3}i)$.

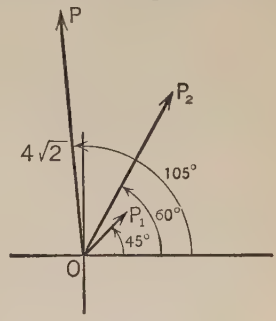


FIG. 103

**59. Division of complex numbers.** The quotient of two complex numbers is a complex number. Let $a + bi$ and $x + yi$ be the numbers. Then

$$\frac{a + bi}{x + yi} = \frac{a + bi}{x + yi} \cdot \frac{x - yi}{x - yi} = \frac{ax + bxi - ayi - byi^2}{x^2 + y^2}$$
$$= \frac{(ax + by) + (bx - ay)i}{x^2 + y^2}$$
$$= \frac{ax + by}{x^2 + y^2} + \frac{bx - ay}{x^2 + y^2} i. \qquad (7)$$

To see the graphical meaning of the division of complex numbers, we put the numbers in the polar form:

$$a + bi = r_1(\cos\theta_1 + i\sin\theta_1)$$
$$x + yi = r_2(\cos\theta_2 + i\sin\theta_2)$$

$$\begin{aligned}\frac{a+bi}{x+yi} &= \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}\\ &= \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}\cdot\frac{r_2(\cos\theta_2 - i\sin\theta_2)}{r_2(\cos\theta_2 - i\sin\theta_2)}\\ &= \frac{r_1r_2[\cos\theta_1\cos\theta_2 + i\sin\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 - i^2\sin\theta_1\sin\theta_2]}{r_2^2(\cos^2\theta_2 + \sin^2\theta_2)}\\ &= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]. \qquad (8)\end{aligned}$$

From this result we see that the absolute value of the quotient of two complex numbers is the quotient of their absolute values and the angle is the difference of their angles.

## EXERCISES

Find the quotients of the following pairs of complex numbers both algebraically and graphically:

**1.** $\dfrac{-2\sqrt{3}+2i}{1+i}$.

SOLUTION:

$$\frac{-2\sqrt{3}+2i}{1+i} = \frac{-2\sqrt{3}+2i}{1+i}\cdot\frac{1-i}{1-i} = \frac{-2\sqrt{3}+2i+2\sqrt{3}i-2i^2}{1-i^2}$$

$$= \frac{(-2\sqrt{3}+2)+(2+2\sqrt{3})i}{2} = (1-\sqrt{3})+(1+\sqrt{3})i.$$

In the polar form, we have

$$-2\sqrt{3}+2i = 4(\cos 150^\circ + i\sin 150^\circ),$$
$$1+i = \sqrt{2}(\cos 45^\circ + i\sin 45^\circ).$$

The quotient of the absolute values is $\dfrac{4}{\sqrt{2}} = 2\sqrt{2}$.

The difference of the angles is $150^\circ - 45^\circ = 105^\circ$.

The polar form of the quotient of the two complex numbers is then

$$2\sqrt{2}(\cos 105^\circ + i\sin 105^\circ).$$

In Fig. 104, $OP_1$ represents $-2\sqrt{3} + 2i$ and $OP_2$ represents $1 + i$. $OP$ represents the quotient.

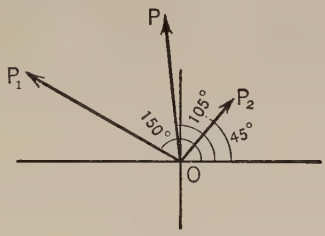


FIG. 104

2. $\dfrac{2 + 2\sqrt{3}i}{1 + i}$.

3. $\dfrac{1 + i}{2 + 2\sqrt{3}i}$.

4. $\dfrac{i}{1 + i}$.

5. $\dfrac{2 + 2i}{3 + \sqrt{3}i}$.

6. $\dfrac{2 + 2i}{2 - 2i}$.

7. $\dfrac{3}{3 + \sqrt{3}i}$.

8. $\dfrac{1 - i}{(1 + i)^2}$.

9. $\dfrac{1}{\sqrt{3} + i}$.

10. $\dfrac{1 + i}{i}$.

**60. De Moivre's theorem.** As a special case under the multiplication of complex numbers, we have for two equal numbers

$$\begin{aligned}&[r(\cos\theta + i\sin\theta)][r(\cos\theta + i\sin\theta)]\\ &\quad = [r(\cos\theta + i\sin\theta)]^2 = r^2(\cos\theta + i\sin\theta)^2\\ &\quad = r^2(\cos 2\theta + i\sin 2\theta).\end{aligned}$$

Multiplying both sides of this identity by $r(\cos\theta + i\sin\theta)$ we have

$$[r(\cos\theta + i\sin\theta)]^3 = r^3(\cos 3\theta + i\sin 3\theta).$$

If $n$ is a positive integer, it easily follows that

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta). \qquad (9)$$

That is, the $n$th power of a complex number is a complex number whose absolute value is the $n$th power of the absolute value of

the given number and whose angle is $n$ times the angle of the given number.

This important theorem is known as **De Moivre's theorem.** It can be proved that the theorem is true if $n$ is any rational fraction positive or negative. For example to prove the theorem if $n$ is a fraction of the form $\frac{1}{p}$ where $p$ is a positive integer, that is to extract the $p$th root of a complex number, we proceed as follows:

$$[r(\cos\theta + i\sin\theta)]^{\frac{1}{p}} = r^{\frac{1}{p}}(\cos\theta + i\sin\theta)^{\frac{1}{p}}$$
$$= r^{\frac{1}{p}}(\cos p\varphi + i\sin p\varphi)^{\frac{1}{p}}$$

if we let $\theta = p\varphi$. Since $p$ is a positive integer we may write from equation (9)

$$\cos p\varphi + i\sin p\varphi = (\cos\varphi + i\sin\varphi)^p.$$

Hence

$$[r(\cos\theta + i\sin\theta)]^{\frac{1}{p}} = r^{\frac{1}{p}}[(\cos\varphi + i\sin\varphi)^p]^{\frac{1}{p}}$$
$$= r^{\frac{1}{p}}(\cos\varphi + i\sin\varphi)$$
$$= r^{\frac{1}{p}}\left(\cos\frac{\theta}{p} + i\sin\frac{\theta}{p}\right). \tag{10}$$

## EXERCISES

Find the indicated powers of the following numbers by means of De Moivre's theorem:

**1.** $(2 + 2\sqrt{3}i)^5$.

SOLUTION: In the polar form $2 + 2\sqrt{3}i$ becomes $4(\cos 60° + i\sin 60°)$. Using De Moivre's theorem, we have

$$(2 + 2\sqrt{3}i)^5 = [4(\cos 60° + i\sin 60°)]^5 = 1024(\cos 300° + i\sin 300°)$$
$$= 1024(\tfrac{1}{2} - \tfrac{1}{2}\sqrt{3}i) = 512(1 - \sqrt{3}i).$$

**2.** $(1 + i)^5$.

**3.** $(3 + \sqrt{3}i)^3$.

**4.** $\left(\frac{3}{2}\sqrt{3} + \frac{3}{2}i\right)^3$.

**5.** $(-\sqrt{3} + i)^4$

**6.** $(1 + \sqrt{3}i)^6$.

**7.** Prove De Moivre's theorem for $n = -1$.

HINT: Write $(\cos\theta + i\cos\theta)^{-1} = \dfrac{1}{\cos\theta + i\sin\theta}$ and multiply both numerator and denominator by $\cos\theta - i\sin\theta$

**8.** Expand the left-hand member of the identity

$$(\cos\theta + i\sin\theta)^3 = \cos 3\,\theta + i\sin 3\,\theta$$

by the binomial theorem and derive formulas for $\cos 3\,\theta$ and $\sin 3\,\theta$.

**9.** Using the results of Exercise 7, prove De Moivre's theorem holds when $n$ is any negative integer.

**10.** Using De Moivre's theorem derive formulas for $\cos 4\,\theta$ and $\sin 4\,\theta$.

**61. Roots of complex numbers.** It has been shown that the $p$th root of a complex number, $x + yi$, is

$$(x + yi)^{\frac{1}{p}} = [r(\cos\theta + i\sin\theta)]^{\frac{1}{p}} = r^{\frac{1}{p}}\left(\cos\frac{\theta}{p} + i\sin\frac{\theta}{p}\right).$$

From Art. 15, it follows, if $n$ is an integer,

$$\begin{aligned}\cos(\theta + n\,360°) &= \cos\theta,\\ \sin(\theta + n\,360°) &= \sin\theta.\end{aligned} \tag{11}$$

Replacing $\cos\theta$ and $\sin\theta$ by their equivalents from (11), we then have

$$\begin{aligned}(x + yi)^{\frac{1}{p}} &= [r(\cos\theta + i\sin\theta)]^{\frac{1}{p}}\\ &= [r\{\cos(\theta + n\,360°) + i\sin(\theta + n\,360°)\}]^{\frac{1}{p}}\\ &= r^{\frac{1}{p}}\left[\cos\frac{\theta + n\,360°}{p} + i\sin\frac{\theta + n\,360°}{p}\right].\end{aligned} \tag{12}$$

Letting $n = 0$, we have

$$r^{\frac{1}{p}}\left[\cos\frac{\theta}{p} + i\sin\frac{\theta}{p}\right]. \tag{13}$$

Letting $n = 1$, we have

$$r^{\frac{1}{p}}\left[\cos\frac{\theta + 360°}{p} + i\sin\frac{\theta + 360°}{p}\right] \tag{14}$$

and so on for $n = 2, 3, \cdots p - 1$.

For $n = p$, the foregoing relation becomes

$$\begin{aligned}&r^{\frac{1}{p}}\left[\cos\frac{\theta + p\,360°}{p} + \sin\frac{\theta + p\,360°}{p}\right]\\ &\quad = r^{\frac{1}{p}}\left[\cos\left(\frac{\theta}{p} + 360°\right) + i\sin\left(\frac{\theta}{p} + 360°\right)\right]\\ &\quad = r^{\frac{1}{p}}\left(\cos\frac{\theta}{p} + i\sin\frac{\theta}{p}\right)\end{aligned} \tag{15}$$

which is the same as the case for $n = 0$. For $n = p + 1, p + 2$, etc., we have the same results as for $n = 1, 2$, etc. We can then say that any number has $p$ distinct $p$th roots.

## EXERCISES

Find all the indicated roots of the following complex numbers:

**1.** $\sqrt[3]{1 + \sqrt{3}i}$.

SOLUTION: Putting $1 + \sqrt{3}i$ in the polar form, we have

$$1 + \sqrt{3}i = 2(\cos 60° + i \sin 60°).$$

Using De Moivre's theorem,

$$\begin{aligned}\sqrt[3]{1 + \sqrt{3}i} &= \sqrt[3]{2(\cos 60° + i \sin 60°)} = \sqrt[3]{2}(\cos 60° + i \sin 60°)^{\frac{1}{3}} \\ &= \sqrt[3]{2}\,[\cos (60° + n\,360°) + i \sin (60° + n\,360°)]^{\frac{1}{3}} \\ &= \sqrt[3]{2}\left[\cos\left(\frac{60° + n\,360°}{3}\right) + i \sin\left(\frac{60° + n\,360°}{3}\right)\right] \\ &= \sqrt[3]{2}\,[\cos (20° + n\,120°) + i \sin (20° + n\,120°)].\end{aligned}$$

Letting $n = 0$, 1 and 2 we find

$$\sqrt[3]{2}(\cos 20° + i \sin 20°),$$
$$\sqrt[3]{2}(\cos 140° + i \sin 140°),$$
$$\sqrt[3]{2}(\cos 260° + i \sin 260°).$$

Each of these numbers is a cube root of $(1 + \sqrt{3}i)$. In Fig. 105 the vector $OP$ represents the number $1 + \sqrt{3}i$, while $OP_1$, $OP_2$, $OP_3$ represent the three cube roots.

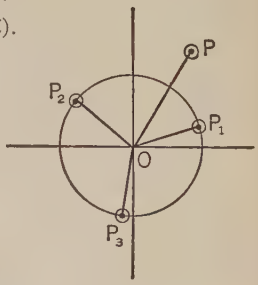


FIG. 105

**2.** $\sqrt[3]{2 + 2i}$.

**3.** $\sqrt[4]{16(\cos 40° + i \sin 40°)}$.

**4.** $\sqrt{1 + i}$. **5.** $\sqrt[3]{-2 + 2i}$.

**6.** $\sqrt[5]{2\sqrt{3} + 2i}$. **7.** $\sqrt[3]{-2 - 2i}$.

**8.** $\sqrt[3]{1}$. HINT: Write in the form $\sqrt[3]{1 + 0i}$.

**9.** $\sqrt[4]{1}$. **10.** $\sqrt[3]{i}$. **11.** $\sqrt[4]{16i}$.

**12.** Represent the fifteen different fifteenth roots of $1 + i$ graphically.

**13.** By actual cubing show that each of the numbers

$$1, \quad -\tfrac{1}{2} + \tfrac{1}{2}\sqrt{3}i, \quad -\tfrac{1}{2} - \tfrac{1}{2}\sqrt{3}i$$

is a cube root of 1.

**14.** Find the three cube roots of $-1$, and by cubing each verify the result.

**62. Series.** A sequence of real numbers connected by plus or minus signs and arranged according to some definite law of succession is called a series. A few examples of series follow:

$$1 + 2 + 3 + 4 + 5 + 6 + 7,$$
$$1 + 2 + 4 + 8 + 16 + 32 + 64,$$
$$1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{6} + \tfrac{1}{7} + \tfrac{1}{8} + \cdots,$$
$$10 - 3\tfrac{1}{3} + 1\tfrac{1}{9} - \tfrac{10}{27} + \tfrac{10}{81} - \tfrac{10}{243} + \cdots.$$

If there is no limit to the number of terms the series is called an **infinite series.** We have had examples of such series in algebra. For example, the series

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} \cdots$$

is an infinite series. If we divide 1 by $1 - x$ we have a never ending process giving an infinite series,

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + x^4 + \cdots.$$

In some infinite series, no matter how many terms we may write down, the sum never increases beyond a certain number. For example, in the above series,

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \cdots,$$

it is easy to see that the sum will never be greater than 2, whatever be the number of terms. If $x = \frac{1}{10}$ in the series arising from the fraction $\dfrac{1}{1 - x}$, then no matter how many terms we may include, the sum will never be greater than $\frac{10}{9}$. On the other hand if $x = 2$ enough terms of the series could be included so that the sum would be greater than any assigned number. Infinite series in which the sum approaches a limiting number as the number of terms increases are called **convergent series.** Other series are called divergent series. In this chapter we are concerned only with series that are convergent.

We have seen that $\dfrac{1}{1 - x}$ can be expressed as an infinite series

of powers of $x$ by simple division. By division, many other algebraic expressions can be thus expressed in series of powers of $x$, for example, $\dfrac{1-x}{1+x+x^2}$, $\dfrac{2+3x}{4+5x}$, $\dfrac{2}{1+2x+3x^2}$, can be so expressed. Expansion by the binomial theorem when the exponent is a fraction or is negative yields infinite series, for example, we have

$$(1+2x)^{\frac{1}{2}} = 1 + x - \tfrac{1}{2}x^2 + \tfrac{1}{2}x^3 - \tfrac{5}{8}x^4 + \cdots$$
$$(2-3x)^{-2} = \tfrac{1}{4} + \tfrac{3}{4}x + \tfrac{27}{16}x^2 + \tfrac{27}{8}x^3 + \tfrac{405}{64}x^4 + \cdots.$$

In more advanced courses in mathematics, it is shown that there is a very large class of functions for which equivalent power series can be written. For example, if $x$ is expressed in radians, we may write

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \tag{16}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \tag{17}$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots, \tag{18}$$

where $3! = 1 \cdot 2 \cdot 3$, $\quad 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ and so on.

It should be noticed that the right-hand members of these identities contain nothing but the four fundamental operations of arithmetic, i.e., addition, subtraction, multiplication and division. This makes computation very easy in many cases. For example to find the sine of 1 radian, we have from the above series when $x$ is replaced by 1,

$$\sin 1 = 1 - \tfrac{1}{6} + \tfrac{1}{120} - \tfrac{1}{5040} + \cdots = .8415.$$

To find $\cos 10°$, we reduce $10°$ to radians

$$\cos 10° = \cos\left(\frac{\pi}{18}\text{ radians}\right) = 1 - \frac{1}{2}\cdot\frac{\pi^2}{18^2} + \frac{1}{4!}\cdot\frac{\pi^4}{18^4} - \frac{1}{6!}\cdot\frac{\pi^6}{18^6}\cdots$$
$$= 0.9848.$$

By means of these series tables of sines and cosines may be constructed to any number of decimal places.

## EXERCISES

**1.** The numbers in the following expression were obtained by throwing two dice and writing down the sum of the faces for each throw:

$$7 + 4 + 4 + 8 + 7 + 6 + 8 + 8 + 4 + 9 + 3$$

Is the expression a series?

**2.** A geometrical progression is a sequence of numbers in which the same quotient is obtained by dividing any term by the preceding term. Write down a series whose terms form a geometrical progression.

**3.** An arithmetical progression is a sequence of numbers each of which differs from the next preceding one by a fixed number. Write down a series whose terms form an arithmetical progression.

**4.** Write down the first few terms of a convergent infinite series whose terms form a geometrical progression.

**5.** Is it possible to find a convergent infinite series whose terms form an arithmetical progression?

**6.** Find the tangent of $\frac{1}{2}$ radian from the series for $\tan x$.

**7.** Find $\cos 5°$ from the series for $\cos x$.

**8.** Find $\sin 10°$ from the series for $\sin x$.

**63. The exponential function.** The expression $e^x$ where $e$ is $2.71828 \cdots$, the base of the Naperian system of logarithms, is called the **exponential function.** There is a very close relation between trigonometric functions and the exponential function which can be easily shown when we express the functions concerned in power series. The series corresponding to $e^x$ is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots. \tag{19}$$

If we have the imaginary number $ix$ for an exponent we have

$$\begin{aligned} e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \cdots \\ &= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} - \frac{ix^5}{5!} + \cdots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right). \end{aligned} \tag{20}$$

The series in the parentheses are the series arising from $\cos x$ and $\sin x$, hence we may write

$$e^{ix} = \cos x + i \sin x. \tag{21}$$

In a similar way, we have

$$e^{-ix} = 1 + (-ix) + \frac{(-ix)^2}{2!} + \frac{(-ix)^3}{3!} + \frac{(-ix)^4}{4!} + \frac{(-ix)^5}{5!} + \cdots$$
$$= \cos x - i \sin x. \tag{22}$$

By adding formulas (21) and (22), we find

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}. \tag{23}$$

Subtracting (22) from (21) we have

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}. \tag{24}$$

Formulas (23) and (24) are often called **Euler's formulas.**

## EXERCISES

**1.** Show that $\tan x = \dfrac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$.

**2.** Test the truth of formulas (23) and (24) by substitution in

$$\sin^2 x + \cos^2 x = 1.$$

**3.** Test the truth of formulas (23) and (24) by substitution in

$$\sin 2x = 2 \sin x \cos x.$$

**4.** Letting $x = 1$ in the series for $e^x$, find the value of $e$ correct to three decimal places.

**5.** Give a graphical representation of the complex number $e^{i\theta}$. What is its absolute value? If the value of $\theta$ is changed, how does it affect the graph?

**6.** Show that any complex number can be represented in the form $re^{i\theta}$.

**7.** Prove $e^{\frac{i\pi}{2}} = i$.

**8.** Prove $\sec x = \dfrac{2e^{ix}}{e^{2ix} + 1}$.

**64. Hyperbolic functions.** In formulas (23) and (24), $\sin x$ and $\cos x$ are represented in terms of $e$ with imaginary exponents. If we replace $x$ by $ix$ in these formulas, we have

$$\cos ix = \frac{e^{i(ix)} + e^{-i(ix)}}{2} = \frac{e^{-x} + e^{x}}{2}, \tag{25}$$

$$\sin ix = \frac{e^{i(ix)} - e^{-i(ix)}}{2i} = -i\frac{e^{-x} - e^{x}}{2},$$

$$= i\frac{e^{x} - e^{-x}}{2}, \tag{26}$$

remembering that $i^2 = -1$, and $\frac{1}{i} = -i$.

We see that the sine of a pure imaginary is a pure imaginary while the cosine of a pure imaginary is a real number.

From (25) and (26) we easily obtain

$$\tan ix = \frac{\sin ix}{\cos ix} = i\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}, \tag{27}$$

which is a pure imaginary number.

The expressions $\frac{e^{x} - e^{-x}}{2}$, $\frac{e^{x} + e^{-x}}{2}$ and $\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$ which appear in formulas (25), (26) and (27) occur so often and are so useful in both pure and applied mathematics that they have received a special name and are called **hyperbolic functions.** The notation is similar to that of the trigonometric functions. The expressions $\frac{e^{x} - e^{-x}}{2}$, $\frac{e^{x} + e^{-x}}{2}$ and $\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$ are called the **hyperbolic sine, hyperbolic cosine** and **hyperbolic tangent,** respectively. Written in symbols we have

$$\sinh x = \frac{e^{x} - e^{-x}}{2} \tag{28}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \tag{29}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}. \tag{30}$$

It has been shown that trigonometric functions are very closely connected with the circle, in fact they are often called circular functions. (See Art. 13.) If we should go deeper into

the study of hyperbolic functions we should find them closely allied with a curve called the equilateral hyperbola (Fig. 106).

The three hyperbolic functions above defined are not the only hyperbolic functions. There are others analogous to the trigonometric functions. For example,

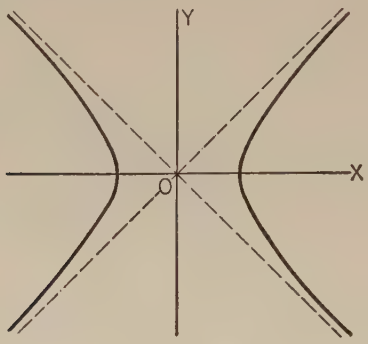


FIG. 106

$$\text{Hyperbolic cotangent} = \coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, \tag{31}$$

$$\text{Hyperbolic secant} = \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}, \tag{32}$$

$$\text{Hyperbolic cosecant} = \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}. \tag{33}$$

Hyperbolic functions are extremely useful in physics, especially in problems connected with electricity.

**65. Relations between hyperbolic functions.** There are many relations between hyperbolic functions which resemble closely relations between trigonometric functions. For example,

$$\cosh^2 x - \sinh^2 x = 1. \tag{34}$$

This is easily proved as follows:
From (29) and (28) we have

$$\cosh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$$

$$\sinh^2 x = \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} - 2 + e^{-2x}}{4},$$

whence

$$\cosh^2 x - \sinh^2 x = \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} = 1.$$

Again, we can prove that

$$\sinh (x + y) = \sinh x \cosh y + \cosh x \sinh y. \tag{35}$$

Substituting $x + y$ for $x$ in (28) we have

$$\sinh (x + y) = \frac{e^{x+y} - e^{-(x+y)}}{2}.$$

The right-hand member of (35) may be written
$\sinh x \cosh y + \cosh x \sinh y$

$$= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$$

$$= \frac{e^x e^y + e^x e^{-y} - e^{-x} e^y - e^{-x} e^{-y}}{4} + \frac{e^x e^y - e^x e^{-y} + e^{-x} e^y - e^{-x} e^{-y}}{4}$$

$$= \frac{2e^x e^y - 2e^{-x} e^{-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2}.$$

To prove,

$$\sinh (-x) = -\sinh x \tag{36}$$

simply replace $x$ in (28) by $(-x)$ and obtain

$$\sinh (-x) = \frac{e^{(-x)} - e^{-(-x)}}{2} = \frac{e^{-x} - e^x}{2}$$

$$= -\frac{e^x - e^{-x}}{2} = -\sinh x.$$

## EXERCISES

Prove the following identities:

1. $\cosh (-x) = \cosh x$.
2. $\tanh (-x) = -\tanh x$.
3. $\sinh 0 = 0$.
4. $\cosh 0 = 1$.
5. $\tanh 0 = 0$.
6. $1 - \tanh^2 x = \operatorname{sech}^2 x$.

**7.** $\coth^2 x - 1 = \operatorname{csch}^2 x.$

**8.** $\sinh (x - y) = \sinh x \cosh y - \cosh x \sinh y.$

**9.** $\cosh (x + y) = \cosh x \cosh y + \sinh x \sinh y.$

**10.** $\cosh (x - y) = \cosh x \cosh y - \sinh x \sinh y.$

**11.** $\tanh (x + y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \cdot \tanh y}.$

**12.** $\tanh (x - y) = \dfrac{\tanh x - \tanh y}{1 - \tanh x \cdot \tanh y}.$

**13.** $\sinh 2x = 2 \sinh x \cosh x.$

**14.** $\cosh 2x = \cosh^2 x + \sinh^2 x = 2 \cosh^2 x - 1 = 1 + 2 \sinh^2 x.$

**66. Graphs of hyperbolic functions.** A three-place table of the three principal hyperbolic functions is given below.

| $x$ | $\sinh x$ | $\cosh x$ | $\tanh x$ |
|---|---|---|---|
| 0.0 | 0.000 | 1.000 | 0.000 |
| .1 | 0.100 | 1.005 | 0.100 |
| .2 | 0.201 | 1.020 | 0.197 |
| .3 | 0.304 | 1.045 | 0.291 |
| .4 | 0.411 | 1.081 | 0.380 |
| .5 | 0.521 | 1.128 | 0.462 |
| .6 | 0.637 | 1.185 | 0.537 |
| .7 | 0.759 | 1.255 | 0.604 |
| .8 | 0.888 | 1.337 | 0.664 |
| .9 | 1.027 | 1.433 | 0.716 |
| 1.0 | 1.175 | 1.543 | 0.762 |
| .1 | 1.336 | 1.669 | 0.800 |
| .2 | 1.509 | 1.811 | 0.834 |
| .3 | 1.698 | 1.971 | 0.862 |
| .4 | 1.904 | 2.151 | 0.885 |
| .5 | 2.129 | 2.352 | 0.905 |
| .6 | 2.376 | 2.577 | 0.922 |
| .7 | 2.646 | 2.828 | 0.935 |
| .8 | 2.942 | 3.107 | 0.947 |
| .9 | 3.268 | 3.418 | 0.956 |
| 2.0 | 3.627 | 3.762 | 0.964 |
| .1 | 4.022 | 4.144 | 0.970 |
| .2 | 4.457 | 4.568 | 0.976 |
| .3 | 4.937 | 5.037 | 0.980 |
| .4 | 5.466 | 5.557 | 0.984 |
| .5 | 6.050 | 6.132 | 0.987 |

| $x$ | $\sinh x$ | $\cosh x$ | $\tanh x$ |
|---|---|---|---|
| 2.6 | 6.695 | 6.769 | 0.989 |
| .7 | 7.406 | 7.473 | 0.991 |
| .8 | 8.192 | 8.253 | 0.993 |
| .9 | 9.060 | 9.115 | 0.994 |
| 3.0 | 10.02 | 10.07 | 0.995 |
| 3.1 | 11.08 | 11.12 | 0.996 |
| .2 | 12.25 | 12.29 | 0.997 |
| .3 | 13.54 | 13.57 | 0.997 |
| .4 | 14.97 | 15.00 | 0.998 |
| .5 | 16.54 | 16.57 | 0.998 |
| .6 | 18.29 | 18.31 | 0.998 |
| .7 | 20.21 | 20.24 | 0.999 |
| .8 | 22.34 | 22.36 | 0.999 |
| .9 | 24.69 | 24.71 | 0.999 |
| 4.0 | 27.29 | 27.31 | 0.999 |
| .1 | 30.16 | 30.18 | 1.000 |
| .2 | 33.34 | 33.35 | 1.000 |
| .3 | 36.84 | 36.86 | 1.000 |
| .4 | 40.72 | 40.73 | |
| .5 | 45.00 | 45.01 | |
| .6 | 49.74 | 49.75 | |
| .7 | 54.97 | 54.98 | |
| .8 | 60.75 | 60.76 | |
| .9 | 67.14 | 67.15 | |
| 5.0 | 74.20 | 74.21 | 1.000 |

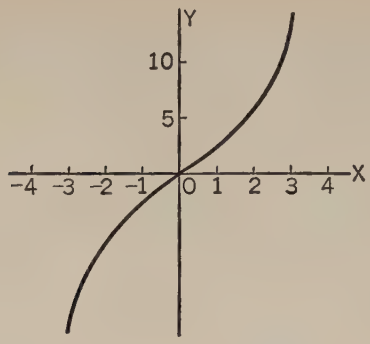


FIG. 107. Graph of sinh $x$

From these tables, the graphs shown in Figs. 107, 108 and 109 were constructed. Comparing these graphs with those of the corresponding trigonometric functions, we note a marked dif-

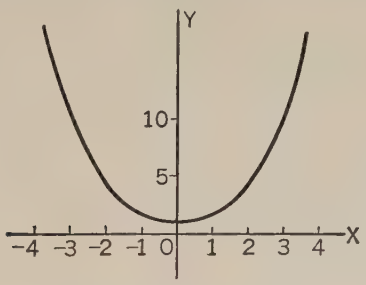


FIG. 108. Graph of cosh $x$

ference between hyperbolic and trigonometric functions. The most notable difference is that the trigonometric functions are periodic while the hyperbolic functions are non-periodic if the

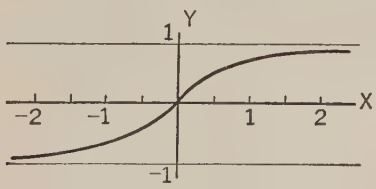


FIG. 109. Graph of tanh $x$

variable $x$ is real. The graph of $\cosh x$ given in Fig. 108 is often called the catenary. It is the curve assumed by a flexible string fastened at two points and allowed to hang freely.

## EXERCISES

**1.** If $x$ is a large positive number, what would you say about the importance of the term $e^{-x}$ in calculating the values of hyperbolic functions?

**2.** For large positive values of $x$, the tables show very little difference between the values of $\sinh x$ and $\cosh x$. Explain why this is to be expected.

**3.** The value of $\tanh x$ lies between $+1$ and $-1$ no matter what the value of $x$. Show that this is true from equation (30).

**4.** Letting $x = 0, 0.5, 1.0, 1.5$, etc., calculate a short table for $\operatorname{sech} x$ from the table on page 134 and construct the graph of $\operatorname{sech} x$.

**5.** Is it possible for a value of $\coth x$ to be between $+1$ and $-1$?

**6.** Why is it unnecessary in a table of hyperbolic functions to tabulate values corresponding to negative values of $x$?

## MISCELLANEOUS EXERCISES

**1.** Perform the operation $\dfrac{2 \cdot 3}{5}$ graphically in one diagram.

**2.** The points 0 and $\sqrt{2}$ are the only points marked on a scale. Give a geometrical method for marking the points 1, 2, 3, etc.

**3.** Give a geometrical interpretation of multiplication by $i^{\frac{1}{4}}$.

**4.** Show that the sum and the product of two conjugate complex numbers are real numbers.

**5.** If $n$ is a positive integer, show that

$$(x + yi)^n + (x - yi)^n$$

is a real number.

**6.** Show that $\dfrac{1 + 2i}{1 + i} + \dfrac{1 - 2i}{1 - i}$ is a real number.

**7.** Show that $\left(\dfrac{x + yi}{x - yi}\right)^2 - \left(\dfrac{x - yi}{x + yi}\right)^2$ is a pure imaginary number.

**8.** Find the angle $\theta$ and the absolute value $r$ of the complex number $-\sqrt{3} + i$.

**9.** Write the complex number $2(\cos 150° + i \sin 150°)$ in the form $a + bi$.

**10.** Show that the absolute value of the complex number $\dfrac{(x + yi)^n}{(x - yi)^n}$ is 1.

**11.** Add graphically the two complex numbers $2(\cos 30° + i \sin 30°)$ and $2(\cos 45° + i \sin 45°)$, and from the diagram find the angle and absolute value of the resulting complex number.

**12.** Perform graphically the following subtraction, and from the diagram find the angle and the absolute value of the resulting complex number:

$$2(\cos 45° + i \sin 45°) - 2(\cos 30° + i \sin 30°).$$

**13.** Perform the multiplication $(1 + i)^2(1 - i)$ both algebraically and graphically.

**14.** Graph the complex number $\frac{3}{2} + \frac{3}{2}\sqrt{3}i$ and its reciprocal.

**15.** Prove De Moivre's theorem when $n = \frac{p}{q}$, where $p$ and $q$ are positive integers.

**16.** If $n$ is a positive integer, prove

$$(\cos \theta - i \sin \theta)^n = \cos n\,\theta - i \sin n\,\theta.$$

**17.** Find the three roots of the equation $x^3 - 27 = 0$.

**18.** Find $\sqrt{e}$ to four places of decimals.

**19.** Assuming the series corresponding to $e^x$ (19), page 129 and assuming that infinite series may be combined according to the laws of algebra, prove

$$\sinh x = x + \frac{x}{3!} + \frac{x}{5!} + \frac{x}{7!} + \cdots,$$

and

$$\cosh x = 1 + \frac{x}{2!} + \frac{x}{4!} + \frac{x}{6!} + \cdots.$$

**20.** The complex number $-\sqrt{3} - i$ can be represented in the form $re^{i\theta}$. Find $r$ and $\theta$.

**21.** Show that

$$\frac{1}{e} = \frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \frac{8}{9!} + \cdots.$$

**22.** Prove

$$\sin i\,\theta = i \sinh \theta,$$

and

$$\cos i\,\theta = \cosh \theta.$$

**23.** Using the series in Exercise 19, check the values of sinh .5 and cosh .5 given in the table on page 134.

**24.** Using the expression for tanh $x$ given by Equation 30, page 131, prove

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}.$$

**25.** Show how a table of values of $e^x$ can be constructed from a table giving values of sinh $x$ and cosh $x$.

**26.** Write the expressions

$$e^{ax} + e^{-ax}, \quad e^{\sqrt{ax}} - e^{-\sqrt{ax}}, \quad \frac{e^{\frac{x}{2}} - e^{-\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}$$

in the hyperbolic function notation.

**27.** The formula

$$E = \frac{E_1}{2}\left[e^{x\sqrt{\frac{r}{r_1}}} + e^{-x\sqrt{\frac{r}{r_1}}}\right] + \frac{rr_1}{2\left(\frac{r}{r_1}\right)^{\frac{1}{3}}}\left[e^{x\left(\frac{r}{r_1}\right)^{\frac{1}{3}}} - e^{-x\left(\frac{r}{r_1}\right)^{\frac{1}{3}}}\right]$$

occurs in certain electrical problems. Rewrite the formula using the notation of hyperbolic functions.

**28.** Calculate the value of $E$ in Exercise 27 when $E_1 = 1$, $x = 2$, $r = 3.2$, $r_1 = .8$.

## CHAPTER VII

## INVERSE TRIGONOMETRIC FUNCTIONS

**67. Inverse functions defined.** One of the first problems in a course in trigonometry is that of finding the value of a function when the angle is given. For example, if $x$ is given as 30°, it is easily found as is done on page 6 that $\sin x = \frac{1}{2}$. However, we often find it necessary to solve the inverse problem, that is, to find the value of the angle when that of the function is given. For example: What is the angle if its sine is equal to $\frac{1}{2}$? There is an indefinite number of solutions to this problem; for, 30°, 150°, 390°, $-$ 210° $\cdots$ are all angles whose sines are equal to $\frac{1}{2}$.

This result may be summed up by considering the equation,

$$x = \sin y. \tag{1}$$

The problem of finding $x$ if $y$ is given, may be called the direct problem, while the inverse problem is that of finding $y$ if $x$ is given. We find this idea of direct and inverse problems connected with most operations in mathematics. Thus, we may consider the finding of the square of a number as the direct problem, and the extraction of the square root as the inverse problem. In symbols, we have $x = y^2$. The problem of finding $x$ if $y$ is given is the direct problem, while the inverse is that of finding $y$ if $x$ is given.

**68. Notation of inverse trigonometric functions.** In equation (1), we consider $x$ as a function of $y$. We may, however, consider $y$ as a function of $x$. That is, we may write

$$y = \text{angle whose sine is } x.$$

Or, since an angle may be measured by the arc of a circle whose center is the vertex of the angle, we may write

$$y = \text{arc whose sine is } x,$$

which is often abbreviated to

$$y = \text{arc sin } x. \tag{2}$$

Another way of writing this equation is

$$y = \sin^{-1} x. \tag{3}$$

The number $(-1)$ is here not to be regarded as an exponent. The whole symbol, $\sin^{-1} x$, is simply a short-hand way of writing "angle whose sine is $x$." If $(-1)$ is to be used as an exponent we shall write $(\sin x)^{-1}$, which means $\frac{1}{\sin x}$. The notations given in (2) and (3) are both in good usage and we shall use both. Arc sin $x$ or $\sin^{-1} x$ are often spoken of as the inverse sine.

In a similar way we may discuss the other inverse trigonometric functions:

$$\begin{array}{lcl} \text{arc cos } x & \text{or} & \cos^{-1} x, \\ \text{arc tan } x & \text{or} & \tan^{-1} x, \\ \text{arc cot } x & \text{or} & \cot^{-1} x, \\ \text{arc sec } x & \text{or} & \sec^{-1} x, \\ \text{arc csc } x & \text{or} & \csc^{-1} x. \end{array}$$

It should be emphasized that the inverse trigonometric functions have more than one value for a given value of $x$, while the direct functions are single valued. In the functions arc sin $x$ and arc cos $x$, the real values of $x$ may not be greater than unity in absolute value.

While the function arc sin $x$ has an indefinite number of values for a given value of $x$ all lying between $-1$ and $+1$ inclusive, the smallest numerical value of the arc sin $x$ for this value of $x$ is the most important and is called its **principal value.** It lies between arc sin $x = -\frac{\pi}{2}$ and arc sin $x = \frac{\pi}{2}$ inclusive. The principal values of arc tan $x$, arc cot $x$, and arc csc $x$

are defined in a similar manner. The principal values of arc cos $x$ and arc sec $x$ lie between 0 and $\pi$ inclusive.

**69. Graphs of the inverse functions.** The two equations

$$x = \sin y,$$

and

$$y = \text{arc} \sin x$$

have the same meaning. We have already graphed the function $y = \sin x$ (page 50). The only difference between this graph and that of $x = \sin y$ is that $x$ and $y$ are interchanged.

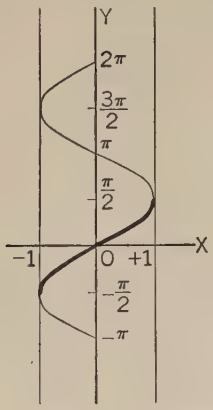


FIG. 110. Graph of $y = \text{arc} \sin x$

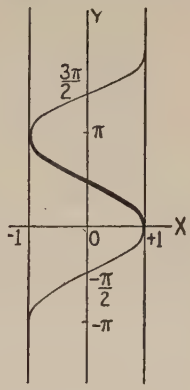


FIG. 111. Graph of $y = \text{arc} \cos x$

In other words, the graph of $x = \sin y$ is a sine curve running along the $Y$-axis (Fig. 110). This curve is also the graph of $y = \text{arc} \sin x$.

From the graph, we clearly see that for $x$ greater than 1 in numerical value we have no $y$ value, and for $x$ less than 1, there is an indefinite number of values of $y$ corresponding to a given value of $x$.

In the same way we may construct the graphs of the other inverse trigonometric functions. In Figs. 111 and 112 the graphs of

$$y = \text{arc} \cos x,$$
$$y = \text{arc} \tan x,$$

are shown. The heavy lines in Figs. 110, 111 and 112 represent the principal values of the functions.

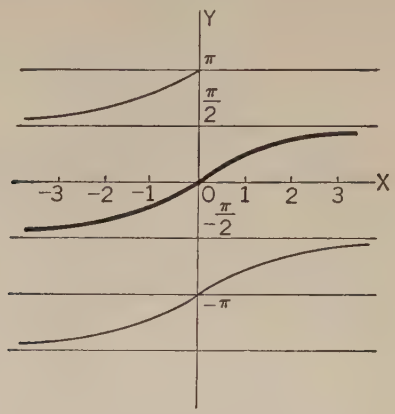


FIG. 112. Graph of $y = \text{arc tan } x$

**70. Identities.** As in the study of direct trigonometric functions, we find many identities connecting the inverse functions. In these identities, it should be remembered that the inverse functions have many values, so that the identities are not true for arbitrary values of all the functions in the equality. The following examples illustrate methods of proving these identities.

EXAMPLE 1. Show that $\text{arc tan } \dfrac{2x}{x^2 - 1} = \text{arc sin } \dfrac{2x}{x^2 + 1}$.

SOLUTION: Construct an angle $\theta$ whose tangent is $\dfrac{2x}{x^2 - 1}$ (Fig. 113). The hypotenuse of this triangle is $1 + x^2$. Hence,

$$\sin \theta = \frac{2x}{x^2 + 1}.$$

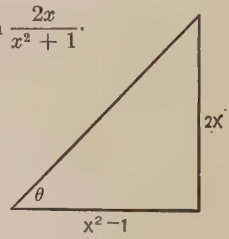


FIG. 113

Or, we have

$$\theta = \text{arc sin } \frac{2x}{x^2 + 1} = \text{arc tan } \frac{2x}{x^2 - 1}.$$

If $x = 2$, write down several values for $\text{arc tan } \dfrac{2x}{x^2 - 1}$ and $\text{arc sin } \dfrac{2x}{x^2 + 1}$ and choose a value for which the identity is true.

In Fig. 113, we see that the identity is true for the acute angle shown in the figure.

EXAMPLE 2. Prove:

$$\text{arc cos } x + \text{arc cos } y = \text{arc cos } (xy - \sqrt{(1 - x^2)(1 - y^2)}).$$

SOLUTION: Let

$$\text{arc cos } x = \theta,$$

then

$$\cos \theta = x, \quad \sin \theta = \sqrt{1 - x^2}.$$

Let

$$\text{arc cos } y = \psi,$$

then

$$\cos \psi = y, \quad \sin \psi = \sqrt{1 - y^2}.$$

We have then

$$\text{arc cos } x + \text{arc cos } y = \theta + \psi,$$

$$\cos (\theta + \psi) = \cos \theta \cos \psi - \sin \theta \sin \psi$$

$$= xy - \sqrt{1 - x^2}\sqrt{1 - y^2}.$$

Hence,

$$\theta + \psi = \text{arc cos } x + \text{arc cos } y = \text{arc cos } (xy - \sqrt{(1 - x^2)(1 - y^2)}).$$

## EXERCISES

Express the following angles in degrees. Give at least four answers to each exercise.

**1.** arc sin 1.

**2.** arc tan $\sqrt{3}$.

**3.** $\cos^{-1} 0$.

**4.** $\cot^{-1} (-1)$.

**5.** $\sec^{-1} 2$.

**6.** arc csc $\sqrt{2}$.

**7.** $\sin^{-1} \dfrac{\sqrt{2}}{2}$.

**8.** $\tan^{-1} \infty$.

**9.** arc cos $\frac{1}{2}$.

**10.** arc cot $\dfrac{\sqrt{3}}{3}$.

From a trigonometric table, express in degrees the following angles:

**11.** arc sin .2924.

**12.** arc cos .9182.

**13.** $\tan^{-1}$ .5243.

**14.** $\cot^{-1}$ 1.9626.

Find the value of the following:

**15.** $\sin \left(\cos^{-1} \dfrac{\sqrt{3}}{2}\right)$.

**16.** tan (arc sin $\frac{1}{2}$).

**17.** cos (arc cos 1).

**18.** cot $(\tan^{-1} (-\sqrt{3}))$.

**19.** sin $(90° - \sin^{-1} \frac{1}{2})$.

**20.** sin $(90° - \text{arc cos } \frac{1}{2})$.

**21.** sec (arc tan 1).

**22.** tan (arc sec $(-2)$).

**23.** sin $(2 \cos^{-1} \frac{1}{2})$.

**24.** cos $(3 \cos^{-1} \frac{1}{2})$.

**25.** $\tan \left(\dfrac{\pi}{2} - \text{arc tan } \dfrac{\sqrt{3}}{3}\right)$.

**26.** $\cot\left(\frac{\pi}{2} - \text{arc}\sin 1\right)$.

**27.** $\sin(\pi - \sin^{-1}\frac{1}{2})$.

**28.** $\cos(\pi - \tan^{-1}(-1))$.

Verify the following equalities and state the conditions under which the equalities hold:

**29.** $\text{arc}\sin\frac{1}{2} + \text{arc}\cos\frac{\sqrt{2}}{2} = 75°$.

**30.** $\tan^{-1} 1 + \cot^{-1} 1 = 90°$.

**31.** $\cos^{-1}\frac{\sqrt{3}}{2} + \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{3}$.

**32.** $\text{arc}\sin 1 - \text{arc}\tan 1 = \frac{\pi}{4}$.

**33.** $\text{arc}\tan\infty - \text{arc}\cos\frac{1}{2} = 30°$.

**34.** $\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{\sqrt{3}}{2} = \tan^{-1}\sqrt{3}$.

**35.** $\text{arc}\sin x + \text{arc}\cos x = 90°$.

**36.** $\text{arc}\tan\frac{x}{\sqrt{1-x^2}} + \text{arc}\cot\frac{x}{\sqrt{1-x^2}} = 90°$.

**37.** $\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$.

**38.** $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right)$.

**39.** $\text{arc}\sin x = -\text{arc}\sin(-x)$.

**40.** $\text{arc}\tan x + \text{arc}\tan\frac{1}{x} = 90°$.

For angles less than 90°, prove the following equations geometrically:

**41.** $\sin\text{arc}\tan x = \frac{x}{\sqrt{1+x^2}}$.

**42.** $\cos\text{arc}\sin x = \sqrt{1-x^2}$.

**43.** $\tan\cot^{-1} x = \frac{1}{x}$.

**44.** $\cot\cos^{-1} x = \frac{x}{\sqrt{1-x^2}}$.

**45.** $\sec\text{arc}\tan\frac{x}{2} = \frac{\sqrt{4+x^2}}{2}$.

**46.** $\sin\text{arc}\cot\frac{x}{3} = \frac{3}{\sqrt{x^2+9}}$.

# CHAPTER VIII

## TRIGONOMETRIC EQUATIONS

**71. Trigonometric equations.** There are two classes of equalities. The trigonometric identities we have considered in Arts. 8, 19 are equalities which are true for any angle or for any consistent value of the unknown. These identities state facts to be proven. For example,

$$\cos^2 x + \sin^2 x = 1$$

can be proved true for all values of the angle $x$, and

$$\text{arc} \sin x + \text{arc} \cos x = 90^\circ$$

can be proved true for all values of $x \gtreqless 1$ if the angles are acute. In this chapter, we shall consider trigonometric equations, or a second class of equalities true only for particular values of an unknown. To solve an equation is to find the particular angle or other unknown which satisfies the equality.

When only a single function of an angle is involved in the equation to be solved, the methods of solving equations given in algebra can be used. The following example involves the solution of a quadratic equation.

EXAMPLE 1. Solve $6 \cos^2 x - 5 \cos x + 1 = 0$ for $x$.

SOLUTION: If we let $a = \cos x$, we may write the given equation as follows:

$$6a^2 - 5a + 1 = 0.$$

Solving this quadratic equation in $a$, we find $a = \frac{1}{2}$ or $\frac{1}{3}$; and hence $\cos x = \frac{1}{2}$ or $\frac{1}{3}$. If $\cos x = \frac{1}{2}$, we have $x = 60^\circ$. If $\cos x = \frac{1}{3}$, we find from the tables $x = 70^\circ\, 31.7'$. Two solutions of the equation are then $60^\circ$, and $70^\circ\, 31.7'$. There are, of course, many other solutions. Any angle whose cosine is $\frac{1}{2}$ or $\frac{1}{3}$ will solve the equation.

If a trigonometric equation contains more than one function, it may be changed to an equation in one function by means of trigonometric identities and then solved as above.

EXAMPLE 2. Find the acute angles satisfying the equation

$$\cos^2 x = \sin^2 x + \sin x.$$

SOLUTION: Replace $\cos^2 x$ by $1 - \sin^2 x$. We then have

$$2 \sin^2 x + \sin x - 1 = 0.$$

This is a quadratic equation in $\sin x$. Its solutions are $\sin x = -1$ or $\frac{1}{2}$. The result $\sin x = -1$ gives no solution, if we limit the problem to acute angles.

From $\sin x = \frac{1}{2}$, we have $x = 30°$.

EXAMPLE 3. Find the acute angle satisfying the equation

$$\sin x = 2 \cos x.$$

SOLUTION: Divide both members of the equation by $\cos x$, thus obtaining

$$\frac{\sin x}{\cos x} = 2,$$

or

$$\tan x = 2,$$

and from the tables we find $x = 63° 26'$.

A trigonometric equation may contain inverse functions.

EXAMPLE 4. Solve $\text{arc tan } x + \text{arc tan } 2x = \text{arc tan } 4x$.

SOLUTION: Let $\text{arc tan } x = \theta$, then $x = \tan \theta$.
Let $\text{arc tan } 2x = \psi$, then $2x = \tan \psi$.

Consequently we have

$$\theta + \psi = \text{arc tan } 4x,$$

and

$$\tan (\theta + \psi) = \frac{\tan \theta + \tan \psi}{1 - \tan \theta \tan \psi} = \frac{x + 2x}{1 - 2x^2} = \frac{3x}{1 - 2x^2}.$$

or

$$\text{arc tan } \frac{3x}{1 - 2x^2} = \text{arc tan } 4x.$$

Hence, we have

$$\frac{3x}{1 - 2x^2} = 4x, \quad \text{or} \quad x(1 - 8x^2) = 0$$

Solving, we obtain

$$x = 0, \quad \text{or} \quad x = \pm \sqrt{\tfrac{1}{8}} = \pm \tfrac{1}{4} \sqrt{2}.$$

## EXERCISES

Find all the angles less than 360° which satisfy the following equations:

1. $2 \tan x = 3.$
2. $\cos (x + 20°) = \frac{1}{2}.$
3. $\sin \left(x + \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}.$
4. $\sec 2x - 2 = 0.$
5. $3 \sin^2 x - 2 \sin x - 1 = 0.$
6. $3 \cos^2 x - 2 \cos x - 1 = 0.$
7. $3 \tan^2 x - 2 \tan x - 1 = 0.$
8. $3 \sec^2 x - 2 \sec x - 1 = 0.$
9. $\sin^2 x - 5 \sin x + 6 = 0.$
10. $6 \cos^2 x - 5 \cos x + 1 = 0.$
11. $\sin x + \cos^2 x = \frac{5}{4}.$
12. $\sin x + \csc x = \frac{5}{2}.$
13. $\tan x + 2 \cot x = 3.$
14. $\cos 2x = 1 - \sin x.$
15. $\tan 2x - 2 \sin x = 0.$
16. $\tan x + \cot x + 2 = 0.$
17. $\tan 2x \tan x = 1.$
18. $2 \cos x \cot 2x = 1.$

Find acute angles $x$ and $y$ which satisfy the following equations:

19. $\sin (x - y) = \frac{1}{2},$
$\cos (x + y) = \frac{1}{2}.$
20. $\tan (2x - y) = 1,$
$\cos (x - 2y) = 1.$
21. Solve $r \cos \theta = 4,$
$r \sin \theta = 3$
for $r$ and $\theta$.
22. Solve $\sin \theta \cos \varphi = \frac{1}{3},$
$\cos \theta \cos \varphi = \frac{1}{4}$
for $\theta$ and $\varphi$.

Solve the following pairs of equations for $x$ and $y$, where $x$ is an acute angle:

23. $y = \cos x,$
$y = \sin x \tan x.$
24. $y = \sin x,$
$y = \sin 2x.$

Solve the following equations for $x$:

25. $2 \tan^{-1} x = 30°.$
26. $\text{arc} \sin (x + \frac{1}{2}) = 50°.$
27. $\text{arc} \tan x + \text{arc} \tan (1 + x) = \text{arc} \tan 1.$
28. $\cos^{-1} x + \cos^{-1} 2x = \cos^{-1} \frac{1}{2}.$
29. $2 \text{ arc} \sin x = \text{arc} \cos \frac{1}{2}\sqrt{3}.$
30. $\tan^{-1} x = 2 \sec^{-1} 2.$

# CHAPTER IX

## RIGHT SPHERICAL TRIANGLES

**72. Spherical trigonometry.** As the name indicates, spherical trigonometry deals with figures on a sphere, in particular with triangles. We shall need certain definitions and theorems from spherical geometry.

A **great circle** is the circle of intersection of the sphere with a plane through the center of the sphere.

The points in which a perpendicular to this plane through the center cuts the sphere are called the **poles** of the great circle.

Two great circles divide the sphere into four parts each of which is called a **lune.**

A spherical triangle is that part of the spherical surface bounded by three arcs of great circles (Fig. 114). The angles and vertices of a spherical triangle are usually denoted by the letters $A$, $B$, $C$ and the sides opposite by $a$, $b$, $c$. Both sides and angles are measured in degrees.

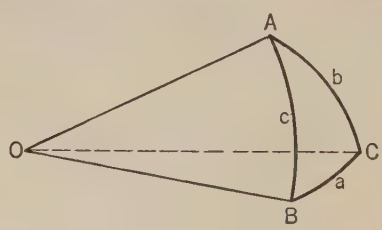


Fig. 114

To every angle of a spherical triangle, there corresponds a dihedral angle whose edge is a line passing through the center of the sphere. Thus, in Fig. 114 the dihedral angle $O-BAC$ corresponds to the angle $A$ of the spherical triangle $ABC$.

The measures of the angles of the spherical triangle are equal to those of the corresponding dihedral angles.

To each spherical triangle, there corresponds a trihedral angle. In Fig. 114 the trihedral angle $O-ABC$ corresponds to $ABC$. The sides $a$, $b$, $c$ of the spherical triangle are measured by the corresponding face angles $BOC$, $AOC$, $AOB$ of the trihedral angle.

Spherical triangles may be constructed with one or more sides or angles greater than 180°, but we shall consider only those with sides and angles each less than 180°.

**73. Polar triangles.** If, with the vertices $ABC$ of a spherical triangle as poles, we construct three great circles, we have other spherical triangles. One of these is $A'B'C'$, (Fig. 115) where $A$ and $A'$, $B$ and $B'$, $C$ and $C'$ lie on the same side of $a$, $b$, $c$ respectively. This spherical triangle is called the **polar triangle** of $ABC$. From a theorem in solid geometry it follows that if $A'B'C'$ is the polar triangle of $ABC$, then $ABC$ is the polar triangle of $A'B'C'$. It is also proved that each angle of a spherical triangle is the supplement of the corresponding side of the polar triangle. Referring to Fig. 115, we have

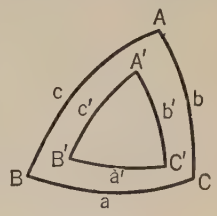

FIG. 115

$$A = 180° - a', \quad B = 180° - b', \quad C = 180° - c',$$
$$a = 180° - A', \quad b = 180° - B', \quad c = 180° - C'.$$

**74. Right Spherical Triangles.** If one of the angles of a spherical triangle is 90°, the triangle is called a **right spherical triangle.** In discussing right spherical triangles, we shall let $C$ mark the right angle. If any two of the parts $a$, $b$, $c$, $A$, $B$ of a right spherical triangle $ABC$ are given, the remaining parts can be found. The process of finding these remaining parts is called **solving the triangle.**

Let $O$ (Fig. (116) be the center of a sphere of radius $OA$, on which the right triangle $ABC$ is drawn. Think of the plane

$OBC$ as vertical; then, since $C$ is a right angle, the plane $OAC$ is horizontal. Through any point $P$ of the radius $OB$ pass a plane $PQR$ perpendicular to $OA$. The lines $PR$ and $QR$ are then perpendicular to $OA$, and the plane $PQR$ is a vertical plane. From this relation, it follows that the plane triangles $OQR$, $OQP$, $PQR$ and $ORP$ are right triangles with the right angles $ORQ$, $OQP$, $PQR$ and $ORP$. Further, from the figure we see that

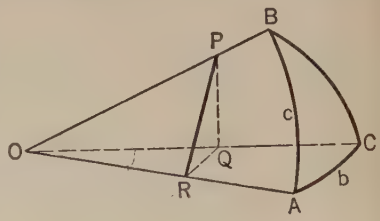

FIG. 116

$$\angle BOC = a,$$
$$\angle AOC = b,$$
$$\angle AOB = c,$$

also $\angle QRP = A$, since $QRP$ measures the dihedral angle between the planes $OAB$ and $OAC$.

We then have

$$\sin A = \sin QRP = \frac{QP}{RP},$$

but

$$QP = OP \sin BOC = OP \sin a,$$

and

$$RP = OP \sin AOB = OP \sin c,$$

whence

$$\sin A = \frac{\sin a}{\sin c}.$$

Similarly, we have

$$\cos A = \frac{RQ}{RP} = \frac{\tan b}{\tan c},$$

$$\tan A = \frac{QP}{RQ} = \frac{\tan a}{\sin b}.$$

From the plane triangle $OPR$, we have

$$\cos c = \frac{OR}{OP},$$

but $OR = OQ \cos b$, and $OP = OQ \sec a$, whence

$$\cos c = \frac{\cos b}{\sec a} = \cos b \cos a.$$

If instead of the plane $PQR$ perpendicular to $OA$, we consider a plane perpendicular to $OB$, we obtain formulas similar to the foregoing in which $B$ replaces $A$, and $b$ and $a$ are interchanged; namely,

$$\sin B = \frac{\sin b}{\sin c}, \qquad \cos B = \frac{\tan a}{\tan c}, \qquad \tan B = \frac{\tan b}{\sin a}.$$

Three other formulas may be obtained by combinations of the above as follows: From the formulas for $\tan A$ and $\tan B$, we have

$$\frac{1}{\tan A}\frac{1}{\tan B} = \frac{\sin b}{\tan a}\frac{\sin a}{\tan b} = \frac{\sin a}{\tan a}\frac{\sin b}{\tan b}$$
$$= \cos a \cos b = \cos c,$$

or

$$\cot A \cot B = \cos c.$$

Also

$$\cos A = \frac{\tan b}{\tan c} = \frac{\sin b}{\cos b}\frac{\cos c}{\sin c} = \frac{\sin b}{\cos b}\frac{\cos a \cos b}{\sin c}$$
$$= \frac{\sin b}{\sin c}\cos a = \sin B \cos a.$$

In a similar way, we obtain

$$\cos B = \sin A \cos b.$$

Collecting the above results, we have ten formulas for the solution of right spherical triangles; namely,

$$\sin a = \sin c \sin A, \tag{1}$$
$$\tan b = \tan c \cos A, \tag{2}$$
$$\tan a = \sin b \tan A, \tag{3}$$
$$\cos c = \cos b \cos a, \tag{4}$$
$$\sin b = \sin c \sin B, \tag{5}$$
$$\tan a = \tan c \cos B, \tag{6}$$
$$\tan b = \sin a \tan B, \tag{7}$$
$$\cos c = \cot A \cot B, \tag{8}$$
$$\cos A = \sin B \cos a, \tag{9}$$
$$\cos B = \cos b \sin A. \tag{10}$$

These formulas have been derived from the spherical triangle in Fig. 116 in which all the parts except the angle $C$ are less than 90°. It can be shown that these formulas hold for parts greater

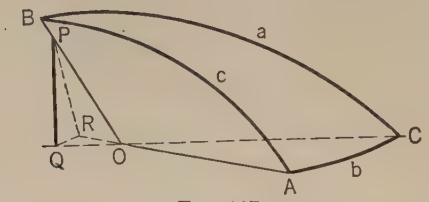


FIG. 117

than 90°. For example, they can be shown to hold for the case in Fig. 117, where $a > 90°$, $c > 90°$.

**75. Napier's Rules.** The ten formulas for the solution of right spherical triangles may be put into a very easily remembered form by means of a rule formulated by Napier, the inventor of logarithms. In the right triangle as we have lettered it, we call $a$, $b$ and the complements of $A$, $c$ and $B$ the **five circular parts.** Instead of lettering the triangle in the usual manner, let it be marked as shown in Fig. 118, where co.$A$, co.$c$, co.$B$, stand for "complement of $A$," "complement of $c$," "complement of $B$," respectively. The right angle $C$ is not marked. In the figure, we have the five circular parts arranged in succession around the triangle. If we designate any one part, we see that it has two adjacent parts and two opposite parts. For example, if we designate $b$, then $a$ and co.$A$ are adjacent parts and co.$B$ and co.$c$ are opposite parts. Napier's rules are:

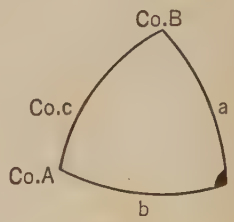


FIG. 118

(1) *The sine of any circular part is equal to the product of the tangents of the adjacent parts.*

(2) *The sine of any circular part is equal to the product of the cosines of the opposite parts.*

Applying these rules to each of the five parts in turn, we obtain ten formulas which reduce to the ten formulas given on page 151. For example, Napier's first rule gives

$$\sin \text{co.} A = \tan b \tan \text{co.} c,$$

or

$$\sin (90° - A) = \tan b \tan (90° - c),$$

or

$$\cos A = \tan b \cot c = \frac{\tan b}{\tan c},$$

or

$$\tan b = \tan c \cos A,$$

which is formula 2, page 151.

## EXERCISES

**1.** What spherical triangle is polar to itself?

**2.** If the side $c$ of a right spherical triangle is less than 90°, show that the sides $a$ and $b$ are either both less than or both greater than 90°.

**3.** If the side $c$ of a right spherical triangle is greater than 90°, show that of the other sides one is less than and the other greater than 90°.

**4.** In a right spherical triangle, show that $A$ and $a$ are both less than or both greater than 90°, the same being true for $B$ and $b$.

**5.** Prove $\tan A = \dfrac{\sin a}{\tan b \cos c}$.

**6.** Prove $\cos A = \dfrac{\sin b \cos a}{\sin c}$.

**7.** If the side $a$ of a right spherical triangle is 90°, what becomes of the triangle $PQR$ in Fig. 116 and what can we say of the other parts of the triangle?

**8.** In a spherical triangle let $c = 90°$, $C = 90°$. Write formulas (1), (2), (4), (5), (6), (8) page 151 for this special case. What do the formulas tell you of the other parts of the triangle?

**9.** What can we tell of the relative sizes of $a$ and $A$ from the formula $\sin a = \sin c \sin A$?

**10.** What can we tell of the relative sizes of $b$ and $c$ from the formula $\cos c = \cos a \cos b$?

**76. Solution of Right Spherical Triangles.** If two parts of a right spherical triangle besides the right angle $C$ are given, it may or may not be possible to find a triangle having these parts. For example, if $a = 50°$ and $A = 100°$ are the given

parts, then the triangle is impossible. (See Ex. 4, p. 153.) If the triangle is solvable, the formulas on page 151 are sufficient. In general, the process consists in choosing one of the formulas which contains the two given parts and one other. Or, Napier's rules may be used by selecting a part and two adjacent parts, or a part and two opposite parts in such a way that two of the three parts are known. The solution of right spherical triangles is illustrated in the following examples. If a side next to the right angle and the angle opposite this side are given, there are in general two solutions.

EXAMPLE 1. Given $a = 62°30'$, $b = 39°46'$; to find $A$, $B$, $c$.

SOLUTION: Use formula (3) in the form $\tan A = \dfrac{\tan a}{\sin b}$.

$$\begin{aligned} \log \tan 62°30' &= 0.28352 \\ \log \sin 39°46' &= \underline{9.80595} \\ \log \tan A &= 0.47757 \\ A &= 71°35'. \end{aligned}$$

Using $\cos c = \cos a \cos b$, we have

$$\begin{aligned} \log \cos 62°30' &= 9.66441 \\ \log \cos 39°46' &= \underline{9.88573} \\ \log \cos c &= 9.55014 \\ c &= 69°12.7'. \end{aligned}$$

Using (7) in the form $\tan B = \dfrac{\tan b}{\sin a}$, we have

$$\begin{aligned} \log \tan 39°46' &= 9.92022 \\ \log \sin 62°30' &= \underline{9.94793} \\ \log \tan B &= 9.97229 \\ B &= 43°10.4'. \end{aligned}$$

Selecting a formula containing $c$, $A$ and $B$ for a check, say

$$\cos c = \cot A \cot B,$$

we have

$$\begin{aligned} \log \cot A &= 9.52242 \\ \log \cot B &= \underline{0.02771} \\ \log \cos c &= 9.55013, \end{aligned}$$

which checks to one unit in the last place with above value of $\log \cos c$.

EXAMPLE 2. Given $a = 25°47'$, $A = 35°19'$; to find the remaining parts of the triangle.

SOLUTION: Use formula (1) in form $\sin c = \dfrac{\sin a}{\sin A}$.

$$\begin{aligned} \log \sin 25°47' &= 9.63846 \\ \log \sin 35°19' &= 9.76200 \\ \log \sin c &= \overline{9.87646} \\ c &= 48°48' \text{ or } 131°12'. \end{aligned}$$

Using (3) in the form $\sin b = \dfrac{\tan a}{\tan A}$, we have

$$\begin{aligned} \log \tan 25°47' &= 9.68400 \\ \log \tan 35°19' &= 9.85033 \\ \log \sin b &= \overline{9.83367} \\ b &= 42°59.2' \text{ or } 137°0.8'. \end{aligned}$$

Using (9) in the form $\sin B = \dfrac{\cos A}{\cos a}$, we have

$$\begin{aligned} \log \cos 35°19' &= 9.91167 \\ \log \cos 25°47' &= \underline{9.95446} \\ \log \sin B &= \overline{9.95721} \\ B &= 64°58.8' \text{ or } 115°1.2'. \end{aligned}$$

We thus have two right triangles for which $a = 25°47'$, $A = 35°19'$. It remains to choose the right sets of the other three parts. From Exercise 4, page 153, $b$ and $B$ must be either both less than 90° or both greater than 90°. Hence, the angles $b = 42°59.2'$, $B = 64°58.8'$, and $b = 137°0.8'$, $B = 115°1.2'$ go together. From Exercise 2, page 153, the value of $c$ which is less than 90° goes with the triangle in which $a$ and $b$ are both less than 90°. The two triangles are

| | |
|---|---|
| $a = 25°47'$ | $a = 25°47'$ |
| $A = 35°19'$ | $A = 35°19'$ |
| $b = 42°59.2'$ | $b = 137°0.8'$ |
| $B = 64°58.8'$ | $B = 115°1.2'$ |
| $c = 48°48'$ | $c = 131°12'$. |

EXAMPLE 3. Given $a = 109°29'$, $A = 135°17'$. Solve the triangle.

SOLUTION: Using $\sin c = \dfrac{\sin a}{\sin A}$, we have

$$\begin{aligned} \log \sin 109°29' &= 9.97439 \\ \log \sin 135°17' &= \underline{9.84733} \\ \log \sin c &= \overline{0.12706}. \end{aligned}$$

Since the sine of an angle is never greater than unity, its logarithm is never greater than 0. Hence, there is no solution for this example.

## EXERCISES

Write down the formulas necessary for a complete solution of the following triangles. Make an outline of the logarithmic work necessary for the solution. Give a formula that can be used as a check.

**1.** Given $a = 35°$, $c = 123°$. To find $b$, $A$, $B$.

SOLUTION: A formula containing $a$, $c$, $b$ is

$$\cos c = \cos a \cos b,$$

or

$$\cos b = \frac{\cos c}{\cos a}.$$

Since $\cos c = \cos 123°$ is negative and $\cos a = \cos 35°$ is positive, $\cos b$ is negative and $b$ is an angle in the second quadrant.

$$\begin{array}{l} \log \cos c = \\ \underline{\log \cos a = \qquad\qquad} \\ \log \cos b = \end{array}$$

and $b$ is 180° minus the acute angle found in the table.

A formula containing $a$, $c$, $A$ is

$$\sin a = \sin c \sin A, \text{ or } \sin A = \frac{\sin a}{\sin c}.$$

Sin $A$ is positive. Hence, $A$ may be an angle in the first or second quadrants. However, since $a$ is less than 90°, $A$ is less than 90°.

$$\begin{array}{l} \log \sin a = \\ \underline{\log \sin c = \qquad\qquad} \\ \log \sin A = \end{array}$$

A formula containing $a$, $c$, $B$ is

$$\tan a = \tan c \cos B, \text{ or } \cos B = \frac{\tan a}{\tan c},$$

where $\cos B$ is negative

$$\begin{array}{l} \log \tan a = \\ \underline{\log \tan c = \qquad\qquad} \\ \log \cos B = \end{array}$$

and $B$ is 180° minus the acute angle given in the table.

The formula $\cos B = \cos b \sin A$ contains the three parts of the triangle just found. Hence, a check on the logarithmic work is

$$\log \cos B = \log \cos b + \log \sin A.$$

**2.** $a = 42°$, $b = 62°$.

**3.** $a = 150°$, $A = 150°$.

**4.** $b = 35°$, $B = 75°$.

**5.** $b = 41°$, $c = 80°$.

**6.** $c = 26°$, $A = 30°$.

**7.** $c = 10°$, $B = 50°$.

**8.** $A = 75°$, $B = 100°$.

**9.** $a = 65°$, $c = 115°$.

**10.** $b = 40°$, $A = 95°$.

Solve the following right spherical triangles:

**11.** $a = 61°13'$, $c = 79°53'$.

**12.** $a = 135°17'$, $A = 109°29'$.

**13.** $b = 47°19'$, $B = 67°15'$.

**14.** $b = 117°17'$, $c = 71°3'$.

**15.** $a = 45°39'$, $b = 67°11'$.

**16.** $a = 31°31'$, $b = 114°57'$.

**17.** $A = 30°$, $B = 60°$.

**18.** $a = 51°14'$, $A = 110°19'$.

**19.** $b = 113°45'$, $B = 113°45'$.

**20.** $a = 49°9'$, $B = 88°46'$.

**21.** $a = 40°$, $A = 80°$.

**22.** $a = 80°$, $A = 40°$.

**23.** $a = 156°30'$, $b = 116°35'$.

**24.** $b = 11°2'$, $A = 36°50'$.

**25.** $b = 19°$, $B = 119°$.

**26.** $c = 65°16'$, $A = 75°13'$.

**27.** $c = 27°47'$, $B = 119°40'$.

**28.** $a = 90°$, $c = 100°$.

**29.** $b = 90°$, $c = 90°$.

**30.** $a = 90°$, $b = 90°$.

**77. Isosceles Spherical Triangles.** The great circle drawn from the vertex of an isosceles triangle to the mid-point of the opposite side divides the triangle into two symmetrical right spherical triangles. The solution of an isosceles triangle then reduces to the solution of a right triangle.

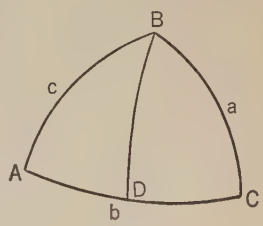


FIG. 119

EXAMPLE. Given $a = c = 75°16'$, $b = 61°8'$. To find the other parts of the triangle.

SOLUTION. A great circle drawn through $B$ to the mid-point of the opposite side divides the triangle into two right triangles. We have for one of these triangles say $ABD$, $c = 75°16'$, $AD = \frac{1}{2}b = 30°34'$; to find the angles $A$ and $ABD = \frac{1}{2}B$.

From formula (2) page 151 we have

$$\cos A = \frac{\tan \frac{1}{2}b}{\tan c}.$$

$$\begin{aligned} \log \tan \tfrac{1}{2}b &= 9.77130 \\ \log \tan c &= 0.58010 \\ \log \cos A &= \overline{9.19120} \\ A &= 81°3.9'. \end{aligned}$$

Also from formula (5) page 151

$$\sin \tfrac{1}{2}B = \frac{\sin \tfrac{1}{2}b}{\sin c}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}b &= 9.70633 \\ \log \sin c &= 9.98548 \\ \log \sin \tfrac{1}{2}B &= \overline{9.72085} \\ \tfrac{1}{2}B &= 31°43.5' \\ B &= 63°27'. \end{aligned}$$

**78. Quadrantal Triangles.** A quadrantal triangle is a spherical triangle with one side measuring 90°. The polar triangle of a quadrantal triangle is a right spherical triangle. From a solution of the polar triangle, we can find the unknown parts of the quadrantal triangle.

EXAMPLE. Given $c = 90°$, $a = 49°30'$, $B = 65°36'$. Solve the triangle.

SOLUTION: Let $A'B'C'$ represent the polar triangle. From the relations on page 149, we have $C' = 90°$, $A' = 130°30'$, $b' = 114°24'$; to find $a', B', c'$. Making use of

$$\tan a' = \sin b' \tan A',$$

we have

$$\begin{aligned} \log \sin\ 114°24' &= 9.95937 \\ \log \tan 130°30' &= \underline{0.06850} \\ \log \tan a' &= \overline{0.02787} \end{aligned}$$

$$a' = 133°9.8' \text{ since } \tan a' \text{ is negative.}$$

From

$$\cos B' = \cos b' \sin A',$$

we obtain

$$\begin{aligned} \log \cos\ 114°24' &= 9.61606 \\ \log \sin\ 130°30' &= \underline{9.88105} \\ \log \cos B' &= \overline{9.49711} \end{aligned}$$

$$B' = 108°18.5', \text{ since } \cos B' \text{ is negative.}$$

The relation

$$\tan c' = \frac{\tan b'}{\cos A'},$$

gives

$$\begin{aligned} \log \tan 114°24' &= 0.34331 \\ \log \cos 130°30' &= \underline{9.81254} \\ \log \tan c' &= \overline{0.53077} \\ c' &= 73°35.1'. \end{aligned}$$

The unknown parts $b$, $A$, $C$ of the quadrantal triangle are:

$$\begin{aligned} b &= 180 - B' = 71°41.5' \\ A &= 180 - a' = 46°50.2' \\ C &= 180 - c' = 106°24.9'. \end{aligned}$$

## EXERCISES

Solve the following triangles:

1. $C = 57° 34'$, $a = b = 65° 16'$.
2. $c = 48° 43'$, $a = b = 70° 3'$.
3. $C = 61° 38'$, $b = c = 70° 3'$.
4. $A = B = C = 75° 36'$.
5. $c = 135° 14'$, $C = 156° 48'$, $a = b$.
6. $A = 110° 30'$, $a = b = 120° 41'$.
7. $c = 90°$, $a = 145° 20'$, $b = 100°$.
8. $c = 90°$, $a = 127° 39'$, $B = 19° 57'$.
9. $c = 90°$, $b = 100°$, $B = 100°$.
10. $c = 90°$, $A = 100°$, $B = 100°$.
11. $c = 90°$, $A = 50° 50'$, $B = 119° 47'$.
12. $b = 45°$, $B = 55°$, $a = c$.

## CHAPTER X

## OBLIQUE SPHERICAL TRIANGLES

**79. The Law of Sines.** Let $ABC$ represent an oblique spherical triangle; that is, a spherical triangle having no right angle. Through the vertex $B$, draw the arc of a great circle cutting the side $b$, or the side $b$ produced, at right angles in the

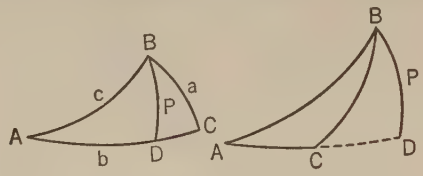


FIG. 120

point $D$. Let $p$ be the length of this perpendicular. We have then two right spherical triangles $ABD$ and $DBC$. For the triangle $ABD$ we have from (1), page 151,

$$\sin p = \sin A \sin c.$$

For the triangle $DBC$, we have

$$\sin p = \sin C \sin a.$$

From these two equations, we obtain immediately

$$\frac{\sin A}{\sin a} = \frac{\sin C}{\sin c}. \tag{1}$$

In a similar manner, by drawing a perpendicular from $A$ to the side $a$, we find

$$\frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}. \tag{2}$$

Combining (1) and (2), we obtain

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}, \tag{3}$$

which is called the **Law of Sines** for spherical triangles.

**80. The Law of Cosines.** From Fig. 120, we have, using formula (4) page 151,

$$\cos a = \cos p \cos DC$$
$$\cos c = \cos p \cos AD,$$

whence

$$\frac{\cos a}{\cos c} = \frac{\cos DC}{\cos AD} = \frac{\cos (b - AD)}{\cos AD} = \frac{\cos b \cos AD + \sin b \sin AD}{\cos AD}$$

$$= \cos b + \frac{\sin b \sin AD}{\cos AD} = \cos b + \sin b \tan AD.$$

But, from formula (6) page 151, we have

$$\tan AD = \tan c \cos A.$$

Hence, we have

$$\frac{\cos a}{\cos c} = \cos b + \sin b \tan c \cos A,$$

or

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \tag{4}$$

In a similar manner, we find

$$\cos b = \cos a \cos c + \sin a \sin c \cos B, \tag{5}$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \tag{6}$$

These three formulas are known as the **Law of Cosines** for spherical triangles.

Considering the polar triangle $A'B'C'$, and applying the law of cosines, we have

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

Remembering the relations

$$a' = 180° - A, \quad b' = 180° - B, \text{ etc.}$$

this expression becomes

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \tag{7}$$

In a similar manner, we find

$$\cos B = -\cos C \cos A + \sin C \sin A \cos b, \tag{8}$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c. \tag{9}$$

**81. Law of tangents.** From the law of sines, we have

$$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}.$$

Subtracting 1 from each member, we obtain

$$\frac{\sin A}{\sin B} - 1 = \frac{\sin a}{\sin b} - 1, \quad \text{or} \quad \frac{\sin A - \sin B}{\sin B} = \frac{\sin a - \sin b}{\sin b}.$$

Adding 1 to each member, gives

$$\frac{\sin A + \sin B}{\sin B} = \frac{\sin a + \sin b}{\sin b}.$$

Dividing the first of these results by the second gives

$$\frac{\sin A - \sin B}{\sin A + \sin B} = \frac{\sin a - \sin b}{\sin a + \sin b}. \tag{10}$$

But from (25) and (24) Art. 28, we have

$$\frac{\sin a - \sin b}{\sin a + \sin b} = \frac{2 \cos \frac{1}{2}(a + b) \sin \frac{1}{2}(a - b)}{2 \sin \frac{1}{2}(a + b) \cos \frac{1}{2}(a - b)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}(a + b)}.$$

Making this transformation for the left-hand member of (10), gives us the equation known as the **Law of Tangents** in spherical trigonometry

$$\frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}(a + b)}. \tag{11}$$

**82. Half-angle Formulas.** The formulas derived in Art. 80 may be used in the solution of spherical triangles. For example, if we have given $a$, $c$ and $B$, we can find $b$ from formula (5); but this formula is inconvenient when logarithm tables are to be used. Expressions adapted to logarithmic computation can be deduced as follows.

From the first equation of the law of cosines, namely

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

we find

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Subtracting each member of this equation from 1, we have

$$1 - \cos A = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$= \frac{\sin b \sin c - \cos a + \cos b \cos c}{\sin b \sin c}$$

$$= \frac{\cos (b - c) - \cos a}{\sin b \sin c}.$$

Adding 1 to each member of the same equation, we have

$$1 + \cos A = \frac{\cos a - \cos (b + c)}{\sin b \sin c}.$$

Using formula (23′) Art. 27, we have

$$\tan \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{\cos (b - c) - \cos a}{\cos a - \cos (b + c)}},$$

and from (27) Art. 28, we may write

$$\cos (b - c) - \cos a = -2 \sin \tfrac{1}{2}(a + b - c) \sin \tfrac{1}{2}(b - c - a),$$
$$\cos a - \cos (b + c) = -2 \sin \tfrac{1}{2}(a + b + c) \sin \tfrac{1}{2}(a - b - c).$$

Now, let

$$2s = a + b + c,$$

and we have

$$a + b - c = 2(s - c),$$
$$b - c - a = -2(s - b),$$
$$a - b - c = -2(s - a).$$

The formula for $\tan \frac{1}{2}A$ then reduces to

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\sin (s - c) \sin (s - b)}{\sin s \sin (s - a)}}. \tag{12}$$

In a similar manner, we obtain

$$\tan \tfrac{1}{2}B = \sqrt{\frac{\sin (s - a) \sin (s - c)}{\sin s \sin (s - b)}}, \tag{13}$$

$$\tan \tfrac{1}{2}C = \sqrt{\frac{\sin (s - a) \sin (s - b)}{\sin s \sin (s - c)}}, \tag{14}$$

These three formulas may be put into more convenient form for computation by using

$$k = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$$

whence

$$\tan \tfrac{1}{2}A = \frac{k}{\sin(s-a)}, \tag{15}$$

$$\tan \tfrac{1}{2}B = \frac{k}{\sin(s-b)}, \tag{16}$$

$$\tan \tfrac{1}{2}C = \frac{k}{\sin(s-c)}. \tag{17}$$

The numerator $k$ is easy to calculate by logarithms, so these formulas are well adapted to finding the angles when the sides are given.

Expressions for $\tan \frac{1}{2}a$, $\tan \frac{1}{2}b$, $\tan \frac{1}{2}c$, may be found by applying formulas (12), (13) and (14) to the polar triangle $A'B'C'$. Taking up first the derivation of a formula for $\tan \frac{1}{2}a$, we have from (12) for the polar triangle,

$$\tan \tfrac{1}{2}A' = \sqrt{\frac{\sin(s-c')\sin(s-b')}{\sin s \sin(s-a')}}, \tag{18}$$

where $2s = a' + b' + c'$.

Recalling the properties of the polar triangle,

$$A' = 180° - a, \quad a' = 180° - A, \text{ etc.},$$

and letting

$$2S = A + B + C,$$

we have

$$\begin{aligned} s = \tfrac{1}{2}(a' + b' + c') &= \tfrac{1}{2}\{540° - (A + B + C)\} \\ &= 270° - S. \end{aligned}$$

Also, we find

$$\begin{aligned} s - a' &= 270° - S - a' = 90° - (S - A), \\ s - b' &= 270° - S - b' = 90° - (S - B), \\ s - c' &= 270° - S - c' = 90° - (S - C). \end{aligned}$$

From the properties of the polar triangle, we have

$$\tfrac{1}{2}A' = 90° - \tfrac{1}{2}a.$$

Hence, it follows that

$$\tan \tfrac{1}{2}A' = \tan (90° - \tfrac{1}{2}a) = \cot \tfrac{1}{2}a;$$

also,

$$\sin (s - a') = \sin \{90° - (S - A)\} = \cos (S - A),$$
$$\sin (s - b') = \sin \{90° - (S - B)\} = \cos (S - B),$$
$$\sin (s - c') = \sin \{90° - (S - C)\} = \cos (S - C),$$
$$\sin s = \sin (270° - S) = - \cos S.$$

Formula (18) now becomes

$$\cot \tfrac{1}{2}a = \sqrt{\frac{\cos (S - C) \cos (S - B)}{- \cos S \cos (S - A)}},$$

or

$$\tan \tfrac{1}{2}a = \sqrt{\frac{- \cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}}. \tag{19}$$

In a similar manner, we find

$$\tan \tfrac{1}{2}b = \sqrt{\frac{- \cos S \cos (S - B)}{\cos (S - A) \cos (S - C)}}, \tag{20}$$

$$\tan \tfrac{1}{2}c = \sqrt{\frac{- \cos S \cos (S - C)}{\cos (S - A) \cos (S - B)}}. \tag{21}$$

If we let

$$K = \sqrt{\frac{- \cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}},$$

we have

$$\tan \tfrac{1}{2}a = K \cos (S - A), \tag{22}$$
$$\tan \tfrac{1}{2}b = K \cos (S - B), \tag{23}$$
$$\tan \tfrac{1}{2}c = K \cos (S - C), \tag{24}$$

which are convenient formulas for finding the sides when the angles are given.

**83. Napier's Analogies.** From formulas (15) and (16) Art. 82, we find

$$\frac{\tan \tfrac{1}{2}A}{\tan \tfrac{1}{2}B} = \frac{\sin (s - b)}{\sin (s - a)}. \tag{25}$$

Adding 1 to each side, we have

$$\frac{\tan \frac{1}{2}A}{\tan \frac{1}{2}B} + 1 = \frac{\sin (s - b)}{\sin (s - a)} + 1,$$

or

$$\frac{\tan \frac{1}{2}A + \tan \frac{1}{2}B}{\tan \frac{1}{2}B} = \frac{\sin (s - b) + \sin (s - a)}{\sin (s - a)}.$$

Subtracting 1 from each side of (25), we find

$$\frac{\tan \frac{1}{2}A - \tan \frac{1}{2}B}{\tan \frac{1}{2}B} = \frac{\sin (s - b) - \sin (s - a)}{\sin (s - a)}.$$

By division, we have

$$\frac{\tan \frac{1}{2}A - \tan \frac{1}{2}B}{\tan \frac{1}{2}A + \tan \frac{1}{2}B} = \frac{\sin (s - b) - \sin (s - a)}{\sin (s - b) + \sin (s - a)}. \tag{26}$$

Since

$$\tan \tfrac{1}{2}A = \frac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A}, \qquad \tan \tfrac{1}{2}B = \frac{\sin \frac{1}{2}B}{\cos \frac{1}{2}B},$$

the left-hand member of (26) reduces to

$$\frac{\sin \frac{1}{2}A \cos \frac{1}{2}B - \cos \frac{1}{2}A \sin \frac{1}{2}B}{\sin \frac{1}{2}A \cos \frac{1}{2}B + \cos \frac{1}{2}A \sin \frac{1}{2}B} = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)}.$$

To reduce the right-hand member of equation (26), we use formulas (25), (24), Art. 28 which may be written

$$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y),$$
$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y).$$

By division, we have

$$\frac{\sin x - \sin y}{\sin x + \sin y} = \frac{\tan \frac{1}{2}(x - y)}{\tan \frac{1}{2}(x + y)}. \tag{27}$$

Applying (27) to the right-hand member of (26), we obtain

$$\frac{\sin (s - b) - \sin (s - a)}{\sin (s - b) + \sin (s - a)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}(2s - a - b)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}c}.$$

Equation (26) now reduces to

$$\frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}c},$$

or

$$\tan \tfrac{1}{2}(a - b) = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} \tan \tfrac{1}{2}c. \tag{28}$$

From the law of tangents for spherical trigonometry, (11), Art. 81, we have

$$\tan \tfrac{1}{2}(a + b) = \frac{\tan \frac{1}{2}(a - b) \tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$$

Substituting in this expression the value of $\tan \frac{1}{2}(a - b)$ from (28), we find

$$\tan \tfrac{1}{2}(a + b) = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} \tan \tfrac{1}{2}c. \tag{29}$$

Applying (28) and (29) to the polar triangle, we easily derive

$$\tan \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} \cot \tfrac{1}{2}C. \tag{30}$$

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \tfrac{1}{2}C. \tag{31}$$

Using the angles $A$ and $C$, and sides $a$ and $c$, we have another set of four equations similar in form to (28), (29), (30), (31) and still another set for $B$, $C$, $b$, $c$. These twelve equations are generally called Napier's Analogies, where the word "analogy" is used in its older sense of "a proportion." The student should write down the other eight of Napier's Analogies.

**84. Solution of Oblique Spherical Triangles.** There are six parts to a spherical triangle. If three of these parts are given, the other three parts may be found. Since there are six essentially different ways of giving three parts of a triangle, there are six different cases for discussion.

CASE I. *Given the three sides.*

CASE II. *Given the three angles.*

CASE III. *Given two sides and the included angle.*

CASE IV. *Given two angles and the included side.*

CASE V. *Given two sides and an angle opposite one.*

CASE VI. *Given two angles and a side opposite one.*

**Case I. Given the three sides.** The half-angle formulas are adapted to the solution of this case; namely,

$$\tan \tfrac{1}{2}A = \frac{k}{\sin (s - a)}, \quad \tan \tfrac{1}{2}B = \frac{k}{\sin (s - b)},$$

$$\tan \tfrac{1}{2}C = \frac{k}{\sin (s - c)},$$

where

$$k = \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}.$$

EXAMPLE. The three sides of a spherical triangle are 77° 36′, 63° 17′, 107° 23′. Find the angles.

SOLUTION:

Let

$$\begin{aligned} a &= 77° 36' \\ b &= 63° 17' \\ c &= 107° 23' \end{aligned}$$

Then

$$\begin{aligned} s &= \tfrac{1}{2}(a + b + c) = 124° 8' \\ s - a &= 46° 32' \\ s - b &= 60° 51' \\ s - c &= 16° 45'. \end{aligned}$$

We first find log $k$ as follows:

$$\begin{aligned} \log \sin (s - a) &= 9.86080 \\ \log \sin (s - b) &= 9.94119 \\ \log \sin (s - c) &= \underline{9.45969} \\ \text{sum} &= 9.26168 \\ \log \sin s &= \underline{9.91789} \\ \log k^2 &= 9.34379 \\ \log k &= 9.67190. \end{aligned}$$

$$\tan \tfrac{1}{2}A = \frac{k}{\sin (s - a)}$$

$$\begin{aligned} \log k &= 9.67190 \\ \log \sin (s - a) &= \underline{9.86080} \\ \log \tan \tfrac{1}{2}A &= 9.81110 \\ \tfrac{1}{2}A &= 32° 54.9' \\ A &= 65° 49.8'. \end{aligned}$$

$$\tan \tfrac{1}{2}B = \frac{k}{\sin (s - b)}$$

$$\begin{aligned} \log k &= 9.67190 \\ \log \sin (s - b) &= \underline{9.94119} \\ \log \tan \tfrac{1}{2}B &= 9.73071 \\ \tfrac{1}{2}B &= 28° 16.57' \\ B &= 56° 33.1' \end{aligned}$$

$$\tan \tfrac{1}{2}C = \frac{k}{\sin (s - c)}$$

$$\begin{aligned} \log k &= 9.67190 \\ \log \sin (s - c) &= \underline{9.45969} \\ \log \tan \tfrac{1}{2}C &= \overline{0.21221} \\ \tfrac{1}{2}C &= 58^\circ 28.34' \\ C &= 116^\circ 56.7' \end{aligned}$$

Using the law of sines as a check; that is,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c},$$

we have

$$\begin{aligned} \log \sin A &= 9.96016 \\ \log \sin a &= \underline{9.98975} \\ & \phantom{=} 9.97041, \end{aligned} \qquad \begin{aligned} \log \sin B &= 9.92137 \\ \log \sin b &= \underline{9.95097} \\ & \phantom{=} 9.97040, \end{aligned} \qquad \begin{aligned} \log \sin C &= 9.95009 \\ \log \sin c &= \underline{9.97970} \\ & \phantom{=} 9.97039. \end{aligned}$$

## EXERCISES

Solve the following spherical triangles:

**1.** $a = 125^\circ, \quad b = 50^\circ, \quad c = 100^\circ.$

**2.** $a = 40^\circ, \quad b = 50^\circ, \quad c = 60^\circ.$

**3.** $a = 100^\circ, \quad b = 110^\circ, \quad c = 120^\circ.$

**4.** $a = 34^\circ 56', \; b = 67^\circ 27', \; c = 73^\circ 20'.$

**Case II. Given the three angles.** The formulas best adapted to this case are (22), (23) and (24) page 165; namely,

$$\tan \tfrac{1}{2}a = K \cos (S - A), \quad \tan \tfrac{1}{2}b = K \cos (S - B),$$
$$\tan \tfrac{1}{2}c = K \cos (S - C),$$

where

$$K = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}$$

and $\cos S$ is negative.

EXAMPLE: The three angles of a spherical triangle are $77^\circ 36'$, $63^\circ 17'$, $107^\circ 23'$. Find the sides.

SOLUTION: Let $A = 77^\circ 36'$, $B = 63^\circ 17'$, $C = 107^\circ 23'$.

Then $S = \frac{1}{2}(A + B + C) = 124^\circ 8'$, ($S$ is always greater than $90^\circ$).

$$\begin{aligned} S - A &= 46^\circ 32', \\ S - B &= 60^\circ 51', \\ S - C &= 16^\circ 45'. \end{aligned}$$

$$\begin{aligned} \log (-\cos S) &= 9.74906 \\ & \phantom{=} \underline{9.50634} \\ \log K^2 &= \overline{0.24272} \\ \log K &= 0.12136. \end{aligned} \qquad \begin{aligned} \log \cos (S - A) &= 9.83755 \\ \log \cos (S - B) &= 9.68762 \\ \log \cos (S - C) &= \underline{9.98117} \\ \text{sum} &= \overline{9.50634}. \end{aligned}$$

$\tan \frac{1}{2}a = K \cos (S - A)$:

$$\begin{aligned} \log K &= 0.12136 \\ \log \cos (S - A) &= \underline{9.83755} \\ \log \tan \tfrac{1}{2}a &= 9.95891 \\ \tfrac{1}{2}a &= 42^\circ 17.6' \\ a &= 84^\circ 35.2' \end{aligned}$$

$\tan \frac{1}{2}b = K \cos (S - B)$:

$$\begin{aligned} \log K &= 0.12136 \\ \log \cos (S - B) &= \underline{9.68762} \\ \log \tan \tfrac{1}{2}b &= 9.80898 \\ \tfrac{1}{2}b &= 32^\circ 47.2' \\ b &= 65^\circ 34.4'. \end{aligned}$$

$\tan \frac{1}{2}c = K \cos (S - C)$:

$$\begin{aligned} \log K &= 0.12136 \\ \log \cos (S - C) &= \underline{9.98117} \\ \log \tan \tfrac{1}{2}c &= 0.10253 \\ \tfrac{1}{2}c &= 51^\circ 42.1' \\ c &= 103^\circ 24.2'. \end{aligned}$$

Using the law of sines as a check, we have

$$\begin{aligned} \log \sin A &= 9.98975 \\ \log \sin a &= \underline{9.99806} \\ & \phantom{=} 9.99169, \end{aligned} \qquad \begin{aligned} \log \sin B &= 9.95097 \\ \log \sin b &= \underline{9.95927} \\ & \phantom{=} 9.99170, \end{aligned} \qquad \begin{aligned} \log \sin C &= 9.97970 \\ \log \sin c &= \underline{9.98800} \\ & \phantom{=} 9.99170. \end{aligned}$$

## EXERCISES

Solve the following spherical triangles:

1. $A = 100^\circ$, $B = 60^\circ$, $C = 70^\circ$.
2. $A = 143^\circ 27'$, $B = 26^\circ 3'$, $C = 36^\circ 7'$.
3. $A = 31^\circ 25'$, $B = 56^\circ 32'$, $C = 60^\circ 30'$.
4. $A = 130^\circ 46.4'$, $B = 108^\circ 0.3'$, $C = 62^\circ 43.9'$.

**Case III. Given two sides and the included angle.** Napier's analogies are useful in this case. If, for example, the given parts are $a$, $b$, $C$, then from the formulas

$$\tan \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} \cot \tfrac{1}{2}C,$$

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \tfrac{1}{2}C,$$

we can find $\frac{1}{2}(A - B)$, $\frac{1}{2}(A + B)$, and then $A$ and $B$. To find $c$ we use

$$\tan \tfrac{1}{2}(a - b) = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} \tan \tfrac{1}{2}c.$$

EXAMPLE. Given $b = 63°17'$, $c = 107°23'$, $A = 65°50'$. Find $a, B, C$.

SOLUTION: To find $B$ and $C$, we use

$$\tan \tfrac{1}{2}(C - B) = \frac{\sin \frac{1}{2}(c - b)}{\sin \frac{1}{2}(c + b)} \cot \tfrac{1}{2}A,$$

$$\tan \tfrac{1}{2}(C + B) = \frac{\cos \frac{1}{2}(c - b)}{\cos \frac{1}{2}(c + b)} \cot \tfrac{1}{2}A.$$

$$\begin{aligned} c - b &= 44°6', & \tfrac{1}{2}(c - b) &= 22°3', \\ c + b &= 170°40', & \tfrac{1}{2}(c + b) &= 85°20', \\ & & \tfrac{1}{2}A &= 32°55'. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(c - b) &= 9.57451 \\ \log \sin \tfrac{1}{2}(c + b) &= \underline{9.99856} \\ \text{difference} &= 9.57595 \\ \log \cot \tfrac{1}{2}A &= 0.18887 \\ \log \tan \tfrac{1}{2}(C - B) &= 9.76482 \\ \tfrac{1}{2}(C - B) &= 30°11.6' \end{aligned}$$

$$\begin{aligned} \log \cos \tfrac{1}{2}(c - b) &= 9.96701 \\ \log \cos \tfrac{1}{2}(c + b) &= \underline{8.91040} \\ \text{difference} &= 1.05661 \\ \log \cot \tfrac{1}{2}A &= 0.18887 \\ \log \tan \tfrac{1}{2}(C + B) &= 1.24548 \\ \tfrac{1}{2}(C + B) &= 86°44.9' \\ \tfrac{1}{2}(C - B) &= 30°11.6' \\ C &= 116°56.5' \\ B &= 56°33.3'. \end{aligned}$$

To find $a$, we use

$$\tan \tfrac{1}{2}(c - b) = \frac{\sin \frac{1}{2}(C - B)}{\sin \frac{1}{2}(C + B)} \tan \tfrac{1}{2}a.$$

$$\begin{aligned} \log \tan \tfrac{1}{2}(c - b) &= 9.60750 \\ \log \sin \tfrac{1}{2}(C + B) &= \underline{9.99930} \\ \text{sum} &= 9.60680 \\ \log \sin \tfrac{1}{2}(C - B) &= \underline{9.70150} \\ \log \tan \tfrac{1}{2}a &= 9.90530 \\ \tfrac{1}{2}a &= 38°48.1' \\ a &= 77°36.2'. \end{aligned}$$

Using the law of sines as a check, we have

$$\begin{aligned} \log \sin A &= 9.96017 \\ \log \sin a &= \underline{9.98976} \\ & \phantom{=} 9.97041, \end{aligned} \qquad \begin{aligned} \log \sin B &= 9.92138 \\ \log \sin b &= \underline{9.95097} \\ & \phantom{=} 9.97041, \end{aligned} \qquad \begin{aligned} \log \sin C &= 9.95010 \\ \log \sin c &= \underline{9.97970} \\ & \phantom{=} 9.97040. \end{aligned}$$

## EXERCISES

Solve the following spherical triangles:

1. $A = 93°17'$, $c = 124°10'$, $b = 63°12'$.
2. $a = 100°$, $b = 65°$, $C = 100°$.
3. $a = 137°8'$, $c = 118°30'$, $B = 24°24'$.
4. $a = 28°7'$, $b = 35°57'$, $C = 49°44'$.

**Case IV. Given two angles and the included side.** The solution is similar to Case III.

EXAMPLE. Given $A = 65°50'$, $C = 116°57'$, $b = 63°17'$. Find $B, a, c$.

SOLUTION: To find $a$ and $c$, we use

$$\tan \tfrac{1}{2}(c - a) = \frac{\sin \frac{1}{2}(C - A)}{\sin \frac{1}{2}(C + A)} \tan \tfrac{1}{2}b,$$

$$\tan \tfrac{1}{2}(c + a) = \frac{\cos \frac{1}{2}(C - A)}{\cos \frac{1}{2}(C + A)} \tan \tfrac{1}{2}b.$$

We have
$$\begin{aligned} C + A &= 182°47', & \tfrac{1}{2}(C + A) &= 91°23.5' \\ C - A &= 51°7', & \tfrac{1}{2}(C - A) &= 25°33.5' \\ & & \tfrac{1}{2}b &= 31°38.5' \end{aligned}$$

NOTE. It should be noticed that since the angle $\frac{1}{2}(C + A)$ is in the second quadrant the cosine is negative, thus making $\tan \frac{1}{2}(c + a)$ negative and $\frac{1}{2}(c + a)$ will be in the second quadrant.

$$\begin{aligned} \log \sin \tfrac{1}{2}(C - A) &= 9.63491 \\ \log \tan \tfrac{1}{2}b &= \underline{9.78973} \\ \text{sum} &= \overline{9.42464} \\ \log \sin \tfrac{1}{2}(C + A) &= \underline{9.99987} \\ \log \tan \tfrac{1}{2}(c - a) &= \overline{9.42477} \\ \tfrac{1}{2}(c - a) &= 14°53.5' \end{aligned}$$

$$\begin{aligned} \log \cos \tfrac{1}{2}(C - A) &= 9.95528 \\ \log \tan \tfrac{1}{2}b &= \underline{9.78973} \\ \text{sum} &= \overline{9.74501} \\ \log \cos \tfrac{1}{2}(C + A) &= \underline{8.38536} \\ \log \tan \tfrac{1}{2}(c + a) &= \overline{1.35965} \\ \tfrac{1}{2}(c + a) &= 92°30.1' \\ \tfrac{1}{2}(c - a) &= 14°53.5' \\ c &= 107°23.6' \\ a &= 77°36.6' \end{aligned}$$

To find $B$, we use

$$\tan \tfrac{1}{2}(C - A) = \frac{\sin \frac{1}{2}(c - a)}{\sin \frac{1}{2}(c + a)} \text{ c t } \tfrac{1}{2}B.$$

$$\begin{aligned} \log \tan \tfrac{1}{2}(C - A) &= 9.67964 \\ \log \sin \tfrac{1}{2}(c + a) &= \underline{9.99959} \\ \text{sum} &= \overline{9.67923} \\ \log \sin \tfrac{1}{2}(c - a) &= \underline{9.40993} \\ \log \cot \tfrac{1}{2}B &= 0.26930 \\ \tfrac{1}{2}B &= 28°16.53' \\ B &= 56°33.1'. \end{aligned}$$

To check, we may use the law of sines or some other formula not used in the solution.

## EXERCISES

Solve the following triangles:

1. $A = 56°18'$, $B = 38°8'$, $c = 35°12'$.
2. $A = 150°$, $C = 125°$, $b = 125°$.
3. $B = 84°35'$, $C = 117°13'$, $a = 64°28'$.
4. $A = 23°32'$, $B = 22°6'$, $c = 147°58'$.

**Case V. Given two sides and the angle opposite one of them.** In this case, there may be no solution, one solution, or two solutions. Suppose the given parts are $a$, $b$, $A$. Then from the law of sines we have

$$\sin B = \frac{\sin b \sin A}{\sin a}.$$

If $\sin b \sin A$ is greater than $\sin a$ then $\sin B$ is greater than unity. Hence, there is no solution. That is, if we calculate $\log \sin B$ from this formula, the characteristic must be negative if a solution is possible. Since

$$\sin B = \sin (180 - B),$$

there will be two values of $B$ to be discussed in case there is a solution. Either value or both values may be solutions. Since the greater side is opposite the greater angle, if $a > b$ and $A$ is greater than both $B$'s there are two solutions. If $A$ is greater than one of the $B$'s only, there is one solution.

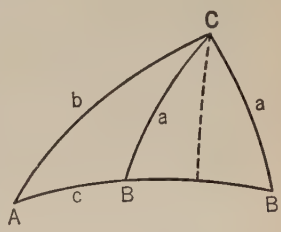

FIG. 121

EXAMPLE. Given $a = 77°36'$, $b = 63°17'$, $B = 56°33'.2$. Find $A$, $C$, $c$.

SOLUTION:

$$\sin A = \frac{\sin a \sin B}{\sin b}.$$

$$\begin{aligned} \log \sin a &= 9.98975 \\ \log \sin B &= \underline{9.92138} \\ \text{sum} &= 9.91113 \\ \log \sin b &= \underline{9.95097} \\ \log \sin A &= 9.96016 \\ A &= 65°49.8' \quad \text{or} \quad A' = 114°10.2'. \end{aligned}$$

Since $a > b$ and $A > B$, $A' > B$, there are two solutions. To find $C$ and $c$, use (30) and (28) Art. 83 in the form,

$$\cot \tfrac{1}{2}C = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} \tan \tfrac{1}{2}(A - B),$$

$$\tan \tfrac{1}{2}c = \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} \tan \tfrac{1}{2}(a - b).$$

$$\begin{aligned} a + b &= 140°53', & \tfrac{1}{2}(a + b) &= 70°26.5'. \\ a - b &= 14°19', & \tfrac{1}{2}(a - b) &= 7°9.5'. \\ A + B &= 122°23', & \tfrac{1}{2}(A + B) &= 61°11.5'. \\ A - B &= 9°16.6', & \tfrac{1}{2}(A - B) &= 4°38.3'. \\ A' + B &= 170°43.4', & \tfrac{1}{2}(A' + B) &= 85°21.7'. \\ A' - B &= 57°37', & \tfrac{1}{2}(A' - B) &= 28°48.5'. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(a + b) &= 9.97419 \\ \log \tan \tfrac{1}{2}(A - B) &= \underline{8.90919} \\ \text{sum} &= \overline{8.88338} \\ \log \sin \tfrac{1}{2}(a - b) &= \underline{9.09556} \\ \log \cot \tfrac{1}{2}C &= \overline{9.78782} \\ \tfrac{1}{2}C &= 58°28.24' \\ C &= 116°56.5'. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(A + B) &= 9.94262 \\ \log \tan \tfrac{1}{2}(a - b) &= \underline{9.09896} \\ \text{sum} &= \overline{9.04158} \\ \log \sin \tfrac{1}{2}(A - B) &= \underline{8.90776} \\ \log \tan \tfrac{1}{2}c &= \overline{0.13382} \\ \tfrac{1}{2}c &= 53°41.44' \\ c &= 107°22.9'. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(a + b) &= 9.97419 \\ \log \tan \tfrac{1}{2}(A' - B) &= \underline{9.74032} \\ \text{sum} &= 9.71451 \\ \log \sin \tfrac{1}{2}(a - b) &= \underline{9.09556} \\ \log \cot \tfrac{1}{2}C' &= 0.61895 \\ \tfrac{1}{2}C' &= 13°31.25' \\ C' &= 27°2.5'. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(A' + B) &= 9.99858 \\ \log \tan \tfrac{1}{2}(a - b) &= \underline{9.09896} \\ \text{sum} &= 9.09754 \\ \log \sin \tfrac{1}{2}(A' - B) &= \underline{9.68294} \\ \log \tan \tfrac{1}{2}c' &= 9.41460 \\ \tfrac{1}{2}c' &= 14°33.73' \\ c' &= 29°7.5'. \end{aligned}$$

The two triangles then are

$a = 77°36'$, $b = 63°17'$, $c = 107°22.9'$, $A = 65°49.8'$, $B = 56°33.2'$, $C = 116°56.5'$; $a = 77°36'$, $b = 63°17'$, $c' = 29°7.5'$, $A' = 114°10.2'$, $B = 56°33.2'$, $C' = 27°2.5'$.

Each triangle may be checked by the law of sines or some other formulas not used in the solution.

**Case VI. Given two angles and a side opposite one of them.** The solution of this case is similar to Case V. There may be no solution, one solution or two solutions.

**85. Area of a spherical triangle.** We have seen that the sum of the three angles of a spherical triangle is greater than 180°. The amount which the sum exceeds 180° is called the *spherical excess* and is usually represented by $E$. That is, we have

$$E = A + B + C - 180°.$$

In solid geometry, it is shown that the areas of two spherical triangles are to each other as their spherical excesses. The

area of a spherical triangle with three right angles is one-eighth the total area of the sphere, or $\frac{\pi R^2}{2}$, where $R$ is the radius of the sphere. The spherical excess is 90°. Hence, we have

$$\frac{\text{area of any spherical triangle}}{\frac{\pi R^2}{2}} = \frac{E}{90},$$

or

$$\textbf{area} = \frac{\pi R^2}{180} E. \tag{32}$$

A well-known formula called **L'Huilier's Theorem** enables us to find $E$ when the sides of the triangle are given; that is,

$$\tan \frac{E}{4} = \sqrt{\tan \tfrac{1}{2}s \tan \tfrac{1}{2}(s-a) \tan \tfrac{1}{2}(s-b) \tan \tfrac{1}{2}(s-c)}. \tag{33}$$

## EXERCISES

Find the number of solutions for each of the following spherical triangles:

**1.** $a = 60°$, $b = 150°$, $A = 45°$.
**2.** $a = 46°$, $b = 30°$, $B = 37°$.
**3.** $a = 30°$, $c = 60°$, $A = 75°$.
**4.** $A = 83°$, $C = 20°$, $a = 30°$.
**5.** $A = 120°$, $B = 30°$, $a = 135°$.
**6.** $B = 132°$, $C = 100°$, $c = 40°$.

Solve the following spherical triangles:

**7.** $b = 46° 20'$, $c = 66°$, $B = 41°$.
**8.** $a = 69° 32'$, $A = 72° 10'$, $b = 102° 45'$.
**9.** $B = 40° 5'$, $C = 96°$, $c = 31° 2'$.
**10.** $A = 147°$, $B = 132°$ $a = 136°$.

Find the areas of the following triangles:

**11.** The triangles in Exercise 7 when $R = 10$.

**12.** The triangles in Exercise 10 when $R = 1$.

**13.** Find the spherical excess for the example worked out under Case I (page 168) by the use of formula (33) and check the result from the values of $A$, $B$, and $C$ obtained in that solution.

**14.** Find the spherical excess for the triangle, $a = 125°$, $b = 50°$, $c = 100°$.

## MISCELLANEOUS EXERCISES

Solve the following spherical triangles:

| | | | |
|---|---|---|---|
| **1.** | $a = 57°$, | $b = 137°$, | $c = 116°$. |
| **2.** | $A = 150°$, | $B = 131°$, | $C = 115°$. |
| **3.** | $A = 70°$, | $B = 131° 20'$, | $c = 116°$. |
| **4.** | $a = 98° 12'$, | $b = 80° 35'$, | $C = 10° 16'$. |
| **5.** | $A = 149° 30'$, | $B = 131°$, | $C = 119° 20'$. |
| **6.** | $b = 105°$, | $c = 153°$, | $B = 69° 50'$. |
| **7.** | $b = 137° 20'$, | $c = 116°$, | $A = 70°$. |
| **8.** | $a = 60°$, | $b = 100°$, | $c = 50°$. |
| **9.** | $a = 58°$, | $b = 138°$, | $B = 134° 50'$. |
| **10.** | $a = 110°$, | $c = 81° 50'$, | $A = 108° 3'$. |
| **11.** | $c = 65° 16'$, | $A = 75° 13'$, | $C = 90°$. |
| **12.** | $B = 135° 15'$, | $C = 50° 30'$, | $a = 69° 34'$. |
| **13.** | $A = 143°$, | $B = 71°$, | $b = 40°$. |
| **14.** | $b = 57° 10'$, | $c = 138° 5'$, | $C = 135°$. |
| **15.** | $B = 160°$, | $C = 37°$, | $b = 10°$. |
| **16.** | $B = 119° 40'$, | $c = 27° 47'$, | $C = 90°$. |
| **17.** | $a = 30°$, | $b = 13° 56'$, | $A = 48° 48'$. |
| **18.** | $A = 110°$, | $B = 131° 20'$, | $b = 137° 20'$. |
| **19.** | $B = 35°$, | $C = 45°$, | $b = 45°$. |
| **20.** | $A = 85° 30'$, | $B = 20° 20'$, | $a = 27° 58'$. |

# CHAPTER XI

## THE TERRESTRIAL AND CELESTIAL TRIANGLES

**86. Distance and direction.** The problems of finding the distance between two points on the surface of the earth and the direction of one point from another are important applications of the solution of spherical triangles.

The **meridian** of a point on the earth is the great circle passing through the point and the two poles. A point is located by giving its latitude and longitude. The **latitude** is the arc of the meridian extending from the equator to the point. Latitude is measured north or south of the equator in degrees, minutes and seconds. The **longitude** of a point is the arc of the equator extending from the meridian through Greenwich to the meridian of the point. Longitude is measured east or west. The **co-latitude** of a point is the arc of the meridian from the point to the north pole. The **bearing** of a point $A$ from $B$ is the acute angle which the arc $BA$ makes with the meridian through $B$. The **distance** between two points means the distance measured along a great circle through the two points.

If the latitude and longitude of each of two places $A$ and $B$ are given, we can find the distance $AB$ and the bearing of one point from the other.

In Fig. 122, $NG$ is the meridian through Greenwich. The arcs, $GA'$ and $GB'$ are the longitudes of $A$ and $B$ respectively. $A'A$ and $B'B$ are the latitudes. The co-latitude of $A$ is $AN$, of $B$ it is $BN$. To find the distance $AB$, we have to solve the triangle $NAB$. If the latitude and longitude of $A$ and $B$ are given, we can easily find the angle $ANB$, the sides $AN$ and $BN$. Thus, we have given two sides and the included angle, to find $AB$ and either $A'AB$ or $ABB'$. In solving for $AB$, the result

is given in angular measure. This must be reduced to linear measure by noting that a nautical mile is one minute of arc of a great circle, or 6080 feet, or 1.1515 statute miles.

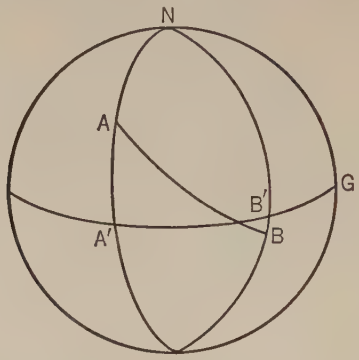


FIG. 122

EXAMPLE. Find the distance in statute miles from New York City (lat. 40° 43′ N, long. 74° 0′ W) to San Francisco (lat. 37° 48′ N, long. 122° 28′ W). Also find the bearings of one city from the other.

SOLUTION: Let $A$ (Fig. 123) be San Francisco, $B$ be New York. Then

$$GNB = \text{longitude of N. Y.} = 74° 0',$$

and

$$GNA = \text{longitude of S. F.} = 122° 28'.$$

Hence,

$$BNA = 122° 28' - 74° 0' = 48° 28' = N,$$
$$NA = 90° - 37° 48' = 52° 12' = b,$$
$$NB = 90° - 40° 43' = 49° 17' = a.$$

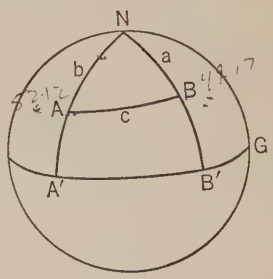


FIG. 123

Consequently, we have to solve the triangle $ANB$. This comes under Case III, using the formulas,

$$\tan \tfrac{1}{2}(B - A) = \frac{\sin \frac{1}{2}(b - a)}{\sin \frac{1}{2}(b + a)} \cot \tfrac{1}{2}N,$$

$$\tan \tfrac{1}{2}(B + A) = \frac{\cos \frac{1}{2}(b - a)}{\cos \frac{1}{2}(b + a)} \cot \tfrac{1}{2}N,$$

$$\tfrac{1}{2}(b + a) = 50° 44.5',$$
$$\tfrac{1}{2}(b - a) = 1° 57.5',$$
$$\tfrac{1}{2}N = 24° 14'.$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(b-a) &= 8.53368 \\ \log \cot \tfrac{1}{2}N &= \underline{0.34667} \\ \text{sum} &= \overline{8.88035} \\ \log \sin \tfrac{1}{2}(b+a) &= 9.88891 \\ \log \tan \tfrac{1}{2}(B-A) &= \overline{8.99144} \\ \tfrac{1}{2}(B-A) &= 5^\circ 36'. \end{aligned}$$

$$\begin{aligned} \log \cos \tfrac{1}{2}(b-a) &= 9.99975 \\ \log \cot \tfrac{1}{2}N &= \underline{0.34667} \\ \text{sum} &= \overline{0.34642} \\ \log \cos \tfrac{1}{2}(b+a) &= \underline{9.80128} \\ \log \tan \tfrac{1}{2}(B+A) &= \overline{0.54514} \\ \tfrac{1}{2}(B+A) &= 74^\circ 5.5' \\ \tfrac{1}{2}(B-A) &= \underline{5^\circ 36'} \\ B &= \overline{79^\circ 41.5'}. \\ A &= 68^\circ 29.5'. \end{aligned}$$

From $\dfrac{\sin c}{\sin N} = \dfrac{\sin a}{\sin A}$, we have

$$\begin{aligned} \log \sin a &= 9.87964 \\ \log \sin N &= \underline{9.87423} \\ \text{sum} &= \overline{9.75387} \\ \log \sin A &= \underline{9.96866} \\ \log \sin c &= \overline{9.78521} \\ c &= 37^\circ 35' = 2255', \text{ to the nearest minute,} \end{aligned}$$

giving

$$2255 \cdot 1.1515 = 2597 \text{ miles.}$$

The bearing of New York from San Francisco is given by the angle $A$ and is usually written N 68° 29.5′ E. The bearing of a line is usually the smaller angle which the line makes with the meridian. The bearing of San Francisco from New York is N 79° 41.5′ W. While San Francisco is south of New York, the angle $B$ is less than 90°, hence the part of a great circle connecting New York with San Francisco starts in a northerly direction.

## EXERCISES

Find the distance in statute miles between the following places. Find the bearings of one from the other.

**1.** New York, (lat. 40° 43′ N, long. 74° 0′ W) and Paris, (lat. 48°50′ N, long. 2° 20′ E).

**2.** New York and Chicago, (lat. 41° 50′ N, long. 87° 35′ W).

**3.** Chicago and San Francisco.

**4.** New York and Manila (14° 36′ N, 121° 5′ E).

**5.** New York and Valparaiso (33° 2′ S, 71° 42′ W).

**6.** New York and Cape of Good Hope (lat. 34° 22′ S, long. 18° 29′ E).

**87. The Celestial Sphere.** A person looking at the heavens finds himself apparently at the center of a great sphere, which is called the celestial sphere of the observer. Stars and other heavenly bodies may be regarded as located on this sphere in

much the same way that points are located on the surface of the earth. There are various imaginary points and circles that have received names. The **zenith,** $Z$, (Fig. 124) is the point on the celestial sphere directly overhead.

The **horizon,** $H'H$, of the observer is the great circle of which the zenith is the pole.

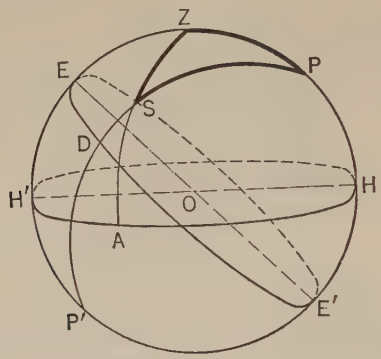


FIG. 124

The **celestial equator,** $EE'$, is the great circle made by the intersection of the plane of the earth's equator with the celestial sphere.

The **celestial poles,** $P$, $P'$, are the poles of the celestial equator

The **celestial meridian** of the observer is the great circle through the zenith and the celestial poles, the circle $ZPHP'$ in Fig. 124.

The **hour circle** $PSP'$ of a star $S$ is the great circle passing through the star and the poles. The hour circle is located by giving the angle between the celestial meridian and the hour circle. This angle $SPZ$ is called the hour angle and is measured in hours, minutes and seconds, westward from the celestial meridian. Since an apparent revolution of the celestial sphere takes place in 24 hours, we have 360 degrees equal to 24 hours or

$$1 \text{ hour} = 15 \text{ degrees},$$
$$1 \text{ minute of time} = 15 \text{ minutes of arc},$$
$$1 \text{ second of time} = 15 \text{ seconds of arc}.$$

The **declination** of a star, $DS$, is the distance in degrees from the celestial equator along the hour circle north or south to the star. The arc $AS$ is the **altitude** of the star, and the angle $PZS$ the **azimuth**. The arc $EZ$ is the latitude of the observer.

The triangle $SPZ$ is called the astronomical triangle. Five of the parts of the triangle are determined as follows:

$$SP = PD - SD = 90° \textit{ minus the declination},$$
$$= \textit{co-declination of the star}.$$
$$SZ = ZA - SA = 90° \textit{ minus the altitude},$$
$$= \textit{co-altitude of the star}.$$
$$PZ = EP - EZ = 90° \textit{ minus latitude of the observer},$$
$$= \textit{co-latitude}.$$
$$SPZ = \text{hour angle of the star}.$$
$$SZP = \text{azimuth of the star}.$$

Given three of these parts the triangle may be solved.

If the altitude and the declination of the sun are given, and the latitude of the observer is known, the three sides of $SPZ$ are known and the hour angle $SPZ$ may be found in degrees. If this be reduced to hours, minutes and seconds, it will be the time before noon or after noon, depending upon whether the observation was made in the morning or afternoon. If at the time of observation the time at Greenwich is known, the difference between the two times will give the longitude of the observer.

EXAMPLE 1. An observation taken at Urbana (lat. 40° 6′ N) at 2:32.65 P.M. Greenwich time gives the altitude of the sun as 42°. Its declination was 19° N. What was the local time at Urbana at the time of the observation? What is the longitude of Urbana?

SOLUTION: Referring to Fig. 125, we have three sides of the triangle given

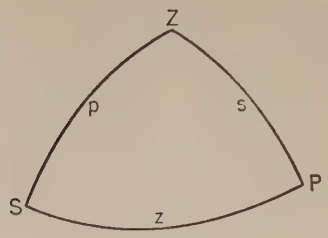


FIG. 125

$$p = 90° - 42° = 48°,$$
$$s = 90° - 40°6' = 49°54',$$
$$z = 90° - 19° = 71°.$$

Let
$$s' = \frac{s + p + z}{2} = 84°27'.$$

We find $P$ from $\tan \frac{1}{2}P = \dfrac{k}{\sin(s' - p)}$, Case I, page 168.

$$\begin{aligned}
s' - z &= 13°27', & \log \sin (s' - z) &= 9.35660 \\
s' - p &= 36°27', & \log \sin (s' - p) &= 9.77387 \\
s' - s &= 34°33', & \log \sin (s' - s) &= 9.75368 \\
& & \text{sum} &= \overline{8.88415} \\
& & \log \sin s' &= 9.99796 \\
& & \log k^2 &= \overline{8.88619} \\
& & \log k &= 9.44310 \\
& & \log \sin (s' - p) &= 9.77387 \\
& & \log \tan \tfrac{1}{2}P &= \overline{9.66923} \\
& & \tfrac{1}{2}P &= 25°1.7' \\
& & P &= 50°3.4' \\
& & &= 3 \text{ hours } 20.2 \text{ minutes.}
\end{aligned}$$

Hence, the time of observation is 8:39.8 A.M.

The difference in time between Greenwich and Urbana is 3 hours 20.2 minutes plus 2 hours 32.65 minutes equal 5 hours 52.85 minutes and the longitude is 88° 13′ W to the nearest minute.

EXAMPLE 2. At what time will the sun set for Chicago (lat. 41° 50′ N) if the sun's declination is 21° N.

SOLUTION: At sunrise or sunset the co-altitude of the sun is 90°. Hence, we have

$$p = 90°, \quad s = 48°10', \quad z = 69°.$$

The triangle $SZP$ is quadrantal. The angles of the polar triangle are

$$P' = 90°, \quad S' = 131°50', \quad Z' = 111°.$$

We have then to solve the polar triangle for $p'$. Using formula (8) Art. 74 we have $\cos p' = \cot S' \cot Z'$, from which we find

$$p' = 69° 54.1'$$

or

$$\begin{aligned} P &= 180° - 69° 54.1' \\ &= 110° 5.9' \\ &= 7 \text{ hours } 20.4 \text{ minutes.} \end{aligned}$$

Therefore, the sun sets at 20.4 minutes after 7.

## PROBLEMS

**1.** An observer in New York (lat. 40° 43′ N) finds the sun's altitude to be 30° and its declination 20°. What was the morning time of observation?

**2.** Observation of the sun taken during the afternoon in Chicago (lat. 41° 50′ N) gives the altitude 31° 35′ and the declination 11° N. What was the time of observation?

**3.** At 7 P.M. Greenwich time, an observer on a vessel finds the altitude of the sun to be 34° 48′ and its declination 10° 56′ S. The latitude of the vessel is 27° 11′ N. Find the afternoon time of the observation and the longitude of the vessel.

**4.** At Washington D. C. (lat. 38° 55′ N) a morning observation gives the sun's altitude as 51° 30′ and declination 19° 25′. A chronometer giving Greenwich time reads 2 hours, 34 minutes. What is the local time and the longitude of Washington?

**5.** Find the time of sunrise at Urbana, Ill. (lat. 40° 6′ N) when the sun's declination is 19° N.

**6.** Find the time of sunset at New York (lat. 40° 43 N) when the sun's declination is 23° 30′.

**7.** At 8:40 A.M. local time, the sun's altitude is 42°, and its declination 19°. Find the latitude of the place of observation.

**8.** Refraction makes the sun on the horizon appear 50′ higher than it really is. Hence at sunrise or sunset the angle $ZS$ is 90° 50′. What effect does this correction have on the result of Example 2, p. 182.

# ANSWERS TO ODD NUMBERED EXERCISES AND PROBLEMS

## Art. 1. Pages 3, 4.

**1.** $\sin 45° = 0.71$, $\cos 45° = 0.71$, $\tan 45° = 1$,
$\cot 45° = 1$, $\sec 45° = 1.4$, $\csc 45° = 1.4$.

**3.** $\sin B = \frac{4}{5}$, $\cos B = \frac{3}{5}$, $\tan B = \frac{4}{3}$, $\cot B = \frac{3}{4}$,
$\sec B = \frac{5}{3}$, $\csc B = \frac{5}{4}$.

**5.** Sine, cosine; secant, cosecant; tangent, cotangent.

**7.** $\sin 15° = 0.26$, $\cos 15° = 0.97$, $\tan 15° = 0.27$,
$\cot 15° = 3.73$, $\sec 15° = 1.03$, $\csc 15° = 3.86$.

**19.** $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\cot A = \frac{4}{3}$, $\sec A = \frac{5}{4}$, $\csc A = \frac{5}{3}$.

## Art. 3. Pages 8, 9.

**1.** $\cos A = \frac{1}{4}\sqrt{7}$, $\tan A = \frac{3}{7}\sqrt{7}$, $\cot A = \frac{1}{3}\sqrt{7}$, $\sec A = \frac{4}{7}\sqrt{7}$, $\csc A = \frac{4}{3}$.

**3.** $\sin A = \frac{1}{4}\sqrt{15}$, $\tan A = \sqrt{15}$, $\cot A = \frac{1}{15}\sqrt{15}$, $\sec A = 4$,
$\csc A = \frac{4}{15}\sqrt{15}$.

**5.** $\sin A = \frac{2}{5}\sqrt{5}$, $\cos A = \frac{1}{5}\sqrt{5}$, $\cot A = \frac{1}{2}$, $\sec A = \sqrt{5}$, $\csc A = \frac{1}{2}\sqrt{5}$.

**7.** $\sin A = \frac{2}{13}\sqrt{13}$, $\cos A = \frac{3}{13}\sqrt{13}$, $\tan A = \frac{2}{3}$, $\sec A = \frac{1}{3}\sqrt{13}$,
$\csc A = \frac{1}{2}\sqrt{13}$.

**9.** $\sin A = \frac{2}{3}\sqrt{2}$, $\cos A = \frac{1}{3}$, $\tan A = 2\sqrt{2}$, $\cot A = \frac{1}{4}\sqrt{2}$, $\csc A = \frac{3}{4}\sqrt{2}$

**11.** $\sin A = \frac{1}{4}$, $\cos A = \frac{1}{4}\sqrt{15}$, $\tan A = \frac{1}{15}\sqrt{15}$, $\cot A = \sqrt{15}$,
$\sec A = \frac{4}{15}\sqrt{15}$.

**13.** $\cos x = \sqrt{1 - a^2}$, $\tan x = \dfrac{a}{\sqrt{1 - a^2}}$, $\cot x = \dfrac{\sqrt{1 - a^2}}{a}$,

$\sec x = \dfrac{1}{\sqrt{1 - a^2}}$, $\csc x = \dfrac{1}{a}$.

**15.** $a = \frac{1}{3}\sqrt{3}$.

**17.** $c = \frac{2}{3}\sqrt{3}$.

**19.** $a = \sqrt{2}$.

**21.** $c = \frac{1}{2}\sqrt{5}$.

**23.** $a = \sqrt{3}$.

**25.** $b = 2\sqrt{2}$.

### Art. 4. Page 11.

**1.** cos 75°, sin 46°, cot 33°, tan 72° 35′, csc 4° 15′, sec 45°.

**5.** $x = 30°$.

**7.** $x = 12\frac{6}{7}° = 12° 51.4'$.

**9.** $A = 30°$.

**11.** $A = 7° 30'$.

### Art. 5. Page 14.

**1.** sin 33° = 0.545, cos 33° = 0.839, tan 33° = 0.649,
cot 33° = 1.540, sec 33° = 1.192, csc 33° = 1.836.

**3.** sin 76° = 0.970, cos 76° = 0.242, tan 76° = 4.011,
cot 76° = 0.249, sec 76° = 4.134, csc 76° = 1.031.

| | | |
|---|---|---|
| **5.** 0.458, 0.458. | **15.** 0.078. | **25.** 77.6°. |
| **7.** 0.271, 0.271. | **17.** 11.270. | **27.** 12.7°. |
| **9.** 2.936, 2.936. | **19.** 1.177. | **29.** 10.8°. |
| **11.** 0.698. | **21.** 2.466. | **31.** 25.8°. |
| **13.** 0.945. | **23.** 30.7°. | **33.** 12.2°. |

### Art. 7. Pages 16–23.

The following answers were found by the use of the three-place table on page 000. In some cases the third significant figure will vary with different methods of solving the problem.

| | | | |
|---|---|---|---|
| **1.** $c = 5$, | $A = 36.9°$, | $B = 53.1°$, | area = 6. |
| **3.** $b = 42.6$, | $A = 46.6°$, | $B = 43.4°$, | area = 959. |
| **5.** $b = 238$, | $c = 263$, | $B = 64.8°$, | area = 13330. |
| **7.** $a = 0.022$, | $c = 0.132$, | $B = 80.5°$, | area = 0.0014. |
| **9.** $b = 17.9$, | $c = 40.2$, | $A = 63.6°$, | area = 323. |
| **11.** $a = 38.5$, | $c = 39.5$, | $A = 77.4°$, | area = 166. |
| **13.** $a = 112$, | $A = 48.2°$, | $B = 41.8°$, | area = 5600. |
| **15.** $a = 17.9$, | $b = 20.2$, | $B = 48.4°$, | area = 181. |
| **17.** $a = 6.96$, | $b = 0.805$, | $A = 83.4°$, | area = 2.8. |

| | | |
|---|---|---|
| **19.** 19.6 feet. | **23.** 33.7°. | **27.** 72 feet. |
| **21.** 19.4°. | **25.** 181 feet. | **29.** 426 feet. |

**31.** 7.81 miles; between S. and S. by W., or 6.1° W. of S.

**33.** 8.02 miles; 14.4 miles. **37.** 5 miles per hour.

**35.** 9.83 inches. **39.** 13.9, 5.75 miles per hour.

**41.** 24.6 pounds.

**43.** 2280 feet.

**45.** 552300 sq. feet.

**47.** 70.5°.

**51.** 3.87 miles.

**53.** Negligible.

**55.** $AB = 343$ feet, $AC = 349$ feet, $C = 70°$, area $= 36050$.

**57.** $AB = 55.3$, $B = 23.4°$, $C = 125.6°$, area $= 384$.

**59.** $CB = 168$, $B = 79.9°$, $C = 58.1°$, area $= 17594$.

### Art. 8. Page 28.

**21.** $\dfrac{\sin A}{\sqrt{1 - \sin^2 A}}$.

**23.** $\dfrac{\cot \theta}{\sqrt{1 + \cot^2 \theta}}$.

**25.** $\sqrt{1 - \cos^2 x}$, $\dfrac{\tan x}{\sqrt{1 + \tan^2 x}}$, $\dfrac{1}{\sqrt{1 + \cot^2 x}}$, $\dfrac{\sqrt{\sec^2 x - 1}}{\sec x}$, $\dfrac{1}{\csc x}$.

### Art. 9. Pages 31, 32.

**1.** Second, first, third, second, fourth, second, fourth, second, first.

**3.** 468°, 828°, 1188°, etc.; − 252°, − 612°, − 972°, etc.

**5.** 60°, 420° and − 300°; 870° and 150°; 330°, − 30° and 690°.

**7.** 1080°; 1560°.

**9.** 9600°; 2880°.

**11.** Second.

**13.** First; first.

**15.** Second.

**17.** First.

### Art. 11. Pages 35–37.

**3.** $\sin 315° = -\frac{1}{2}\sqrt{2}$, $\cos 315° = \frac{1}{2}\sqrt{2}$, $\tan 315° = -1$,
$\cot 315° = -1$, $\sec 315° = \sqrt{2}$, $\csc 315° = -\sqrt{2}$.

**5.**

| | | |
|---|---|---|
| $\sin 120° = \frac{1}{2}\sqrt{3}$ | $\sin 210° = -\frac{1}{2}$ | $\sin 240° = -\frac{1}{2}\sqrt{3}$ |
| $\cos 120° = -\frac{1}{2}$ | $\cos 210° = -\frac{1}{2}\sqrt{3}$ | $\cos 240° = -\frac{1}{2}$ |
| $\tan 120° = -\sqrt{3}$ | $\tan 210° = \frac{1}{3}\sqrt{3}$ | $\tan 240° = \sqrt{3}$ |
| $\cot 120° = -\frac{1}{3}\sqrt{3}$ | $\cot 210° = \sqrt{3}$ | $\cot 240° = \frac{1}{3}\sqrt{3}$ |
| $\sec 120° = -2$ | $\sec 210° = -\frac{2}{3}\sqrt{3}$ | $\sec 240° = -2$ |
| $\csc 120° = \frac{2}{3}\sqrt{3}$ | $\csc 210° = -2$ | $\csc 240° = -\frac{2}{3}\sqrt{3}$ |

| | |
|---|---|
| $\sin 300° = -\frac{1}{2}\sqrt{3}$ | $\sin 330° = -\frac{1}{2}$ |
| $\cos 300° = \frac{1}{2}$ | $\cos 330° = \frac{1}{2}\sqrt{3}$ |
| $\tan 300° = -\sqrt{3}$ | $\tan 330° = -\frac{1}{3}\sqrt{3}$ |
| $\cot 300° = -\frac{1}{3}\sqrt{3}$ | $\cot 330° = -\sqrt{3}$ |
| $\sec 300° = 2$ | $\sec 330° = \frac{2}{3}\sqrt{3}$ |
| $\csc 300° = -\frac{2}{3}\sqrt{3}$ | $\csc 330° = -2$. |

**7.** 2.

**9.** $\text{Sin } A = -\frac{2}{3}\sqrt{2}$, $\tan A = 2\sqrt{2}$, $\cot A = \frac{1}{4}\sqrt{2}$, $\sec A = -3$,
$\csc A = -\frac{3}{4}\sqrt{2}$.

**11.** Third or fourth.

**13.** First or third.

**15.** Second.

**17.** First.

**19.** Sin $B = \pm \frac{24}{25}$, tan $B = \mp \frac{24}{7}$, cot $B = \mp \frac{7}{24}$, sec $B = -\frac{25}{7}$, csc $B = \pm \frac{25}{24}$.

### Art. 12. Page 38.

**1.** 0.

**3.** 2.

**5.** $-1$.

### Art. 13. Page 41.

**1.** 90°, 270°, 0°, 180°.

**3.** Two; four.

### Art. 14. Pages 43, 44.

**1.** $\frac{5\pi}{12}$ radians.

**3.** $\frac{11\pi}{12}$ radians.

**5.** $\frac{7\pi}{4}$ radians.

**7.** $0.18\pi = 0.565$ radians.

**9.** $-0.183$ radians.

**11.** 120°.

**13.** 225°.

**15.** 343° 46′ 29″.

**17.** $\frac{\pi}{3}$ radians; 60°.

**19.** 3.5 radians; 200.5°.

**21.** 0.3915; 0.9060; 31.416.

### Art. 15. Page 47.

**1.** sin 15° or cos 75°.

**3.** cos 74° or sin 16°.

**5.** cot 35° or tan 55°.

**7.** − tan 35° or − cot 55°.

**9.** cos 55° or sin 35°.

**11.** csc 80° or sec 10°.

**13.** sin 40°.

**15.** cot 25°.

**17.** − 0.3907.

### Art. 21. Pages 57, 58.

**1.** (1, 0), (3, 0), (− 1, 0), (− 4, 0), (−5, 0), (2, 0); (0, 3), (0,1), (0, 2), (0, − 2), (0, − 6).

**5.** $\frac{a}{2}\sqrt{2}$.

**7.** 5.69, 3.79.

### Art. 25. Pages 64, 65.

**3.** $\frac{1}{4}(\sqrt{6}+\sqrt{2})$, $\frac{1}{4}(\sqrt{6}-\sqrt{2})$, $2+\sqrt{3}$.

**5.** $\frac{1}{2}\cos x - \frac{1}{2}\sqrt{3}\sin x$.

**7.** $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$, $\cos 15° = \frac{1}{4}(\sqrt{6} + \sqrt{2})$, $\tan 15° = 2 - \sqrt{3}$, $\cot 15° = 2 + \sqrt{3}$, $\sec 15° = \sqrt{6} - \sqrt{2}$, $\csc 15° = \sqrt{6} + \sqrt{2}$.

**9.** $\frac{56}{65}, \frac{33}{65}, \frac{56}{33}$.

**19.** $\cos A$, $-\sin A$.

**27.** $\tan B = \frac{1}{5}$.

### Art. 27. Pages 67, 68.

**7.** $4\sqrt{1 - \sin^2 x}\,(\sin x - 2\sin^3 x)$.

**21.** $\frac{24}{7}$.

**23.** $\sin 2x = -\frac{120}{169}$, $\cos 2x = \frac{119}{169}$, $\tan 2x = -\frac{120}{119}$.

$\cot 2x = -\frac{119}{120}$, $\sec 2x = \frac{169}{119}$, $\csc 2x = -\frac{169}{120}$.

**25.** $\sin \frac{x}{2} = \frac{1}{10}\sqrt{10}$, $\cos \frac{x}{2} = \frac{3}{10}\sqrt{10}$, $\tan \frac{x}{2} = \frac{1}{3}$.

### Art. 28. Page 70.

**1.** $2 \sin 30° \cos 20°$. **3.** $-2 \sin 50° \sin 30°$. **5.** $2 \cos 3x \cos x$.

### Art. 32. Pages 74, 75.

**1.** $16.25 < 16.3 < 16.35$; $16.295 < 16.30 < 16.305$; $16.2995 < 16.300 < 16.3005$.

**3.** $0.00002727 \cdots$, $0.0001$; $0.0002727 \cdots$, $0.0001$; $0.2727 \cdots$, $0.0001$.

**5.** 3.14159265, 3.1415927, 3.141593, 3.14159, 3.1416, 3.142, 3.14, 3.1.

**7.** Between 3.113625 and 3.149025.

**11.** Between $53.121 \cdots$ and $53.262 \cdots$.

**13.** 503 centimeters.

**15.** $0.005$, $0.003300+$; $0.005$, $0.00001634+$; $0.005$, $0.00005469-$; $0.005$, $0.04348-$.

### Art. 33. Page 76.

**1.** 2; 3; 7; $-2$; 0.

**3.** $2^4 = 16$; $10^2 = 100$; $3^2 = 9$; $9^{-\frac{1}{2}} = \frac{1}{3}$; $10^0 = 1$.

**5.** Any power of 1 is 1.

### Art. 34. Page 78.

**1.** 0.60206; 0.95424; $-0.12494$ or $9.87506 - 10$; 1.44716; 1.50515.

### Art. 38. Page 84.

**1.** 3.51135. **5.** 3.54518. **9.** 9.52866 − 10.
**3.** 0.53224. **7.** 5.48926.

### Art. 39. Page 85.

**1.** 2.52288. **5.** 8.53866 − 10. **9.** 6.52172.
**3.** 4.50755. **7.** 3.50694.

### Art. 40. Page 86.

**1.** 30.12 **5.** 0.3376. **9.** 0.00030206.
**3.** 326.36. **7.** 0.034112.

### Art. 41. Pages 87, 88.

**1.** 10.09. **9.** 90.46. **15.** 1.638.
**3.** 2.866. **11.** 39.20. **17.** 2.8845.
**5.** 7.368 **13.** 1.210. **19.** 0.2786.
**7.** 10390000.

### Art. 44. Page 91.

**1.** $B = 88°11'$, $a = 27.03$, $c = 57.97$.
**3.** $A = 42°55'$, $a = 29.29$, $b = 41.09$.
**5.** $a = 2740$, $b = 7272$, $C = 61°41'$.
**7.** $AC = 566.7$ feet.

### Art. 45. Pages 94, 95.

**1.** $b = 1065$, $B = 61°32'$, $C = 90°$.
**3.** No solution.
**5.** $A = 156°40.8'$, $C = 6°24.2'$, $a = 139.2$.
**7.** $A = 62°38.9'$, $B = 66°12.1'$, $b = 83.63$.
$A' = 117°21.1'$, $B' = 11°29.9'$, $b' = 18.22$.
**9.** $A = 30°15.8'$, $B = 49°44.2'$, $a = 555.8$.
**11.** No solution. **13.** 10872.

### Art. 47. Pages 96, 97.

**1.** $a = 17.45$, $B = 21°29.6'$, $C = 120°23.4'$.
**3.** $A = 5°53.1'$, $B = 66°44.9'$, $c = 9710$.
**5.** $A = 16°11.1,'$ $C = 88°48.9'$, $b = 429.4$.
**7.** $AB = 6123$ feet.

### Art. 48. Pages 98, 99.

**1.** $c = 7.96$. **3.** $b = 47.5$. **5.** $7\frac{3}{4}$ miles.

### Art. 49. Pages 101, 102.

**1.** 6953. **3.** 639. **5.** 264700.

### Art. 50. Pages 103, 104.

**1.** $A = 51°53.2'$, $B = 59°30.8'$, $C = 68°36'$.
**3.** $A = 8°20.2'$, $B = 33°41.7'$, $C = 137°58.1'$.
**5.** $28°57'$, $46°34'$, $104°29'$; 2.905.

### Art. 51. Pages 106–111.

*Problems*

**1.** 6246 feet. **3.** 401 feet.
**5.** Two solutions, 44.9 feet, or 329.2 feet.
**7.** 1460 feet. **31.** 33.1 feet.
**11.** 55.36 feet. **33.** 6283 feet.
**13.** $78°27.8'$, $44°24.9'$, $57°7.3'$. **35.** $41°48.6'$, $138°11.4'$.
**15.** 21874 feet. **37.** 22.98 feet.
**17.** 16 feet, 2 inches. **39.** 0.18 square inches.
**19.** 6.946, 13.054 feet. **41.** 52.24 square feet.
**21.** 636.8 feet, $160°44'$. **43.** 65.72 feet.
**25.** 7851 square feet. **45.** 1194 feet.
**27.** 2062 feet

### Art. 56. Pages 117, 118.

**17.** $1 + \sqrt{3}i$. **21.** $0 + 4i$.
**19.** $\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{2}i$. **23.** $0 - 5i$.
**27.** $x - 2y = 2$, $2x + 3y = -1$.
**29.** $x + 3y - 3 = 0$, $x - 3y + 1 = 0$.

### Art. 57. Page 119.

**1.** $3 + 5i$. **5.** $3 + 3i$. **9.** $3 + 3i$.
**3.** $3 + i$. **7.** $1 + 3i$.

### Art. 58. Page 121.

**3.** $4 + 4\sqrt{3}i$. **7.** $-2\sqrt{3} + 2 - (2\sqrt{3} + 2)i$.
**5.** $-1 + i$. **9.** $-2 + 2i$.

### Art. 59. Page 123.

**3.** $\dfrac{(1+\sqrt{3})+(1-\sqrt{3})i}{8}$.

**5.** $(3+\sqrt{3})+(3-\sqrt{3})i$.

**7.** $\dfrac{3-\sqrt{3}i}{4}$.

**9.** $\dfrac{\sqrt{3}-i}{4}$.

### Art. 60. Pages 124, 125.

**3.** $24\sqrt{3}i$.

**5.** $-8-8\sqrt{3}i$.

### Art. 61. Page 126.

**3.** $2(\cos 10^\circ + i \sin 10^\circ)$, $2(\cos 100^\circ + i \sin 100^\circ)$, $2(\cos 190^\circ + i \sin 190^\circ)$, $2(\cos 280^\circ + i \sin 280^\circ)$.

**5.** $\sqrt{2}(\cos 45^\circ + i \sin 45^\circ) = 1 + i$, $\sqrt{2}(\cos 165^\circ + i \sin 165^\circ)$, $\sqrt{2}(\cos 285^\circ + i \sin 285^\circ)$.

**7.** $\sqrt{2}(\cos 75^\circ + i \sin 75^\circ)$, $\sqrt{2}(\cos 195^\circ + i \sin 195^\circ)$, $\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$.

**9.** $\pm 1$, $\pm i$.

**11.** $2(\cos 22.5^\circ + i \sin 22.5^\circ)$ with three other similar expressions with $112.5^\circ$, $202.5^\circ$, $292.5^\circ$.

### Art. 62. Page 129.

**1.** No; there is no definite law in the sequence.

**5.** No.

**7.** 0.996+.

### Art. 66. Pages 136–138.

*Miscellaneous Exercises*

**9.** $-\sqrt{3}+1$.

**11.** $37^\circ 30'$; 3.815.

**13.** $2+2i$.

**17.** $3$, $-\frac{3}{2}+\frac{3}{2}\sqrt{3}i$, $-\frac{3}{2}-\frac{3}{2}\sqrt{3}i$.

**27.** $E = E_1 \cosh\left\{x\sqrt{\dfrac{r}{r_1}}\right\} + \dfrac{rr_1}{\left(\dfrac{r}{r_1}\right)^{\frac{1}{3}}} \sinh\left\{x\left(\dfrac{r}{r_1}\right)^{\frac{1}{3}}\right\}$.

### Art. 70. Pages 143, 144.

**1.** $90^\circ$, $450^\circ$, $-270^\circ$, $-630^\circ$.

**3.** $\pm 90^\circ$, $\pm 270^\circ$.

**5.** $\pm 60^\circ$, $300^\circ$, $420^\circ$.

**7.** $45^\circ$, $135^\circ$, $-225^\circ$, $-315^\circ$.

**9.** $\pm 60^\circ$, $\pm 300^\circ$.

**11.** $17^\circ$.

**13.** $27° 40'$. **17.** $\frac{1}{2}\sqrt{2}$. **21.** $\sqrt{2}$. **25.** $\sqrt{3}$.

**15.** $\frac{1}{2}$. **19.** $\frac{1}{2}\sqrt{3}$. **23.** $\frac{1}{2}\sqrt{3}$. **27.** $\frac{1}{2}$.

### Art. 71. Page 147.

**1.** $56° 18.6'$, $236° 18.6'$.

**3.** $0°$, $60°$.

**5.** $90°$, $340° 31.7'$.

**7.** $45°$, $109° 31.7'$, $250° 31.7'$.

**9.** No solution.

**11.** $30°$, $150°$.

**13.** $45°$, $63° 26.1'$, $225°$, $243° 26.1'$.

**15.** $0°$, $120°$, $180°$, $240°$.

**17.** $30°$, $210°$.

**19.** $x = 45°$, $y = 15°$.

**21.** $r = 5$, $\theta = 36° 52.2'$.

**23.** $x = 45°$, $y = \frac{1}{2}\sqrt{2}$.

**25.** 0.26795.

**27.** 0, $-3$.

**29.** 0.25882.

### Art. 75. Page 153.

**1.** The triangle whose angles are right angles and each of whose sides measure $90°$.

### Art. 76. Pages 156, 157.

**11.** $A = 62° 54.7'$, $b = 68° 36.2'$, $B = 71° 2.8'$.

**13.** $a = 27° 2.7'$, $A = 34° 46.7'$, $c = 52° 51.3'$;
$a' = 152° 57.3'$, $A' = 145° 13.3'$, $c' = 127° 8.7'$.

**15.** $A = 47° 58.7'$, $B = 73° 15.4'$, $c = 74° 16.3'$.

**17.** No solution.

**19.** $a = 90°$, $A = 90°$, $c = 90°$.

**21.** $b = 8° 30.5'$, $B = 13° 6.1'$, $c = 40° 44.8'$;
$b' = 171° 29.5'$, $B' = 166° 53.9'$, $c' = 139° 15.2'$.

**23.** $A = 154° 4.2'$, $B = 101° 17.1'$, $c = 65° 46.2'$.

**25.** No solution.

**27.** $a = 165° 23.1'$, $b = 156° 6.4'$, $A = 147° 13.5'$.

**29.** $B = 90°$, $A = a$ where $a$ may have any value between $0°$ and $180°$.

### Art. 78. Page 159.

**1.** $A = B = 77° 3.3'$, $c = 51° 52.1'$.

**3.** $a = 105° 14.4'$, $A = 115° 25.2'$, $B = 61° 38'$.

**5.** $a = b = 70° 43.4'$, $A = B = 31° 52.3'$;
$a' = b' = 109° 16.6'$, $A' = B' = 148° 7.7'$.

**7.** $A = 146° 38'$, $B = 107° 46.5'$, $C = 104° 46.3'$.

**9.** $a = 90°$, $A = 90°$, $C = 90°$.

**11.** $a = 54° 44.4'$, $b = 113° 55.7'$, $C = 71° 43'$.

### Art. 84. Case I. Page 169.

**1.** $A = 127° 45.6'$, $B = 47° 40.6'$, $C = 70° 53.5'$.
**3.** $A = 115° 3.4'$, $B = 120° 11.2'$, $C = 127° 11.4'$.

### Art. 84. Case II. Page 170.

**1.** $a = 90° 11.1'$, $b = 61° 34'$, $c = 72° 35.4'$.
**3.** No solution.

### Art. 84. Case III. Page 171.

**1.** $a = 72° 48.8'$, $B = 68° 52.3'$, $C = 120° 9.3'$.
**3.** $b = 26° 35.8'$, $A = 141° 7.1'$, $C = 54° 11.1'$.

### Art. 84. Case IV. Page 172.

**1.** $a = 28° 39.7'$, $b = 20° 51.2'$, $C = 89° 5'$.
**3.** $b = 97° 7.6'$, $c = 117° 34.7'$, $A = 64° 51.9'$.

### Art. 85. Page 175.

**1.** One solution. **3.** No solution. **5.** One solution.
**7.** $a = 89° 51.7'$, $A = 114° 55'$, $C = 55° 56.9'$;
$a' = 29° 3.9'$, $A' = 26° 8.6'$, $C' = 124° 3.1'$.
**9.** $a = 22° 35.9'$, $A = 47° 50.5'$, $b = 19° 29.9'$;
**11.** 19.54 or 55.62. **13.** $E = 59° 19.6'$.

### Miscellaneous Exercises, Page 176.

**1.** $A = 68° 33.7'$, $B = 130° 48.3'$, $C = 94° 0.8'$.
**3.** $a = 57° 56.7'$, $b = 137° 22.3'$, $C = 94° 47.2'$.
**5.** $a = 145° 11.6'$, $b = 121° 55.3'$, $c = 78° 38.9'$.
**7.** $a = 57° 57.1'$, $B = 131° 17.4'$, $C = 94° 48.6'$.
**9.** $A = 63° 59.8'$, $C = 84° 45.8'$, $c = 110° 1.4'$;
$A' = 116° 0.2'$, $C' = 151° 20.4'$, $c' = 153° 5.6'$.
**11.** $a = 61° 25.6'$, $b = 28° 59'$, $B = 32° 14.4'$.
**13.** $a = 155° 51'$, $c = 137° 52'$, $C = 97° 19.2'$.
**15.** No solution.
**17.** $B = 21° 14.7'$, $c = 37° 34'$, $C = 113° 26.6'$.
**19.** $a = 90° 48.4'$, $A = 125° 48.4'$, $c = 60° 39.7'$;
$a' = 159° 43.2'$, $A' = 163° 40.4'$, $c' = 119° 20.3'$.

### Art. 86. Page 179.

**1.** 3626 miles. Bearing of Paris from New York N 53° 44.5′ E; New York from Paris N 68° 12′ W.

**3.** 1859 miles. Chicago from S. F. N 70° 20′ E; S. F. from Chicago N 86° 55 W.

**5.** 5107 miles. Valparaiso from New York S 2° E; New York from Valparaiso N 1° 49′ W.

### Art. 87. Page 183.

**1.** 31.5 minutes after 7.

**3.** 44.3 minutes after 3; longitude = 48° 54.5′.

**5.** 53 minutes after 4. **7.** 40° 6′.

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## Table I. Squares of Numbers

**1. Finding the square of a number.** This table gives directly the approximate values of the squares of the numbers 1, 1.01, 1.02 . . . 9.99. Indirectly, the square of any given number may be found more or less approximately from the table. The use of the table is illustrated by the following examples.

Example 1. Find $(3.61)^2$.

In the column headed **N**, page 2, find 3.6, then across to the column headed 1 where we find the number 13.03. Hence, $(3.61)^2 = 13.03$ approximately.

Example 2. Find $(36.1)^2$.

We may write $36.1 = 10 \times 3.61$,

$$(36.1)^2 = 10^2 \times (3.61)^2$$

Hence, from Example 1, we find

$$(36.1)^2 = 100 \times 13.03 = 1303 \text{ approximately.}$$

Example 3. Find $(4.773)^2$.

In the column headed **N**, page 2, find 4.7, then across the page to the column headed 7 and we find $(4.77)^2 = 22.75$. We also find from the table $(4.78)^2 = 22.85$, there being a difference of .10 between 22.75 and 22.85. Now we have to interpolate. Since 4.773 is three-tenths of the way from 4.77 to 4.78 we should find $(4.773)^2$ approximately three-tenths of the way from 22.75 to 22.85, that is $22.75 + .3 \times .10 = 22.78$.

Example 4. Find $(8676)^2$.

We may write, $8676 = 1000 \times 8.676$

$$(8676)^2 = 1000000 \times (8.676)^2.$$

From the table we find after interpolation

$$(8.676)^2 = 75.27$$

Hence, $(8676)^2 = 75270000$ approximately.

**2. Extraction of square roots.** Table 1 may also be considered as a table of square roots if the table be read backwards. The square roots of the numbers in the body of the table are the numbers whose first two digits are in the column **N** and the third digit a number at the top of a column. For example, we

note that $(2.76)^2 = 7.618$. Hence $\sqrt{7.618} = 2.76$. The numbers in the body of the table run from 1 to practically 100, so we may find the square root of any number between 1 and 100 either directly from the table or after interpolation.

EXAMPLE 1. Find $\sqrt{65.74}$.

We find directly from the table, $\sqrt{65.61} = 8.10$ and $\sqrt{65.77} = 8.11$. The number 65.74 lies $\frac{13}{16}$ of the way from 65.61 to 65.77, hence $\sqrt{65.74}$ lies $\frac{13}{16}$ of the way from 8.10 to 8.11 or

$$\sqrt{65.74} = 8.108 \text{ approximately.}$$

From this table it is easy to find the approximate square root of a number which does not lie between 1 and 100. Any number may be expressed as a product of an even power of 10 and some number between 1 and 100.
For example,

$$\begin{aligned} 6574 &= 10^2 \times 65.74, \\ 657.4 &= 10^2 \times 6.574, \\ .6574 &= 10^{-2} \times 65.74 = .01 \times 65.74, \\ .06574 &= 10^{-2} \times 6.574 = .01 \times 6.574. \end{aligned}$$

EXAMPLE 2. Find $\sqrt{6574}$.

$$6574 = 10^2 \times 65.74, \text{ hence } \sqrt{6574} = 10 \times \sqrt{65.74}$$
$$= 10 \times 8.108 = 81.08, \text{ using the result of Example 1.}$$

EXAMPLE 3. Find $\sqrt{657.4}$.

$$\sqrt{657.4} = 10 \times \sqrt{6.574} = 10 \times 2.564 = 25.64.$$

EXAMPLE 4. Find $\sqrt{.6574}$.

$$\sqrt{.6574} = 10^{-1} \times \sqrt{65.74} = .1 \times 8.108 = .8108.$$

EXAMPLE 5. Find $\sqrt{.06574}$.

$$\sqrt{.06574} = 10^{-1} \times \sqrt{6.574} = .1 \times 2.564 = .2564.$$

EXAMPLE 6. Find $\sqrt{8569300}$.

$$8569300 = 10^6 \times 8.5693.$$
$$\sqrt{8569300} = 10^3 \times \sqrt{8.5693} = 1000 \times 2.927 = 2927 \text{ approximately.}$$

## Table II. Four-Place Values of Functions and of Radians

**3. Use of the table.** This table, (pages 4–8) gives the natural values of the six trigonometric functions for the angles 0° to 90° at intervals of 10′. In the first column we find the angles 0° to 45° reading downwards. In the last column are the angles 45° to 90° reading upwards. For angles between 0° and 45° the headings at the tops of the columns should be used. For angles between 45° and 90° the headings are at the bottom of the columns.

The number of radians in an angle given in degrees and minutes may also be found from the table. The use of the table is illustrated in the following examples.

Example 1. Find sin 9° 20′.

This number is given directly in the table where on page 5 in the column headed "Sin" at the top of the page we find for 9° 20′ the value .1622. Hence sin 9° 20′ = .1622.

Example 2. Find sin 66° 50′.

This number is found in the table on page 6. Locate 66° 50′ in the last column, then go across the page to the column having "Sin" at the bottom and find sin 66° 50′ = .9194.

Example 3. Find sin 23° 16′.

On page 6 we find sin 23° 10′ = .3934 and sin 23° 20′ = .3961. The difference is .0027. The angle 23° 16′ is $\frac{6}{10}$ of the way from 23° 10 to 23° 20′, hence sin 23° 16′ will be $\frac{6}{10}$ of the way from .3924 to .3961 or $.3924 + \frac{6}{10} \times .0027 = .3934 + .0016 = .3950$. We usually neglect the decimal point and call the difference 27 and add $\frac{6}{10}$ of it to 3924. The final result should always be given to four decimal places.

Example 4. Find cos 23° 16′.

On page 6 we find cos 23° 10′ = .9194 and cos 23° 20′ = .9182, a difference of .0012. Hence cos 23° 16 = $.9194 - \frac{6}{10} \times .0012 = .9187$. Note that the cosine decreases as the angle increases hence subtraction is used in the interpolation. This is true also for cotangents and cosecants.

Example 5. Find tan 78° 18′.

From page 5 we find tan 78° 10′ = 4.773 and tan 78° 20′ = 4.843 and after interpolation tan 78° 18′ = 4.829.

EXAMPLE 6. Given $\sin A = .4111$, find $A$.

On page 6 we find $\sin 24° 10' = .4094$ and $\sin 24° 20' = .4120$ with $\sin A$ lying between $\frac{17}{26}$ of the way from .4094 to .4120. The angle $A$ is then approximately $\frac{17}{26}$ of the way from $24° 10'$ to $24° 20'$, or $A = 24° 16'$ to the nearest minute.

EXAMPLE 7. Given $\cot B = .6000$, find $B$.

On page 7 we find $\cot 59° = .6009$ and $\cot 59° 10' = .5969$ with $\cot B$ lying between $\frac{9}{40}$ of the way from $59°$ to $59° 10'$. The angle $B$ is then $59° + \frac{9}{40} \times 10' = 59° 2'$ to the nearest minute.

EXAMPLE 8. Express $43° 43'$ in radians.

From page 8 we find

$$43° 40' = .7621 \text{ radians},$$
$$43° 50' = .7650 \text{ radians}.$$

By interpolation we find $43° 43' = .7630$ radians.

EXAMPLE 9. Express 1.5 radians in degrees and minutes.

From page 4 we find

$$1.4981 \text{ radians} = 85° 50'$$
$$1.5010 \text{ radians} = 86°$$

with 1.5 lying $\frac{19}{29}$ of the way from 1.4981 to 1.5010, from which we find 1.5 radians $= 85° 57'$ to the nearest minute.

## TABLE III. FOUR-PLACE LOGARITHMS OF NUMBERS

**4. To find the logarithm of a number.** Table III, pages 10, 11, gives directly the mantissas of the logarithms of the integers 100 to 999. The characteristic of a logarithm is to be obtained from the rules for characteristics. (See page 80, Crathorne and Lytle's *Trigonometry*.) The table is a four-place table though the last three digits are usually the only ones given. The first digit is omitted except for the first number in the row after the first digit changes. For example, in the row for **N** = 2.5 we find 979, 997, *014, etc. If all the digits were written these numbers would be 3979, 3997, 4014, etc. The * denotes a change in the first digit. The method of finding the logarithm of a number is illustrated in the following examples.

EXAMPLE 1. Find log 475 to four decimal places.

On page 10 of the tables we find the mantissa by looking down the column headed **N** until we have 4.7 and then across the page to the column

headed 5, where we find 6767. The characteristic is 2. Hence log 475 = 2.6767.

EXAMPLE 2. Find log .796.

On page 11 we pick out 7.9 in the N column, then across the page to the column headed 6 where we find the mantissa *009. This means 9009. The * calls attention to the fact that the first digit has changed and that we are to take the first digit of the next row. The characteristic is − 1, and as usually written we have log .796 = 9.9009 − 10.

EXAMPLE 3. Find log 3647.

The characteristic is 3. Directly from the table we find

$$\log 3640 = 3.5611,$$
$$\log 3650 = 3.5623.$$

The difference is 12 disregarding decimals. The number 3647 is seven-tenths of the way from 3640 to 3650. Log 3647 is then approximately seven-tenths of the way from log 3640 to log 3650. We then add to the mantissa for log 3640, seven-tenths of 12, obtaining 5611 + 8 = 5619. Hence log 3647 = 3.5619.

Using the table of proportional parts given at the right of the table, log 3647 is found as follows. From the table obtain directly log 3640 = 3.5611. Look at the number 7 in the top row under the words **Proportional Parts.** Follow down the column under 7 until the row 3.6 is reached. Here we find the number 8. Add 8 to the mantissa for log 3640 and get log 3647 = 3.5619.

**5. To find the number corresponding to a given logarithm.** If the logarithm of a number is given the number may be found by working the table backwards as illustrated in the following example.

EXAMPLE 4. Find the number whose logarithm is 2.3334.

The mantissa 3334 is not given in the table, but it lies between the two adjacent mantissas 3324 and 3345 of the table. The mantissa 3324 corresponds to log 215 and 3345 to log 216. The number 3334 is $\frac{10}{21}$ of the way from 3324 to 3345. By the principle of proportional parts the number we seek is $\frac{10}{21}$ of the way from 215 to 216, or 215.5. Hence log 215.5 = 2.3334.

Using the table of proportional parts this example is solved as follows. Find the mantissa in the table next below 3334. It is 3324. This corresponds to log 215. The difference between 3334 and 3324 is 10. Look for 10 in the row of proportional parts corresponding to N = 2.1. Follow up the column in which 10 lies until the top figure is reached, which is 5. Write the 5 at the end of 215 making 2155. Placing the decimal point according to the rules for characteristics, we have log 215.5 = 2.3334.

## Table IV. Four-Place Logarithms of Trigonometric Functions

**6. To find the logarithm of a function.** This table, pages 12–17, gives four-place logarithms of the sine, cosine, tangent and cotangent at intervals of 10′. For the angles 0° to 45° which are found in the first column, we read the headings at the tops of the other columns; for the angles 45° to 90° found in the last column, we read the headings at the bottom. The columns headed **d 1′**, **c d 1′** are for aid in interpolation. The third column headed **d 1′** gives the change in the logarithm of the sine for a change of 1′ in the angle. The fifth column headed **c d 1′** shows the common difference of the logarithms of the tangent and the cotangent for a change of 1′ in the angle. The eighth column headed **d 1′** gives the corresponding difference for the logarithm of the cosine.

For simplicity in printing, all the characteristics in this table have been increased by 10. Hence 10 must be subtracted from each tabulated value of a logarithm.

Example 1. Find log tan 38° 48′.

On page 17, in the column headed **L Tan** we find

$$\log \tan 38° 40' = 9.9032.$$

In the corresponding place in the **c d 1′** column we find 2.6. This is the difference for 1′ in this part of the table. The correction for 8′ is then $8 \times 2.6 = 21$ approximately. Since **L Tan** increases when the angle increases this correction is added to 9032 and

$$\log \tan 38° 48' = 9.9053 - 10.$$

Example 2. Find log cos 81° 33′.

On page 13, in the column with **L Cos** at the bottom we find

$$\log \cos 81° 30' = 9.1697.$$

In the **d 1′** column we find 8.5. The correction for 3′ is $3 \times 8.5 = 26$. (See Art. 31, page 72, Crathorne and Lytle's *Trigonometry*.) Since **L Cos** decreases when the angle increases the correction is subtracted and

$$\log \cos 81° 33' = 9.1671.$$

**7. To find the angle when the logarithm of the function is given.** If the logarithm of a trigonometric function of an angle

is given and we wish to find the angle, the steps of Art. 6 are reversed as shown in the following examples.

EXAMPLE 3. Given $\log \tan A = 9.9053 - 10$, find $A$.

On page 17 in the column headed **L Tan** we find

$$\log \tan 38° 40' = 9.9032$$
$$\log \tan 38° 50' = 9.9058.$$

Hence $A$ lies between $38° 40'$ and $38° 50'$. The difference between $\log \tan 38° 40'$ and $\log \tan A$ is 21. From the table the difference for $1'$ is 2.6. Hence the correction to add to $38° 40'$ is $\frac{21}{2.6} = 8$, and $A = 38° 48'$.

EXAMPLE 4. Given $\log \cot B = 9.7921 - 10$, find $B$.

On page 16 in the column having **L Cot** at the bottom we find

$$\log \cot 58° 20' = 9.7902$$
$$\log \cot 58° 10' = 9.7930.$$

Hence $B$ lies between $58° 10'$ and $58° 20'$. The difference between $\log \cot 58° 10'$ and $\log \cot B$ is 9. From the table the difference for $1'$ is 2.8. Hence the correction to add to $58° 10'$ is $\frac{9}{2.8} = 3$ and $B = 58° 13'$.

## TABLE V*a*. AUXILIARY TABLE OF *S* AND *T*

**8. Angles near 0° or 90°.** The tabular differences in Table VI for angles near 0° or 90° are either too small or too large to make interpolation at this part of the table very accurate. Table V*a* will give better results. When the angle is small it is desirable to use a formula, when possible, which involves the sine, tangent or cotangent of the angle; when the angle is near 90° use a formula involving cosine or tangent. The use of this table is illustrated by the following examples.

EXAMPLE 1. To find $\log \tan 0° 27' 32''$ by Table V*a*.

At the head of the table we find the formula

$$\log \tan A = T + \log A'$$

where $A'$ is the angle expressed in minutes and $T$ is a constant given in the table. Here, $A' = 27.533$ minutes and by Table VII

$$\log A' = 1.43985$$

and by Table V*a*

$$T = 6.46374 - 10$$

and therefore

$$\log \tan 0° 27' 32'' = 7.90359 - 10.$$

EXAMPLE 2. To find angle $B$, by Table V$a$, when given

$$\log \tan B = 3.21431.$$

Since $B$ is near 90° we use formula

$$\log \cot B = \log B'_1 + T_1,$$

given at the top of table, where $B'_1 = 90° - B$ expressed in minutes and $T_1$ is given in the table. We have

$$\log \cot B = 10 - \log \tan B - 10 = 6.78569 - 10.$$

Now by Table VI

$$B = 89° \, 57' \text{ approximately,}$$
$$B'_1 = \phantom{8}0° \, 3' \text{ approximately.}$$

By Table V$a$

$$T_1 = 6.46373 - 10.$$

Since we have here

$$\log B'_1 = \log \cot B - T_1 = .32196,$$

then by Table VII

$$B'_1 = 2.0987' = 2' \, 6''.$$

Therefore

$$B = 89° \, 57' \, 54''.$$

## TABLE V$b$. ANGLES NEAR 0° AND 90°

**9. Second consideration of angles near 0° or 90°.** This table gives values of log sin and a certain constant $C$; we can find the values of log cos and log tan from the formulas

$$\log \cos A = 10 - C - 10$$
$$\log \tan A = \log \sin A + C$$

given in the right margin of the table. For an angle near 90° use the cofunction of the complementary angle.

EXAMPLE 1. To find log tan 0° 27′ 32″ by Table V$b$.

From Table V$b$ we get

$$\log \tan 0° \, 27' \, 30'' = 7.90306 - 10$$
$$\log \tan 0° \, 27' \, 40'' = 7.90569 - 10$$
$$\text{Difference for } 10'' = .00263$$
$$\text{Correction for } 2'' = \tfrac{2}{10} 263 = 53.$$

Therefore

$$\log \tan 0° \, 27' \, 32'' = 7.90359 - 10.$$

EXAMPLE 2. To find angle $B$ by Table V$b$, when given log tan $B$ = 3.21431.

The angle $B$ is near 90°, so let $A = 90° - B$. Then

$$\log \cot A = 3.21431$$

and

$$\log \tan A = 10 - \log \cot A - 10 = 6.78569 - 10.$$

From Table V*b* we get

$$\begin{aligned} \log \tan 0^\circ\, 2'\ \ 0'' &= 6.76476 - 10 \\ \log \tan 0^\circ\, 2'\, 10'' &= 6.79952 - 10 \\ \text{Difference for } 10'' &= .03476. \end{aligned}$$

Therefore

$$\begin{aligned} A &= 0^\circ\, 2' + \frac{2093}{3476}\, 10'' \\ &= 0^\circ\, 2'\, 6'' \end{aligned}$$

and

$$B = 89^\circ\, 57'\, 54''.$$

## Table VI. Five-Place Logarithms of Functions

**10. To find the logarithm when the angle is given.** Table VI gives the five-place logarithms of sines, cosines, tangents and cotangents for angles at intervals of one minute. For angles less than 45° use top and left side of pages and when angles are greater than 45° use bottom and right side of pages. Parts of a minute require interpolation.

Example 1. To find log sin 36° 41′ 18″. From Table VI we find the logarithms in the following form and then compute the differences exhibited.

$$60''\left[\begin{array}{l} 18''\left[\begin{array}{ll} \log \sin 36^\circ\, 41' & = 9.77626 - 10 \\ \log \sin 36^\circ\, 41'\, 18'' & = x \end{array}\right] y \\ \quad \log \sin 36^\circ\, 42' \qquad = 9.77643 - 10 \end{array}\right] 17$$

Then

$$\frac{y}{17} = \frac{18}{60},\quad y = \frac{(17)(18)}{60} = 5.1$$

and

$$x = 9.77626 + .00005 = 9.77631.$$

To find $y$ by Prop. Pts. we note that under the column marked 17 on the right margin of the page

correction for 10″ is 2.8
correction for 8″ is 2.3,

therefore the sum 5.1 is the correction for 18″.

Five-place tables usually will not give results correct to the second, but can be depended upon for tenths of a minute. (See

Art. 37, page 82, Crathorne and Lytle's *Trigonometry*.) If tenths of minutes are used in the data, the interpolation is illustrated in the following example.

EXAMPLE 2. Find log cos 34° 36.7′.

Find the logarithms in the form below from the tables.

$$1\left[\begin{array}{l} .7\left[\begin{array}{ll} \log\cos 34^\circ\, 36' & = 9.91547 \\ \log\cos 34^\circ\, 36.7' & = x \\ \log\cos 34^\circ\, 37' & = 9.91538 \end{array}\right]y \end{array}\right]9$$

Then

$$\frac{y}{9} = \frac{.7}{1},\ y = \frac{9(.7)}{10} = 6.3$$

and therefore

$$\log\cos 34^\circ\, 36.7' = x = 9.91547 - .00006 = 9.91541.$$

**11. To find the angle when the logarithm of some function is given.** This is illustrated by an example.

EXAMPLE 1. To find the acute angle $B$ when given log tan $B$ = 10.27546.

We look for 10.27546 in the tangent column of the table and see that

$$60''\left[\begin{array}{l} y\left[\begin{array}{ll} \log\tan 62^\circ\, 3' & = 10.27524 \\ \log\tan B & = 10.27546 \end{array}\right]22 \\ \quad \log\tan 62^\circ\, 4' = 10.27555 \end{array}\right]31$$

and by proportion

$$y = \frac{22}{31}60'' = 42.6,\quad B'' = 62^\circ\, 3'\, 42.6'' = 62^\circ\, 3.7'.$$

To get the correction $y$ by Prop. Pts., in column headed 31 on side of page 54 we note that the

correction for 40″ is 20.7
correction for 3″ is 1.5

and by adding we see that there is a correction of approximately 43″ for a tabular difference of 22.2.

## TABLE VII. FIVE-PLACE LOGARITHMS OF NUMBERS

**12. How to use a five-place table.** This table gives the mantissas of logarithms of four digit numbers to five decimal places. The characteristics are obtained by rule. Interpolation should be made when the numbers have more than four significant digits.

EXAMPLE 1. To find the value of log 52787.

Find the page in the table having 527 in the column marked **N**; opposite 527 and in column headed 8 we find the mantissa 72247. We then note that

$$10\left[\begin{array}{l} 7\left[\begin{array}{l}\log 52780 = 4.72247\\ \log 52787 = x\end{array}\right]y \\ \quad\log 52790 = 4.72255\end{array}\right]8$$

and then

$$\frac{y}{8} = \frac{7}{10},\ y = \frac{56}{10} = 5.6$$

and

$$x = 4.72247 + .00006 = 4.72253.$$

The correction $y$ could have been obtained from the Prop. Pts. tables on the right margin of the page where in column headed 8 in row 7 we find the proper correction 5.6.

EXAMPLE 2. Given $\log F = 2.14725$ to find $F$.

Find the page in the table having the mantissa nearest to 14725. We then note that

$$.1\left[\begin{array}{l} y\left[\begin{array}{l}\log 140.3 = 2.14706\\ \log F \quad\ = 2.14725\end{array}\right]19 \\ \quad\log 140.4 = 2.14737\end{array}\right]31$$

then

$$\frac{y}{.1} = \frac{19}{31},\ y = \frac{1.9}{31} = .061$$

and

$$F = 140.3 + .06 = 140.36.$$

To get the correction $y$ by Prop. Pts. run down the column headed 31 on the margin of page until you come to the number nearest 19 and the fifth digit 6 is opposite 18.6.

Most of the interpolation can be done mentally after drill and experience.

## TABLE VIII. CONSTANTS WITH THEIR LOGARITHMS

**13. Some useful constants.** This table lists the constants which are important in trigonometry with their logarithms. No explanation is necessary.

## TABLE IX. NATURAL LOGARITHMS OF NUMBERS

**14. Natural logarithms and exponentials.** This table gives natural logarithms, that is, logarithms to the base $e$. The laws

for finding the characteristic used for common logarithms to the base 10 (Table VII) do not hold for natural logarithms, so this table gives both characteristics and mantissas. To find a natural logarithm of a number not within the range of this table follow the note at the head of the table. Interpolations in this table are performed just as in other tables. This table may be used to find values of exponentials, that is, values of $e^x$ for given values of $x$.

EXAMPLE 1. To find $\log_e 5.48$, a number within the range of the table.

In the column marked **N** find 5.4 and opposite this in column headed 8 we find 0111 as the last four digits of the logarithm, the first two digits 1.7 being given in first column and not repeated each time. The * indicates that the first two digits follow instead of precede as usual. We have then

$$\log_e 5.48 = 1.70111.$$

EXAMPLE 2. To find $\log_e 372$, a number outside the range of the table.

By the note at the beginning of the table we see that

$$\begin{aligned}\log_e 372 &= \log_e 3.72 + \log_e 10^2\\ &= 1.31372 + 2(2.30259)\\ &= 5.91890.\end{aligned}$$

EXAMPLE 3. To find the value of $e^{2.12346}$

Let $$\log_e N = 2.12346.$$

Then from the tables

$$N = 8.36 = e^{2.12346}.$$

# FOUR-PLACE TABLES

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1.0** | 1.000 | 1.020 | 1.040 | 1.061 | 1.082 | 1.103 | 1.124 | 1.145 | 1.166 | 1.188 |
| 1.1 | 1.210 | 1.232 | 1.254 | 1.277 | 1.300 | 1.323 | 1.346 | 1.369 | 1.392 | 1.416 |
| 1.2 | 1.440 | 1.464 | 1.488 | 1.513 | 1.538 | 1.563 | 1.588 | 1.613 | 1.638 | 1.664 |
| 1.3 | 1.690 | 1.716 | 1.742 | 1.769 | 1.796 | 1.823 | 1.850 | 1.877 | 1.904 | 1.932 |
| 1.4 | 1.960 | 1.988 | 2.016 | 2.045 | 2.074 | 2.103 | 2.132 | 2.161 | 2.190 | 2.220 |
| 1.5 | 2.250 | 2.280 | 2.310 | 2.341 | 2.372 | 2.403 | 2.434 | 2.465 | 2.496 | 2.528 |
| 1.6 | 2.560 | 2.592 | 2.624 | 2.657 | 2.690 | 2.723 | 2.756 | 2.789 | 2.822 | 2.856 |
| 1.7 | 2.890 | 2.924 | 2.958 | 2.993 | 3.028 | 3.063 | 3.098 | 3.133 | 3.168 | 3.204 |
| 1.8 | 3.240 | 3.276 | 3.312 | 3.349 | 3.386 | 3.423 | 3.460 | 3.497 | 3.534 | 3.572 |
| 1.9 | 3.610 | 3.648 | 3.686 | 3.725 | 3.764 | 3.803 | 3.842 | 3.881 | 3.920 | 3.960 |
| **2.0** | 4.000 | 4.040 | 4.080 | 4.121 | 4.162 | 4.203 | 4.244 | 4.285 | 4.326 | 4.368 |
| 2.1 | 4.410 | 4.452 | 4.494 | 4.537 | 4.580 | 4.623 | 4.666 | 4.709 | 4.752 | 4.796 |
| 2.2 | 4.840 | 4.884 | 4.928 | 4.973 | 5.018 | 5.063 | 5.108 | 5.153 | 5.198 | 5.244 |
| 2.3 | 5.290 | 5.336 | 5.382 | 5.429 | 5.476 | 5.523 | 5.570 | 5.617 | 5.664 | 5.712 |
| 2.4 | 5.760 | 5.808 | 5.856 | 5.905 | 5.954 | 6.003 | 6.052 | 6.101 | 6.150 | 6.200 |
| 2.5 | 6.250 | 6.300 | 6.350 | 6.401 | 6.452 | 6.503 | 6.554 | 6.605 | 6.656 | 6.708 |
| 2.6 | 6.760 | 6.812 | 6.864 | 6.917 | 6.970 | 7.023 | 7.076 | 7.129 | 7.182 | 7.236 |
| 2.7 | 7.290 | 7.344 | 7.398 | 7.453 | 7.508 | 7.563 | 7.618 | 7.673 | 7.728 | 7.784 |
| 2.8 | 7.840 | 7.896 | 7.952 | 8.009 | 8.066 | 8.123 | 8.180 | 8.237 | 8.294 | 8.352 |
| 2.9 | 8.410 | 8.468 | 8.526 | 8.585 | 8.644 | 8.703 | 8.762 | 8.821 | 8.880 | 8.940 |
| **3.0** | 9.000 | 9.060 | 9.120 | 9.181 | 9.242 | 9.303 | 9.364 | 9.425 | 9.486 | 9.548 |
| 3.1 | 9.610 | 9.672 | 9.734 | 9.797 | 9.860 | 9.923 | 9.986 | 10.05 | 10.11 | 10.18 |
| 3.2 | 10.24 | 10.30 | 10.37 | 10.43 | 10.50 | 10.56 | 10.63 | 10.69 | 10.76 | 10.82 |
| 3.3 | 10.89 | 10.96 | 11.02 | 11.09 | 11.16 | 11.22 | 11.29 | 11.36 | 11.42 | 11.49 |
| 3.4 | 11.56 | 11.63 | 11.70 | 11.76 | 11.83 | 11.90 | 11.97 | 12.04 | 12.11 | 12.18 |
| 3.5 | 12.25 | 12.32 | 12.39 | 12.46 | 12.53 | 12.60 | 12.67 | 12.74 | 12.82 | 12.89 |
| 3.6 | 12.96 | 13.03 | 13.10 | 13.18 | 13.25 | 13.32 | 13.40 | 13.47 | 13.54 | 13.62 |
| 3.7 | 13.69 | 13.76 | 13.84 | 13.91 | 13.99 | 14.06 | 14.14 | 14.21 | 14.29 | 14.36 |
| 3.8 | 14.44 | 14.52 | 14.59 | 14.67 | 14.75 | 14.82 | 14.90 | 14.98 | 15.05 | 15.13 |
| 3.9 | 15.21 | 15.29 | 15.37 | 15.44 | 15.52 | 15.60 | 15.68 | 15.76 | 15.84 | 15.92 |
| **4.0** | 16.00 | 16.08 | 16.16 | 16.24 | 16.32 | 16.40 | 16.48 | 16.56 | 16.65 | 16.73 |
| 4.1 | 16.81 | 16.89 | 16.97 | 17.06 | 17.14 | 17.22 | 17.31 | 17.39 | 17.47 | 17.56 |
| 4.2 | 17.64 | 17.72 | 17.81 | 17.89 | 17.98 | 18.06 | 18.15 | 18.23 | 18.32 | 18.40 |
| 4.3 | 18.49 | 18.58 | 18.66 | 18.75 | 18.84 | 18.92 | 19.01 | 19.10 | 19.18 | 19.27 |
| 4.4 | 19.36 | 19.45 | 19.54 | 19.62 | 19.71 | 19.80 | 19.89 | 19.98 | 20.07 | 20.16 |
| 4.5 | 20.25 | 20.34 | 20.43 | 20.52 | 20.61 | 20.70 | 20.79 | 20.88 | 20.98 | 21.07 |
| 4.6 | 21.16 | 21.25 | 21.34 | 21.44 | 21.53 | 21.62 | 21.72 | 21.81 | 21.90 | 22.00 |
| 4.7 | 22.09 | 22.18 | 22.28 | 22.37 | 22.47 | 22.56 | 22.66 | 22.75 | 22.85 | 22.94 |
| 4.8 | 23.04 | 23.14 | 23.23 | 23.33 | 23.43 | 23.52 | 23.62 | 23.72 | 23.81 | 23.91 |
| 4.9 | 24.01 | 24.11 | 24.21 | 24.30 | 24.40 | 24.50 | 24.60 | 24.70 | 24.80 | 24.90 |
| **5.0** | 25.00 | 25.10 | 25.20 | 25.30 | 25.40 | 25.50 | 25.60 | 25.70 | 25.81 | 25.91 |
| 5.1 | 26.01 | 26.11 | 26.21 | 26.32 | 26.42 | 26.52 | 26.63 | 26.73 | 26.83 | 26.94 |
| 5.2 | 27.04 | 27.14 | 27.25 | 27.35 | 27.46 | 27.56 | 27.67 | 27.77 | 27.88 | 27.98 |
| 5.3 | 28.09 | 28.20 | 28.30 | 28.41 | 28.52 | 28.62 | 28.73 | 28.84 | 28.94 | 29.05 |
| 5.4 | 29.16 | 29.27 | 29.38 | 29.48 | 29.59 | 29.70 | 29.81 | 29.92 | 30.03 | 30.14 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5.5 | 30.25 | 30.36 | 30.47 | 30.58 | 30.69 | 30.80 | 30.91 | 31.02 | 31.14 | 31.25 |
| 5.6 | 31.36 | 31.47 | 31.58 | 31.70 | 31.81 | 31.92 | 32.04 | 32.15 | 32.26 | 32.38 |
| 5.7 | 32.49 | 32.60 | 32.72 | 32.83 | 32.95 | 33.06 | 33.18 | 33.29 | 33.41 | 33.52 |
| 5.8 | 33.64 | 33.76 | 33.87 | 33.99 | 34.11 | 34.22 | 34.34 | 34.46 | 34.57 | 34.69 |
| 5.9 | 34.81 | 34.93 | 35.05 | 35.16 | 35.28 | 35.40 | 35.52 | 35.64 | 35.76 | 35.88 |
| **6.0** | 36.00 | 36.12 | 36.24 | 36.36 | 36.48 | 36.60 | 36.72 | 36.84 | 36.97 | 37.09 |
| 6.1 | 37.21 | 37.33 | 37.45 | 37.58 | 37.70 | 37.82 | 37.95 | 38.07 | 38.19 | 38.32 |
| 6.2 | 38.44 | 38.56 | 38.69 | 38.81 | 38.94 | 39.06 | 39.19 | 39.31 | 39.44 | 39.56 |
| 6.3 | 39.69 | 39.82 | 39.94 | 40.07 | 40.20 | 40.32 | 40.45 | 40.58 | 40.70 | 40.83 |
| 6.4 | 40.96 | 41.09 | 41.22 | 41.34 | 41.47 | 41.60 | 41.73 | 41.86 | 41.99 | 42.12 |
| 6.5 | 42.25 | 42.38 | 42.51 | 42.64 | 42.77 | 42.90 | 43.03 | 43.16 | 43.30 | 43.43 |
| 6.6 | 43.56 | 43.69 | 43.82 | 43.96 | 44.09 | 44.22 | 44.36 | 44.49 | 44.62 | 44.76 |
| 6.7 | 44.89 | 45.02 | 45.16 | 45.29 | 45.43 | 45.56 | 45.70 | 45.83 | 45.97 | 46.10 |
| 6.8 | 46.24 | 46.38 | 46.51 | 46.65 | 46.79 | 46.92 | 47.06 | 47.20 | 47.33 | 47.47 |
| 6.9 | 47.61 | 47.75 | 47.89 | 48.02 | 48.16 | 48.30 | 48.44 | 48.58 | 48.72 | 48.86 |
| **7.0** | 49.00 | 49.14 | 49.28 | 49.42 | 49.56 | 49.70 | 49.84 | 49.98 | 50.13 | 50.27 |
| 7.1 | 50.41 | 50.55 | 50.69 | 50.84 | 50.98 | 51.12 | 51.27 | 51.41 | 51.55 | 51.70 |
| 7.2 | 51.84 | 51.98 | 52.13 | 52.27 | 52.42 | 52.56 | 52.71 | 52.85 | 53.00 | 53.14 |
| 7.3 | 53.29 | 53.44 | 53.58 | 53.73 | 53.88 | 54.02 | 54.17 | 54.32 | 54.46 | 54.61 |
| 7.4 | 54.76 | 54.91 | 55.06 | 55.20 | 55.35 | 55.50 | 55.65 | 55.80 | 55.95 | 56.10 |
| 7.5 | 56.25 | 56.40 | 56.55 | 56.70 | 56.85 | 57.00 | 57.15 | 57.30 | 57.46 | 57.61 |
| 7.6 | 57.76 | 57.91 | 58.06 | 58.22 | 58.37 | 58.52 | 58.68 | 58.83 | 58.98 | 59.14 |
| 7.7 | 59.29 | 59.44 | 59.60 | 59.75 | 59.91 | 60.06 | 60.22 | 60.37 | 60.53 | 60.68 |
| 7.8 | 60.84 | 61.00 | 61.15 | 61.31 | 61.47 | 61.62 | 61.78 | 61.94 | 62.09 | 62.25 |
| 7.9 | 62.41 | 62.57 | 62.73 | 62.88 | 63.04 | 63.20 | 63.36 | 63.52 | 63.68 | 63.84 |
| **8.0** | 64.00 | 64.16 | 64.32 | 64.48 | 64.64 | 64.80 | 64.96 | 65.12 | 65.29 | 65.45 |
| 8.1 | 65.61 | 65.77 | 65.93 | 66.10 | 66.26 | 66.42 | 66.59 | 66.75 | 66.91 | 67.08 |
| 8.2 | 67.24 | 67.40 | 67.57 | 67.73 | 67.90 | 68.06 | 68.23 | 68.39 | 68.56 | 68.72 |
| 8.3 | 68.89 | 69.06 | 69.22 | 69.39 | 69.56 | 69.72 | 69.89 | 70.06 | 70.22 | 70.39 |
| 8.4 | 70.56 | 70.73 | 70.90 | 71.06 | 71.23 | 71.40 | 71.57 | 71.74 | 71.91 | 72.08 |
| 8.5 | 72.25 | 72.42 | 72.59 | 72.76 | 72.93 | 73.10 | 73.27 | 73.44 | 73.62 | 73.79 |
| 8.6 | 73.96 | 74.13 | 74.30 | 74.48 | 74.65 | 74.82 | 75.00 | 75.17 | 75.34 | 75.52 |
| 8.7 | 75.69 | 75.86 | 76.04 | 76.21 | 76.39 | 76.56 | 76.74 | 76.91 | 77.08 | 77.26 |
| 8.8 | 77.44 | 77.62 | 77.79 | 77.97 | 78.15 | 78.32 | 78.50 | 78.68 | 78.85 | 79.03 |
| 8.9 | 79.21 | 79.39 | 79.57 | 79.74 | 79.92 | 80.10 | 80.28 | 80.46 | 80.64 | 80.82 |
| **9.0** | 81.00 | 81.18 | 81.36 | 81.54 | 81.72 | 81.90 | 82.08 | 82.26 | 82.45 | 82.63 |
| 9.1 | 82.81 | 82.99 | 83.17 | 83.36 | 83.54 | 83.72 | 83.91 | 84.09 | 84.27 | 84.46 |
| 9.2 | 84.64 | 84.82 | 85.01 | 85.19 | 85.38 | 85.56 | 85.75 | 85.93 | 86.12 | 86.30 |
| 9.3 | 86.49 | 86.68 | 86.86 | 87.05 | 87.24 | 87.42 | 87.61 | 87.80 | 87.98 | 88.17 |
| 9.4 | 88.36 | 88.55 | 88.74 | 88.92 | 89.11 | 89.30 | 89.49 | 89.68 | 89.87 | 90.06 |
| 9.5 | 90.25 | 90.44 | 90.63 | 90.82 | 91.01 | 91.20 | 91.39 | 91.58 | 91.78 | 91.97 |
| 9.6 | 92.16 | 92.35 | 92.54 | 92.74 | 92.93 | 93.12 | 93.32 | 93.51 | 93.70 | 93.90 |
| 9.7 | 94.09 | 94.28 | 94.48 | 94.67 | 94.87 | 95.06 | 95.26 | 95.45 | 95.65 | 95.84 |
| 9.8 | 96.04 | 96.24 | 96.43 | 96.63 | 96.83 | 97.02 | 97.22 | 97.42 | 97.61 | 97.81 |
| 9.9 | 98.01 | 98.21 | 98.41 | 98.60 | 98.80 | 99.00 | 99.20 | 99.40 | 99.60 | 99.80 |

| Degrees | Radians | Sin | Cos | Tan | Cot | Sec | Csc | | |
|---|---|---|---|---|---|---|---|---|---|
| **0° 00′** | .0000 | .0000 | 1.0000 | .0000 | —— | 1.000 | —— | 1.5708 | **90° 00′** |
| 10 | 029 | 029 | 000 | 029 | 343.8 | 000 | 343.8 | 679 | 50 |
| 20 | 058 | 058 | 000 | 058 | 171.9 | 000 | 171.9 | 650 | 40 |
| 30 | .0087 | .0087 | 1.0000 | .0087 | 114.6 | 1.000 | 114.6 | 1.5621 | 30 |
| 40 | 116 | 116 | .9999 | 116 | 85.94 | 000 | 85.95 | 592 | 20 |
| 50 | 145 | 145 | 999 | 145 | 68.75 | 000 | 68.76 | 563 | 10 |
| **1° 00′** | .0175 | .0175 | .9998 | .0175 | 57.29 | 1.000 | 57.30 | 1.5533 | **89° 00′** |
| 10 | 204 | 204 | 998 | 204 | 49.10 | 000 | 49.11 | 504 | 50 |
| 20 | 233 | 233 | 997 | 233 | 42.96 | 000 | 42.98 | 475 | 40 |
| 30 | .0262 | .0262 | .9997 | .0262 | 38.19 | 1.000 | 38.20 | 1.5446 | 30 |
| 40 | 291 | 291 | 996 | 291 | 34.37 | 000 | 34.38 | 417 | 20 |
| 50 | 320 | 320 | 995 | 320 | 31.24 | 001 | 31.26 | 388 | 10 |
| **2° 00′** | .0349 | .0349 | .9994 | .0349 | 28.64 | 1.001 | 28.65 | 1.5359 | **88° 00′** |
| 10 | 378 | 378 | 993 | 378 | 26.43 | 001 | 26.45 | 330 | 50 |
| 20 | 407 | 407 | 992 | 407 | 24.54 | 001 | 24.56 | 301 | 40 |
| 30 | .0436 | .0436 | .9990 | .0437 | 22.90 | 1.001 | 22.93 | 1.5272 | 30 |
| 40 | 465 | 465 | 989 | 466 | 21.47 | 001 | 21.49 | 243 | 20 |
| 50 | 495 | 494 | 988 | 495 | 20.21 | 001 | 20.23 | 213 | 10 |
| **3° 00′** | .0524 | .0523 | .9986 | .0524 | 19.08 | 1.001 | 19.11 | 1.5184 | **87° 00′** |
| 10 | 553 | 552 | 985 | 553 | 18.07 | 002 | 18.10 | 155 | 50 |
| 20 | 582 | 581 | 983 | 582 | 17.17 | 002 | 17.20 | 126 | 40 |
| 30 | .0611 | .0610 | .9981 | .0612 | 16.35 | 1.002 | 16.38 | 1.5097 | 30 |
| 40 | 640 | 640 | 980 | 641 | 15.60 | 002 | 15.64 | 068 | 20 |
| 50 | 669 | 669 | 978 | 670 | 14.92 | 002 | 14.96 | 039 | 10 |
| **4° 00′** | .0698 | .0698 | .9976 | .0699 | 14.30 | 1.002 | 14.34 | 1.5010 | **86° 00′** |
| 10 | 727 | 727 | 974 | 729 | 13.73 | 003 | 13.76 | 981 | 50 |
| 20 | 756 | 756 | 971 | 758 | 13.20 | 003 | 13.23 | 952 | 40 |
| 30 | .0785 | .0785 | .9969 | .0787 | 12.71 | 1.003 | 12.75 | 1.4923 | 30 |
| 40 | 814 | 814 | 967 | 816 | 12.25 | 003 | 12.29 | 893 | 20 |
| 50 | 844 | 843 | 964 | 846 | 11.83 | 004 | 11.87 | 864 | 10 |
| **5° 00′** | .0873 | .0872 | .9962 | .0875 | 11.43 | 1.004 | 11.47 | 1.4835 | **85° 00′** |
| 10 | 902 | 901 | 959 | 904 | 11.06 | 004 | 11.10 | 806 | 50 |
| 20 | 931 | 929 | 957 | 934 | 10.71 | 004 | 10.76 | 777 | 40 |
| 30 | .0960 | .0958 | .9954 | .0963 | 10.39 | 1.005 | 10.43 | 1.4748 | 30 |
| 40 | 989 | 987 | 951 | 992 | 10.08 | 005 | 10.13 | 719 | 20 |
| 50 | .1018 | .1016 | 948 | .1022 | 9.788 | 005 | 9.839 | 690 | 10 |
| **6° 00′** | .1047 | .1045 | .9945 | .1051 | 9.514 | 1.006 | 9.567 | 1.4661 | **84° 00′** |
| 10 | 076 | 074 | 942 | 080 | 9.255 | 006 | 9.309 | 632 | 50 |
| 20 | 105 | 103 | 939 | 110 | 9.010 | 006 | 9.065 | 603 | 40 |
| 30 | .1134 | .1132 | .9936 | .1139 | 8.777 | 1.006 | 8.834 | 1.4573 | 30 |
| 40 | 164 | 161 | 932 | 169 | 8.556 | 007 | 8.614 | 544 | 20 |
| 50 | 193 | 190 | 929 | 198 | 8.345 | 007 | 8.405 | 515 | 10 |
| **7° 00′** | .1222 | .1219 | .9925 | .1228 | 8.144 | 1.008 | 8.206 | 1.4486 | **83° 00′** |
| 10 | 251 | 248 | 922 | 257 | 7.953 | 008 | 8.016 | 457 | 50 |
| 20 | 280 | 276 | 918 | 287 | 7.770 | 008 | 7.834 | 428 | 40 |
| 30 | .1309 | .1305 | .9914 | .1317 | 7.596 | 1.009 | 7.661 | 1.4399 | 30 |
| 40 | 338 | 334 | 911 | 346 | 7.429 | 009 | 7.496 | 370 | 20 |
| 50 | 367 | 363 | 907 | 376 | 7.269 | 009 | 7.337 | 341 | 10 |
| **8° 00′** | .1396 | .1392 | .9903 | .1405 | 7.115 | 1.010 | 7.185 | 1.4312 | **82° 00′** |
| 10 | 425 | 421 | 899 | 435 | 6.968 | 010 | 7.040 | 283 | 50 |
| 20 | 454 | 449 | 894 | 465 | 6.827 | 011 | 6.900 | 254 | 40 |
| 30 | .1484 | .1478 | .9890 | .1495 | 6.691 | 1.011 | 6.765 | 1.4224 | 30 |
| 40 | 513 | 507 | 886 | 524 | 6.561 | 012 | 6.636 | 195 | 20 |
| 50 | 542 | 536 | 881 | 554 | 6.435 | 012 | 6.512 | 166 | 10 |
| **9° 00′** | .1571 | .1564 | .9877 | .1584 | 6.314 | 1.012 | 6.392 | 1.4137 | **81° 00′** |
| | | Cos | Sin | Cot | Tan | Csc | Sec | Radians | Degrees |

| Degrees | Radians | Sin | Cos | Tan | Cot | Sec | Csc | | |
|---|---|---|---|---|---|---|---|---|---|
| **9° 00′** | .1571 | .1564 | .9877 | .1584 | 6.314 | 1.012 | 6.392 | 1.4137 | **81° 00′** |
| 10 | 600 | 593 | 872 | 614 | 197 | 013 | 277 | 108 | 50 |
| 20 | 629 | 622 | 868 | 644 | 084 | 013 | 166 | 079 | 40 |
| 30 | .1658 | .1650 | .9863 | .1673 | 5.976 | 1.014 | 6.059 | 1.4050 | 30 |
| 40 | 687 | 679 | 858 | 703 | 871 | 014 | 5.955 | 1.4021 | 20 |
| 50 | 716 | 708 | 853 | 733 | 769 | 015 | 855 | 992 | 10 |
| **10° 00′** | .1745 | .1736 | .9848 | .1763 | 5.671 | 1.015 | 5.759 | 1.3963 | **80° 00′** |
| 10 | 774 | 765 | 843 | 793 | 576 | 016 | 665 | 934 | 50 |
| 20 | 804 | 794 | 838 | 823 | 485 | 016 | 575 | 904 | 40 |
| 30 | .1833 | .1822 | .9833 | .1853 | 5.396 | 1.017 | 5.487 | 1.3875 | 30 |
| 40 | 862 | 851 | 827 | 883 | 309 | 018 | 403 | 846 | 20 |
| 50 | 891 | 880 | 822 | 914 | 226 | 018 | 320 | 817 | 10 |
| **11° 00′** | .1920 | .1908 | .9816 | .1944 | 5.145 | 1.019 | 5.241 | 1.3788 | **79° 00′** |
| 10 | 949 | 937 | 811 | 974 | 066 | 019 | 164 | 759 | 50 |
| 20 | 978 | 965 | 805 | .2004 | 4.989 | 020 | 089 | 730 | 40 |
| 30 | .2007 | .1994 | .9799 | .2035 | 4.915 | 1.020 | 5.016 | 1.3701 | 30 |
| 40 | 036 | .2022 | 793 | 065 | 843 | 021 | 4.945 | 672 | 20 |
| 50 | 065 | 051 | 787 | 095 | 773 | 022 | 876 | 643 | 10 |
| **12 00′** | .2094 | .2079 | .9781 | .2126 | 4.705 | 1.022 | 4.810 | 1.3614 | **78° 00′** |
| 10 | 123 | 108 | 775 | 156 | 638 | 023 | 745 | 584 | 50 |
| 20 | 153 | 136 | 769 | 186 | 574 | 024 | 682 | 555 | 40 |
| 30 | .2182 | .2164 | .9763 | .2217 | 4.511 | 1.024 | 4.620 | 1.3526 | 30 |
| 40 | 211 | 193 | 757 | 247 | 449 | 025 | 560 | 497 | 20 |
| 50 | 240 | 221 | 750 | 278 | 390 | 026 | 502 | 468 | 10 |
| **13° 00′** | .2269 | .2250 | .9744 | .2309 | 4.331 | 1.026 | 4.445 | 1.3439 | **77° 00′** |
| 10 | 298 | 278 | 737 | 339 | 275 | 027 | 390 | 410 | 50 |
| 20 | 327 | 306 | 730 | 370 | 219 | 028 | 336 | 381 | 40 |
| 30 | .2356 | .2334 | .9724 | .2401 | 4.165 | 1.028 | 4.284 | 1.3352 | 30 |
| 40 | 385 | 363 | 717 | 432 | 113 | 029 | 232 | 323 | 20 |
| 50 | 414 | 391 | 710 | 462 | 061 | 030 | 182 | 294 | 10 |
| **14° 00′** | .2443 | .2419 | .9703 | .2493 | 4.011 | 1.031 | 4.134 | 1.3265 | **76° 00′** |
| 10 | 473 | 447 | 696 | 524 | 3.962 | 031 | 086 | 235 | 50 |
| 20 | 502 | 476 | 689 | 555 | 914 | 032 | 039 | 206 | 40 |
| 30 | .2531 | .2504 | .9681 | .2586 | 3.867 | 1.033 | 3.994 | 1.3177 | 30 |
| 40 | 560 | 532 | 674 | 617 | 821 | 034 | 950 | 148 | 20 |
| 50 | 589 | 560 | 667 | 648 | 776 | 034 | 906 | 119 | 10 |
| **15° 00′** | .2618 | .2588 | .9659 | .2679 | 3.732 | 1.035 | 3.864 | 1.3090 | **75° 00′** |
| 10 | 647 | 616 | 652 | 711 | 689 | 036 | 822 | 061 | 50 |
| 20 | 676 | 644 | 644 | 742 | 647 | 037 | 782 | 032 | 40 |
| 30 | .2705 | .2672 | .9636 | .2773 | 3.606 | 1.038 | 3.742 | 1.3003 | 30 |
| 40 | 734 | 700 | 628 | 805 | 566 | 039 | 703 | 974 | 20 |
| 50 | 763 | 728 | 621 | 836 | 526 | 039 | 665 | 945 | 10 |
| **16° 00′** | .2793 | .2756 | .9613 | .2867 | 3.487 | 1.040 | 3.628 | 1.2915 | **74° 00′** |
| 10 | 822 | 784 | 605 | 899 | 450 | 041 | 592 | 886 | 50 |
| 20 | 851 | 812 | 596 | 931 | 412 | 042 | 556 | 857 | 40 |
| 30 | .2880 | .2840 | .9588 | .2962 | 3.376 | 1.043 | 3.521 | 1.2828 | 30 |
| 40 | 909 | 868 | 580 | 994 | 340 | 044 | 487 | 799 | 20 |
| 50 | 938 | 896 | 572 | .3026 | 305 | 045 | 453 | 770 | 10 |
| **17° 00′** | .2967 | .2924 | .9563 | .3057 | 3.271 | 1.046 | 3.420 | 1.2741 | **73° 00′** |
| 10 | 996 | 952 | 555 | 089 | 237 | 047 | 388 | 712 | 50 |
| 20 | .3025 | 979 | 546 | 121 | 204 | 048 | 357 | 683 | 40 |
| 30 | .3054 | .3007 | .9537 | .3153 | 3.172 | 1.048 | 3.326 | 1.2654 | 30 |
| 40 | 083 | 035 | 528 | 185 | 140 | 049 | 295 | 625 | 20 |
| 50 | 113 | 062 | 520 | 217 | 108 | 050 | 265 | 595 | 10 |
| **18° 00′** | .3142 | .3090 | .9511 | .3249 | 3.078 | 1.051 | 3.236 | 1.2566 | **72° 00′** |
| | | Cos | Sin | Cot | Tan | Csc | Sec | Radians | Degrees |

| Degrees | Radians | Sin | Cos | Tan | Cot | Sec | Csc | | |
|---|---|---|---|---|---|---|---|---|---|
| **18° 00′** | .3142 | .3090 | .9511 | .3249 | 3.078 | 1.051 | 3.236 | 1.2566 | **72° 00′** |
| 10 | 171 | 118 | 502 | 281 | 047 | 052 | 207 | 537 | 50 |
| 20 | 200 | 145 | 492 | 314 | 018 | 053 | 179 | 508 | 40 |
| 30 | .3229 | .3173 | .9483 | .3346 | 2.989 | 1.054 | 3.152 | 1.2479 | 30 |
| 40 | 258 | 201 | 474 | 378 | 960 | 056 | 124 | 450 | 20 |
| 50 | 287 | 228 | 465 | 411 | 932 | 057 | 098 | 421 | 10 |
| **19° 00′** | .3316 | .3256 | .9455 | .3443 | 2.904 | 1.058 | 3.072 | 1.2392 | **71° 00′** |
| 10 | 345 | 283 | 446 | 476 | 877 | 059 | 046 | 363 | 50 |
| 20 | 374 | 311 | 436 | 508 | 850 | 060 | 021 | 334 | 40 |
| 30 | .3403 | .3338 | .9426 | .3541 | 2.824 | 1.061 | 2.996 | 1.2305 | 30 |
| 40 | 432 | 365 | 417 | 574 | 798 | 062 | 971 | 275 | 20 |
| 50 | 462 | 393 | 407 | 607 | 773 | 063 | 947 | 246 | 10 |
| **20° 00′** | .3491 | .3420 | .9397 | .3640 | 2.747 | 1.064 | 2.924 | 1.2217 | **70° 00′** |
| 10 | 520 | 448 | 387 | 673 | 723 | 065 | 901 | 188 | 50 |
| 20 | 549 | 475 | 377 | 706 | 699 | 066 | 878 | 159 | 40 |
| 30 | .3578 | .3502 | .9367 | .3739 | 2.675 | 1.068 | 2.855 | 1.2130 | 30 |
| 40 | 607 | 529 | 356 | 772 | 651 | 069 | 833 | 101 | 20 |
| 50 | 636 | 557 | 346 | 805 | 628 | 070 | 812 | 072 | 10 |
| **21° 00′** | .3665 | .3584 | .9336 | .3839 | 2.605 | 1.071 | 2.790 | 1.2043 | **69° 00′** |
| 10 | 694 | 611 | 325 | 872 | 583 | 072 | 769 | 1.2014 | 50 |
| 20 | 723 | 638 | 315 | 906 | 560 | 074 | 749 | 985 | 40 |
| 30 | .3752 | .3665 | .9304 | .3939 | 2.539 | 1.075 | 2.729 | 1.1956 | 30 |
| 40 | 782 | 692 | 293 | 973 | 517 | 076 | 709 | 926 | 20 |
| 50 | 811 | 719 | 283 | .4006 | 496 | 077 | 689 | 897 | 10 |
| **22° 00′** | .3840 | .3746 | .9272 | .4040 | 2.475 | 1.079 | 2.669 | 1.1868 | **68° 00′** |
| 10 | 869 | 773 | 261 | 074 | 455 | 080 | 650 | 839 | 50 |
| 20 | 898 | 800 | 250 | 108 | 434 | 081 | 632 | 810 | 40 |
| 30 | .3927 | .3827 | .9239 | .4142 | 2.414 | 1.082 | 2.613 | 1.1781 | 30 |
| 40 | 956 | 854 | 228 | 176 | 394 | 084 | 595 | 752 | 20 |
| 50 | 985 | 881 | 216 | 210 | 375 | 085 | 577 | 723 | 10 |
| **23° 00′** | .4014 | .3907 | .9205 | .4245 | 2.356 | 1.086 | 2.559 | 1.1694 | **67° 00′** |
| 10 | 043 | 934 | 194 | 279 | 337 | 088 | 542 | 665 | 50 |
| 20 | 072 | 961 | 182 | 314 | 318 | 089 | 525 | 636 | 40 |
| 30 | .4102 | .3987 | .9171 | .4348 | 2.300 | 1.090 | 2.508 | 1.1606 | 30 |
| 40 | 131 | .4014 | 159 | 383 | 282 | 092 | 491 | 577 | 20 |
| 50 | 160 | 041 | 147 | 417 | 264 | 093 | 475 | 548 | 10 |
| **24° 00′** | .4189 | .4067 | .9135 | .4452 | 2.246 | 1.095 | 2.459 | 1.1519 | **66° 00′** |
| 10 | 218 | 094 | 124 | 487 | 229 | 096 | 443 | 490 | 50 |
| 20 | 247 | 120 | 112 | 522 | 211 | 097 | 427 | 461 | 40 |
| 30 | .4276 | .4147 | .9100 | .4557 | 2.194 | 1.099 | 2.411 | 1.1432 | 30 |
| 40 | 305 | 173 | 088 | 592 | 177 | 100 | 396 | 403 | 20 |
| 50 | 334 | 200 | 075 | 628 | 161 | 102 | 381 | 374 | 10 |
| **25° 00′** | .4363 | .4226 | .9063 | .4663 | 2.145 | 1.103 | 2.366 | 1.1345 | **65° 00′** |
| 10 | 392 | 253 | 051 | 699 | 128 | 105 | 352 | 316 | 50 |
| 20 | 422 | 279 | 038 | 734 | 112 | 106 | 337 | 286 | 40 |
| 30 | .4451 | .4305 | .9026 | .4770 | 2.097 | 1.108 | 2.323 | 1.1257 | 30 |
| 40 | 480 | 331 | 013 | 806 | 081 | 109 | 309 | 228 | 20 |
| 50 | 509 | 358 | 001 | 841 | 066 | 111 | 295 | 199 | 10 |
| **26° 00′** | .4538 | .4384 | .8988 | .4877 | 2.050 | 1.113 | 2.281 | 1.1170 | **64° 00′** |
| 10 | 567 | 410 | 975 | 913 | 035 | 114 | 268 | 141 | 50 |
| 20 | 596 | 436 | 962 | 950 | 020 | 116 | 254 | 112 | 40 |
| 30 | .4625 | .4462 | .8949 | .4986 | 2.006 | 1.117 | 2.241 | 1.1083 | 30 |
| 40 | 654 | 488 | 936 | .5022 | 1.991 | 119 | 228 | 054 | 20 |
| 50 | 683 | 514 | 923 | 059 | 977 | 121 | 215 | 1.1025 | 10 |
| **27° 00′** | .4712 | .4540 | .8910 | .5095 | 1.963 | 1.122 | 2.203 | 1.0996 | **63° 00′** |
| | | Cos | Sin | Cot | Tan | Csc | Sec | Radians | Degrees |

| Degrees | Radians | Sin | Cos | Tan | Cot | Sec | Csc | | |
|---|---|---|---|---|---|---|---|---|---|
| **27° 00′** | .4712 | .4540 | .8910 | .5095 | 1.963 | 1.122 | 2.203 | 1.0996 | **63° 00′** |
| 10 | 741 | 566 | 897 | 132 | 949 | 124 | 190 | 966 | 50 |
| 20 | 771 | 592 | 884 | 169 | 935 | 126 | 178 | 937 | 40 |
| 30 | .4800 | .4617 | .8870 | .5206 | 1.921 | 1.127 | 2.166 | 1.0908 | 30 |
| 40 | 829 | 643 | 857 | 243 | 907 | 129 | 154 | 879 | 20 |
| 50 | 858 | 669 | 843 | 280 | 894 | 131 | 142 | 850 | 10 |
| **28° 00′** | .4887 | .4695 | .8829 | .5317 | 1.881 | 1.133 | 2.130 | 1.0821 | **62° 00′** |
| 10 | 916 | 720 | 816 | 354 | 868 | 134 | 118 | 792 | 50 |
| 20 | 945 | 746 | 802 | 392 | 855 | 136 | 107 | 763 | 40 |
| 30 | .4974 | .4772 | .8788 | .5430 | 1.842 | 1.138 | 2.096 | 1.0734 | 30 |
| 40 | .5003 | 797 | 774 | 467 | 829 | 140 | 085 | 705 | 20 |
| 50 | 032 | 823 | 760 | 505 | 816 | 142 | 074 | 676 | 10 |
| **29° 00′** | .5061 | .4848 | .8746 | .5543 | 1.804 | 1.143 | 2.063 | 1.0647 | **61° 00′** |
| 10 | 091 | 874 | 732 | 581 | 792 | 145 | 052 | 617 | 50 |
| 20 | 120 | 899 | 718 | 619 | 780 | 147 | 041 | 588 | 40 |
| 30 | .5149 | .4924 | .8704 | .5658 | 1.767 | 1.149 | 2.031 | 1.0559 | 30 |
| 40 | 178 | 950 | 689 | 696 | 756 | 151 | 020 | 530 | 20 |
| 50 | 207 | 975 | 675 | 735 | 744 | 153 | 010 | 501 | 10 |
| **30° 00′** | .5236 | .5000 | .8660 | .5774 | 1.732 | 1.155 | 2.000 | 1.0472 | **60° 00′** |
| 10 | 265 | 025 | 646 | 812 | 720 | 157 | 1.990 | 443 | 50 |
| 20 | 294 | 050 | 631 | 851 | 709 | 159 | 980 | 414 | 40 |
| 30 | .5323 | .5075 | .8616 | .5890 | 1.698 | 1.161 | 1.970 | 1.0385 | 30 |
| 40 | 352 | 100 | 601 | 930 | 686 | 163 | 961 | 356 | 20 |
| 50 | 381 | 125 | 587 | 969 | .675 | 165 | 951 | 327 | 10 |
| **31° 00′** | .5411 | .5150 | .8572 | .6009 | 1.664 | 1.167 | 1.942 | 1.0297 | **59° 00′** |
| 10 | 440 | 175 | 557 | 048 | 653 | 169 | 932 | 268 | 50 |
| 20 | 469 | 200 | 542 | 088 | 643 | 171 | 923 | 239 | 40 |
| 30 | .5498 | .5225 | .8526 | .6128 | 1.632 | 1.173 | 1.914 | 1.0210 | 30 |
| 40 | 527 | 250 | 511 | 168 | 621 | 175 | 905 | 181 | 20 |
| 50 | 556 | 275 | 496 | 208 | 611 | 177 | 896 | 152 | 10 |
| **32° 00′** | .5585 | .5299 | .8480 | .6249 | 1.600 | 1.179 | 1.887 | 1.0123 | **58° 00′** |
| 10 | 614 | 324 | 465 | 289 | 590 | 181 | 878 | 094 | 50 |
| 20 | 643 | 348 | 450 | 330 | 580 | 184 | 870 | 065 | 40 |
| 30 | .5672 | .5373 | .8434 | .6371 | 1.570 | 1.186 | 1.861 | 1.0036 | 30 |
| 40 | 701 | 398 | 418 | 412 | 560 | 188 | 853 | 1.0007 | 20 |
| 50 | 730 | 422 | 403 | 453 | 550 | 190 | 844 | 977 | 10 |
| **33° 00′** | .5760 | .5446 | .8387 | .6494 | 1.540 | 1.192 | 1.836 | .9948 | **57° 00′** |
| 10 | 789 | 471 | 371 | 536 | 530 | 195 | 828 | 919 | 50 |
| 20 | 818 | 495 | 355 | 577 | 520 | 197 | 820 | 890 | 40 |
| 30 | .5847 | .5519 | .8339 | .6619 | 1.511 | 1.199 | 1.812 | .9861 | 30 |
| 40 | 876 | 544 | 323 | 661 | 501 | 202 | 804 | .832 | 20 |
| 50 | 905 | 568 | 307 | 703 | 1.492 | 204 | 796 | 803 | 10 |
| **34° 00′** | .5934 | .5592 | .8290 | .6745 | 1.483 | 1.206 | 1.788 | .9774 | **56° 00′** |
| 10 | 963 | 616 | 274 | 787 | 473 | 209 | 781 | 745 | 50 |
| 20 | 992 | 640 | 258 | 830 | 464 | 211 | 773 | 716 | 40 |
| 30 | .6021 | .5664 | .8241 | .6873 | 1.455 | 1.213 | 1.766 | .9687 | 30 |
| 40 | 050 | 688 | 225 | 916 | 446 | 216 | 758 | 657 | 20 |
| 50 | 080 | 712 | 208 | 959 | 437 | 218 | 751 | 628 | 10 |
| **35° 00′** | .6109 | .5736 | .8192 | .7002 | 1.428 | 1.221 | 1.743 | .9599 | **55° 00′** |
| 10 | 138 | 760 | 175 | 046 | 419 | 223 | 736 | 570 | 50 |
| 20 | 167 | 783 | 158 | 089 | 411 | 226 | 729 | 541 | 40 |
| 30 | .6196 | .5807 | .8141 | .7133 | 1.402 | 1.228 | 1.722 | .9512 | 30 |
| 40 | 225 | 831 | 124 | 177 | .393 | 231 | 715 | 483 | 20 |
| 50 | 254 | 854 | 107 | 221 | 385 | 233 | 708 | 454 | 10 |
| **36° 00′** | .6283 | .5878 | .8090 | .7265 | 1.376 | 1.236 | 1.701 | .9425 | **54° 00′** |
| | | Cos | Sin | Cot | Tan | Csc | Sec | Radians | Degrees |

| Degrees | Radians | Sin | Cos | Tan | Cot | Sec | Csc | | |
|---|---|---|---|---|---|---|---|---|---|
| **36° 00′** | .6283 | .5878 | .8090 | .7265 | 1.376 | 1.236 | 1.701 | .9425 | **54° 00′** |
| 10 | 312 | 901 | 073 | 310 | 368 | 239 | 695 | 396 | 50 |
| 20 | 341 | 925 | 056 | 355 | 360 | 241 | 688 | 367 | 40 |
| 30 | .6370 | .5948 | .8039 | .7400 | 1.351 | 1.244 | 1.681 | .9338 | 30 |
| 40 | 400 | 972 | 021 | 445 | 343 | 247 | 675 | 308 | 20 |
| 50 | 429 | 995 | 004 | 490 | 335 | 249 | 668 | 279 | 10 |
| **37° 00′** | .6458 | .6018 | .7986 | .7536 | 1.327 | 1.252 | 1.662 | .9250 | **53° 00′** |
| 10 | 487 | 041 | 969 | 581 | 319 | 255 | 655 | 221 | 50 |
| 20 | 516 | 065 | 951 | 627 | 311 | 258 | 649 | 192 | 40 |
| 30 | .6545 | .6088 | .7934 | .7673 | 1.303 | 1.260 | 1.643 | .9163 | 30 |
| 40 | 574 | 111 | 916 | 720 | 295 | 263 | 636 | 134 | 20 |
| 50 | 603 | 134 | 898 | 766 | 288 | 266 | 630 | 105 | 10 |
| **38° 00′** | .6632 | .6157 | .7880 | .7813 | 1.280 | 1.269 | 1.624 | .9076 | **52° 00′** |
| 10 | 661 | 180 | 862 | 860 | 272 | 272 | 618 | 047 | 50 |
| 20 | 690 | 202 | 844 | 907 | 265 | 275 | 612 | .9018 | 40 |
| 30 | .6720 | .6225 | .7826 | .7954 | 1.257 | 1.278 | 1.606 | .8988 | 30 |
| 40 | 749 | 248 | 808 | .8002 | 250 | 281 | 601 | 959 | 20 |
| 50 | 778 | 271 | 790 | 050 | 242 | 284 | 595 | 930 | 10 |
| **39° 00′** | .6807 | .6293 | .7771 | .8098 | 1.235 | 1.287 | 1.589 | .8901 | **51° 00′** |
| 10 | 836 | 316 | 753 | 146 | 228 | 290 | 583 | 872 | 50 |
| 20 | 865 | 338 | 735 | 195 | 220 | 293 | 578 | 843 | 40 |
| 30 | .6894 | .6361 | .7716 | .8243 | 1.213 | 1.296 | 1.572 | .8814 | 30 |
| 40 | 923 | 383 | 698 | 292 | 206 | 299 | 567 | 785 | 20 |
| 50 | 952 | 406 | 679 | 342 | 199 | 302 | 561 | 756 | 10 |
| **40° 00′** | .6981 | .6428 | .7660 | .8391 | 1.192 | 1.305 | 1.556 | .8727 | **50° 00′** |
| 10 | .7010 | 450 | 642 | 441 | 185 | 309 | 550 | 698 | 50 |
| 20 | 039 | 472 | 623 | 491 | 178 | 312 | 545 | 668 | 40 |
| 30 | .7069 | .6494 | .7604 | .8541 | 1.171 | 1.315 | 1.540 | .8639 | 30 |
| 40 | 098 | 517 | 585 | 591 | 164 | 318 | 535 | 610 | 20 |
| 50 | 127 | 539 | 566 | 642 | 157 | 322 | 529 | 581 | 10 |
| **41° 00′** | .7156 | .6561 | .7547 | .8693 | 1.150 | 1.325 | 1.524 | .8552 | **49° 00′** |
| 10 | 185 | 583 | 528 | 744 | 144 | 328 | 519 | 523 | 50 |
| 20 | 214 | 604 | 509 | 796 | 137 | 332 | 514 | 494 | 40 |
| 30 | .7243 | .6626 | .7490 | .8847 | 1.130 | 1.335 | 1.509 | .8465 | 30 |
| 40 | 272 | 648 | 470 | 899 | 124 | 339 | 504 | 436 | 20 |
| 50 | 301 | 670 | 451 | 952 | 117 | 342 | 499 | 407 | 10 |
| **42° 00′** | .7330 | .6691 | .7431 | .9004 | 1.111 | 1.346 | 1.494 | .8378 | **48° 00′** |
| 10 | 359 | 713 | 412 | 057 | 104 | 349 | 490 | 348 | 50 |
| 20 | 389 | 734 | 392 | 110 | 098 | 353 | 485 | 319 | 40 |
| 30 | .7418 | .6756 | .7373 | .9163 | 1.091 | 1.356 | 1.480 | .8290 | 30 |
| 40 | 447 | 777 | 353 | 217 | 085 | 360 | 476 | 261 | 20 |
| 50 | 476 | 799 | 333 | 271 | 079 | 364 | 471 | 232 | 10 |
| **43° 00′** | .7505 | .6820 | .7314 | .9325 | 1.072 | 1.367 | 1.466 | .8203 | **47° 00′** |
| 10 | 534 | 841 | 294 | 380 | 066 | 371 | 462 | 174 | 50 |
| 20 | 563 | 862 | 274 | 435 | 060 | 375 | 457 | 145 | 40 |
| 30 | .7592 | .6884 | .7254 | .9490 | 1.054 | 1.379 | 1.453 | .8116 | 30 |
| 40 | 621 | 905 | 234 | 545 | 048 | 382 | 448 | 087 | 20 |
| 50 | 650 | 926 | 214 | 601 | 042 | 386 | 444 | 058 | 10 |
| **44° 00′** | .7679 | .6947 | .7193 | .9657 | 1.036 | 1.390 | 1.440 | .8029 | **46° 00′** |
| 10 | 709 | 967 | 173 | 713 | 030 | 394 | 435 | 999 | 50 |
| 20 | 738 | 988 | 153 | 770 | 024 | 398 | 431 | 970 | 40 |
| 30 | .7767 | .7009 | .7133 | .9827 | 1.018 | 1.402 | 1.427 | .7941 | 30 |
| 40 | 796 | 030 | 112 | 884 | 012 | 406 | 423 | 912 | 20 |
| 50 | 825 | 050 | 092 | 942 | 006 | 410 | 418 | 883 | 10 |
| **45° 00′** | .7854 | .7071 | .7071 | 1.000 | 1.000 | 1.414 | 1.414 | .7854 | **45° 00′** |
| | | Cos | Sin | Cot | Tan | Csc | Sec | Radians | Degrees |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Proportional Parts 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.0** | 0000 | 043 | 086 | 128 | 170 | 212 | 253 | 294 | 334 | 374 | 4 | 8 | 12 | 17 | 21 | 25 | Use direct interpolation for greater accuracy for numbers between 1.000 and 2.000 | | |
| 1 | 414 | 453 | 492 | 531 | 569 | 607 | 645 | 682 | 719 | 755 | 4 | 8 | 11 | 15 | 19 | 23 | | | |
| 2 | 792 | 828 | 864 | 899 | 934 | 969 | *004 | *038 | *072 | *106 | 3 | 7 | 10 | 14 | 17 | 21 | | | |
| 3 | 1139 | 173 | 206 | 239 | 271 | 303 | 335 | 367 | 399 | 430 | 3 | 6 | 10 | 13 | 16 | 19 | | | |
| 4 | 461 | 492 | 523 | 553 | 584 | 614 | 644 | 673 | 703 | 732 | 3 | 6 | 9 | 12 | 15 | 18 | | | |
| **1.5** | 761 | 790 | 818 | 847 | 875 | 903 | 931 | 959 | 987 | *014 | 3 | 6 | 8 | 11 | 14 | 17 | | | |
| 6 | 2041 | 068 | 095 | 122 | 148 | 175 | 201 | 227 | 253 | 279 | 3 | 5 | 8 | 11 | 13 | 16 | | | |
| 7 | 304 | 330 | 355 | 380 | 405 | 430 | 455 | 480 | 504 | 529 | 2 | 5 | 7 | 10 | 12 | 15 | | | |
| 8 | 553 | 577 | 601 | 625 | 648 | 672 | 695 | 718 | 742 | 765 | 2 | 5 | 7 | 9 | 12 | 14 | | | |
| 9 | 788 | 810 | 833 | 856 | 878 | 900 | 923 | 945 | 967 | 989 | 2 | 4 | 7 | 9 | 11 | 13 | | | |
| **2.0** | 3010 | 032 | 054 | 075 | 096 | 118 | 139 | 160 | 181 | 201 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 1 | 222 | 243 | 263 | 284 | 304 | 324 | 345 | 365 | 385 | 404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 2 | 424 | 444 | 464 | 483 | 502 | 522 | 541 | 560 | 579 | 598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 17 |
| 3 | 617 | 636 | 655 | 674 | 692 | 711 | 729 | 747 | 766 | 784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 4 | 802 | 820 | 838 | 856 | 874 | 892 | 909 | 927 | 945 | 962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| **2.5** | 979 | 997 | *014 | *031 | *048 | *065 | *082 | *099 | *116 | *133 | 2 | 4 | 5 | 7 | 9 | 10 | 12 | 14 | 16 |
| 6 | 4150 | 166 | 183 | 200 | 216 | 232 | 249 | 265 | 281 | 298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 7 | 314 | 330 | 346 | 362 | 378 | 393 | 409 | 425 | 440 | 456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 8 | 472 | 487 | 502 | 518 | 533 | 548 | 564 | 579 | 594 | 609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 9 | 624 | 639 | 654 | 669 | 683 | 698 | 713 | 728 | 742 | 757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| **3.0** | 771 | 786 | 800 | 814 | 829 | 843 | 857 | 871 | 886 | 900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 1 | 914 | 928 | 942 | 955 | 969 | 983 | 997 | *011 | *024 | *038 | 1 | 3 | 4 | 5 | 7 | 8 | 10 | 11 | 12 |
| 2 | 5051 | 065 | 079 | 092 | 105 | 119 | 132 | 145 | 159 | 172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 3 | 185 | 198 | 211 | 224 | 237 | 250 | 263 | 276 | 289 | 302 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 4 | 315 | 328 | 340 | 353 | 366 | 378 | 391 | 403 | 416 | 428 | 1 | 2 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| **3.5** | 441 | 453 | 465 | 478 | 490 | 502 | 514 | 527 | 539 | 551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 6 | 563 | 575 | 587 | 599 | 611 | 623 | 635 | 647 | 658 | 670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 7 | 682 | 694 | 705 | 717 | 729 | 740 | 752 | 763 | 775 | 786 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 11 |
| 8 | 798 | 809 | 821 | 832 | 843 | 855 | 866 | 877 | 888 | 899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9 | 911 | 922 | 933 | 944 | 955 | 966 | 977 | 988 | 999 | *010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| **4.0** | 6021 | 031 | 042 | 053 | 064 | 075 | 085 | 096 | 107 | 117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 1 | 128 | 138 | 149 | 160 | 170 | 180 | 191 | 201 | 212 | 222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 232 | 243 | 253 | 263 | 274 | 284 | 294 | 304 | 314 | 325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 3 | 335 | 345 | 355 | 365 | 375 | 385 | 395 | 405 | 415 | 425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 435 | 444 | 454 | 464 | 474 | 484 | 493 | 503 | 513 | 522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **4.5** | 532 | 542 | 551 | 561 | 571 | 580 | 590 | 599 | 609 | 618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 628 | 637 | 646 | 656 | 665 | 675 | 684 | 693 | 702 | 712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 7 | 721 | 730 | 739 | 749 | 758 | 767 | 776 | 785 | 794 | 803 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 8 | 812 | 821 | 830 | 839 | 848 | 857 | 866 | 875 | 884 | 893 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 9 | 902 | 911 | 920 | 928 | 937 | 946 | 955 | 964 | 972 | 981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| **5.0** | 990 | 998 | *007 | *016 | *024 | *033 | *042 | *050 | *059 | *067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 7076 | 084 | 093 | 101 | 110 | 118 | 126 | 135 | 143 | 152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | 160 | 168 | 177 | 185 | 193 | 202 | 210 | 218 | 226 | 235 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 7 |
| 3 | 243 | 251 | 259 | 267 | 275 | 284 | 292 | 300 | 308 | 316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 4 | 324 | 332 | 340 | 348 | 356 | 364 | 372 | 380 | 388 | 396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| **N** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Proportional Parts | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | 1 2 3 | 4 5 6 | 7 8 9 |
| **5.5** | 7404 | 412 | 419 | 427 | 435 | 443 | 451 | 459 | 466 | 474 | 1 2 2 | 3 4 5 | 5 6 7 |
| 6 | 482 | 490 | 497 | 505 | 513 | 520 | 528 | 536 | 543 | 551 | 1 2 2 | 3 4 5 | 5 6 7 |
| 7 | 559 | 566 | 574 | 582 | 589 | 597 | 604 | 612 | 619 | 627 | 1 1 2 | 3 4 5 | 5 6 7 |
| 8 | 634 | 642 | 649 | 657 | 664 | 672 | 679 | 686 | 694 | 701 | 1 1 2 | 3 4 4 | 5 6 7 |
| 9 | 709 | 716 | 723 | 731 | 738 | 745 | 752 | 760 | 767 | 774 | 1 1 2 | 3 4 4 | 5 6 7 |
| **6.0** | 782 | 789 | 796 | 803 | 810 | 818 | 825 | 832 | 839 | 846 | 1 1 2 | 3 4 4 | 5 6 6 |
| 1 | 853 | 860 | 868 | 875 | 882 | 889 | 896 | 903 | 910 | 917 | 1 1 2 | 3 3 4 | 5 6 6 |
| 2 | 924 | 931 | 938 | 945 | 952 | 959 | 966 | 973 | 980 | 987 | 1 1 2 | 3 3 4 | 5 5 6 |
| 3 | 993 | *000 | *007 | *014 | *021 | *028 | *035 | *041 | *048 | *055 | 1 1 2 | 3 3 4 | 5 6 6 |
| 4 | 8062 | 069 | 075 | 082 | 089 | 096 | 102 | 109 | 116 | 122 | 1 1 2 | 3 3 4 | 5 5 6 |
| **6.5** | 129 | 136 | 142 | 149 | 156 | 162 | 169 | 176 | 182 | 189 | 1 1 2 | 3 3 4 | 5 5 6 |
| 6 | 195 | 202 | 209 | 215 | 222 | 228 | 235 | 241 | 248 | 254 | 1 1 2 | 3 3 4 | 5 5 6 |
| 7 | 261 | 267 | 274 | 280 | 287 | 293 | 299 | 306 | 312 | 319 | 1 1 2 | 3 3 4 | 5 5 6 |
| 8 | 325 | 331 | 338 | 344 | 351 | 357 | 363 | 370 | 376 | 382 | 1 1 2 | 3 3 4 | 4 5 6 |
| 9 | 388 | 395 | 401 | 407 | 414 | 420 | 426 | 432 | 439 | 445 | 1 1 2 | 3 3 4 | 4 5 6 |
| **7.0** | 451 | 457 | 463 | 470 | 476 | 482 | 488 | 494 | 500 | 506 | 1 1 2 | 3 3 4 | 4 5 6 |
| 1 | 513 | 519 | 525 | 531 | 537 | 543 | 549 | 555 | 561 | 567 | 1 1 2 | 3 3 4 | 4 5 6 |
| 2 | 573 | 579 | 585 | 591 | 597 | 603 | 609 | 615 | 621 | 627 | 1 1 2 | 3 3 4 | 4 5 6 |
| 3 | 633 | 639 | 645 | 651 | 657 | 663 | 669 | 675 | 681 | 686 | 1 1 2 | 2 3 4 | 4 5 5 |
| 4 | 692 | 698 | 704 | 710 | 716 | 722 | 727 | 733 | 739 | 745 | 1 1 2 | 2 3 4 | 4 5 5 |
| **7.5** | 751 | 756 | 762 | 768 | 774 | 779 | 785 | 791 | 797 | 802 | 1 1 2 | 2 3 3 | 4 5 5 |
| 6 | 808 | 814 | 820 | 825 | 831 | 837 | 842 | 848 | 854 | 859 | 1 1 2 | 2 3 3 | 4 4 5 |
| 7 | 865 | 871 | 876 | 882 | 887 | 893 | 899 | 904 | 910 | 915 | 1 1 2 | 2 3 3 | 4 4 5 |
| 8 | 921 | 927 | 932 | 938 | 943 | 949 | 954 | 960 | 965 | 971 | 1 1 2 | 2 3 3 | 4 4 5 |
| 9 | 976 | 982 | 987 | 993 | 998 | *004 | *009 | *015 | *020 | *025 | 1 1 2 | 2 3 3 | 4 4 5 |
| **8.0** | 9031 | 036 | 042 | 047 | 053 | 058 | 063 | 069 | 074 | 079 | 1 1 2 | 2 3 3 | 4 4 5 |
| 1 | 085 | 090 | 096 | 101 | 106 | 112 | 117 | 122 | 128 | 133 | 1 1 2 | 2 3 3 | 4 4 5 |
| 2 | 138 | 143 | 149 | 154 | 159 | 165 | 170 | 175 | 180 | 186 | 1 1 2 | 2 3 3 | 4 4 5 |
| 3 | 191 | 196 | 201 | 206 | 212 | 217 | 222 | 227 | 232 | 238 | 1 1 2 | 2 3 3 | 4 4 5 |
| 4 | 243 | 248 | 253 | 258 | 263 | 269 | 274 | 279 | 284 | 289 | 1 1 2 | 2 3 3 | 4 4 5 |
| **8.5** | 294 | 299 | 304 | 309 | 315 | 320 | 325 | 330 | 335 | 340 | 1 1 2 | 2 3 3 | 4 4 5 |
| 6 | 345 | 350 | 355 | 360 | 365 | 370 | 375 | 380 | 385 | 390 | 1 1 2 | 2 3 3 | 4 4 5 |
| 7 | 395 | 400 | 405 | 410 | 415 | 420 | 425 | 430 | 435 | 440 | 1 1 2 | 2 3 3 | 4 4 5 |
| 8 | 445 | 450 | 455 | 460 | 465 | 469 | 474 | 479 | 484 | 489 | 0 1 1 | 2 2 3 | 3 4 4 |
| 9 | 494 | 499 | 504 | 509 | 513 | 518 | 523 | 528 | 533 | 538 | 0 1 1 | 2 2 3 | 3 4 4 |
| **9.0** | 542 | 547 | 552 | 557 | 562 | 566 | 571 | 576 | 581 | 586 | 0 1 1 | 2 2 3 | 3 4 4 |
| 1 | 590 | 595 | 600 | 605 | 609 | 614 | 619 | 624 | 628 | 633 | 0 1 1 | 2 2 3 | 3 4 4 |
| 2 | 638 | 643 | 647 | 652 | 657 | 661 | 666 | 671 | 675 | 680 | 0 1 1 | 2 2 3 | 3 4 4 |
| 3 | 685 | 689 | 694 | 699 | 703 | 708 | 713 | 717 | 722 | 727 | 0 1 1 | 2 2 3 | 3 4 4 |
| 4 | 731 | 736 | 741 | 745 | 750 | 754 | 759 | 763 | 768 | 773 | 0 1 1 | 2 2 3 | 3 4 4 |
| **9.5** | 777 | 782 | 786 | 791 | 795 | 800 | 805 | 809 | 814 | 818 | 0 1 1 | 2 2 3 | 3 4 4 |
| 6 | 823 | 827 | 832 | 836 | 841 | 845 | 850 | 854 | 859 | 863 | 0 1 1 | 2 2 3 | 3 4 4 |
| 7 | 868 | 872 | 877 | 881 | 886 | 890 | 894 | 899 | 903 | 908 | 0 1 1 | 2 2 3 | 3 4 4 |
| 8 | 912 | 917 | 921 | 926 | 930 | 934 | 939 | 943 | 948 | 952 | 0 1 1 | 2 2 3 | 3 3 4 |
| 9 | 956 | 961 | 965 | 969 | 974 | 978 | 983 | 987 | 991 | 996 | 0 1 1 | 2 2 3 | 3 3 4 |
| **N** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | 1 2 3 | 4 5 6 | 7 8 9 |

# TABLE IV

## FOUR-PLACE LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

NOTE 1. — For simplicity in printing, all characteristics have been increased by 10. Hence 10 must be subtracted from each tabulated value of a logarithm.

NOTE 2. — To avoid interpolating for angles between 0° and 3° or 87° and 90° use Tables V *a* or V *b*.

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **0° 0′** | | | | | | 10.0000 | | **90° 0′** |
| 10′ | 7.4637 | | 7.4637 | | 12.5363 | .0000 | .0 | 50′ |
| 20′ | .7648 | 301.1 | .7648 | 301.1 | .2352 | .0000 | .0 | 40′ |
| 30′ | .9408 | 176.0 | .9409 | 176.1 | .0591 | .0000 | .0 | 30′ |
| 40′ | 8.0658 | 125.0 | 8.0658 | 124.9 | 11.9342 | .0000 | .0 | 20′ |
| 50′ | .1627 | 96.9 | .1627 | 96.9 | .8373 | .0000 | .0 | 10′ |
| **1° 0′** | 8.2419 | 79.2 | 8.2419 | 79.2 | 11.7581 | 9.9999 | .1 | **89° 0′** |
| 10′ | .3088 | 66.9 | .3089 | 67.0 | .6911 | .9999 | .0 | 50′ |
| 20′ | .3668 | 58.0 | .3669 | 58.0 | .6331 | .9999 | .0 | 40′ |
| 30′ | .4179 | 51.1 | .4181 | 51.2 | .5819 | .9999 | .0 | 30′ |
| 40′ | .4637 | 45.8 | .4638 | 45.7 | .5362 | .9998 | .1 | 20′ |
| 50′ | .5050 | 41.3 | .5053 | 41.5 | .4947 | .9998 | .0 | 10′ |
| **2° 0′** | 8.5428 | 37.8 | 8.5431 | 37.8 | 11.4569 | 9.9997 | .1 | **88° 0′** |
| 10′ | .5776 | 34.8 | .5779 | 34.8 | .4221 | .9997 | .0 | 50′ |
| 20′ | .6097 | 32.1 | .6101 | 32.2 | .3899 | .9996 | .1 | 40′ |
| 30′ | .6397 | 30.0 | .6401 | 30.0 | .3599 | .9996 | .0 | 30′ |
| 40′ | .6677 | 28.0 | .6682 | 28.1 | .3318 | .9995 | .1 | 20′ |
| 50′ | .6940 | 26.3 | .6945 | 26.3 | .3055 | .9995 | .0 | 10′ |
| **3° 0′** | 8.7188 | 24.8 | 8.7194 | 24.9 | 11.2806 | 9.9994 | .1 | **87° 0′** |
| 10′ | .7423 | 23.5 | .7429 | 23.5 | .2571 | .9993 | .1 | 50′ |
| 20′ | .7645 | 22.2 | .7652 | 22.3 | .2348 | .9993 | .0 | 40′ |
| 30′ | .7857 | 21.2 | .7865 | 21.3 | .2135 | .9992 | .1 | 30′ |
| 40′ | .8059 | 20.2 | .8067 | 20.2 | .1933 | .9991 | .1 | 20′ |
| 50′ | .8251 | 19.2 | .8261 | 19.4 | .1739 | .9990 | .1 | 10 |
| **4° 0′** | 8.8436 | 18.5 | 8.8446 | 18.5 | 11.1554 | 9.9989 | .1 | **86° 0′** |
| 10′ | .8613 | 17.7 | .8624 | 17.8 | .1376 | .9989 | .0 | 50′ |
| 20′ | .8783 | 17.0 | .8795 | 17.1 | .1205 | .9988 | .1 | 40′ |
| 30′ | .8946 | 16.3 | .8960 | 16.5 | .1040 | .9987 | .1 | 30′ |
| 40′ | .9104 | 15.8 | .9118 | 15.8 | .0882 | .9986 | .1 | 20′ |
| 50′ | .9256 | 15.2 | .9272 | 15.4 | .0728 | .9985 | .1 | 10′ |
| **5° 0′** | 8.9403 | 14.7 | 8.9420 | 14.8 | 11.0580 | 9.9983 | .2 | **85° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **5° 0′** | 8.9403 | 14.2 | 8.9420 | 14.3 | 11.0580 | 9.9983 | .1 | **85° 0′** |
| 10′ | .9545 | 13.7 | .9563 | 13.8 | .0437 | .9982 | .1 | 50′ |
| 20′ | .9682 | 13.4 | .9701 | 13.5 | .0299 | .9981 | .1 | 40′ |
| 30′ | .9816 | 12.9 | .9836 | 13.0 | .0164 | .9980 | .1 | 30′ |
| 40′ | .9945 | 12.5 | .9966 | 12.7 | .0034 | .9979 | .2 | 20′ |
| 50′ | 9.0070 | 12.2 | 9.0093 | 12.3 | 10.9907 | .9977 | .1 | 10′ |
| **6° 0′** | 9.0192 | 11.9 | 9.0216 | 12.0 | 10.9784 | 9.9976 | .1 | **84° 0′** |
| 10′ | .0311 | 11.5 | .0336 | 11.7 | .9664 | .9975 | .2 | 50′ |
| 20′ | .0426 | 11.3 | .0453 | 11.4 | .9547 | .9973 | .1 | 40′ |
| 30′ | .0539 | 10.9 | .0567 | 11.1 | .9433 | .9972 | .1 | 30′ |
| 40′ | .0648 | 10.7 | .0678 | 10.8 | .9322 | .9971 | .2 | 20′ |
| 50′ | .0755 | 10.4 | .0786 | 10.5 | .9214 | .9969 | .1 | 10′ |
| **7° 0′** | 9.0859 | 10.2 | 9.0891 | 10.4 | 10.9109 | 9.9968 | .2 | **83° 0′** |
| 10′ | .0961 | 9.9 | .0995 | 10.1 | .9005 | .9966 | .2 | 50′ |
| 20′ | .1060 | 9.7 | .1096 | 9.8 | .8904 | .9964 | .1 | 40′ |
| 30′ | .1157 | 9.5 | .1194 | 9.7 | .8806 | .9963 | .2 | 30′ |
| 40′ | .1252 | 9.3 | .1291 | 9.4 | .8709 | .9961 | .2 | 20′ |
| 50′ | .1345 | 9.1 | .1385 | 9.3 | .8615 | .9959 | .1 | 10′ |
| **8° 0′** | 9.1436 | 8.9 | 9.1478 | 9.1 | 10.8522 | 9.9958 | .2 | **82° 0′** |
| 10′ | .1525 | 8.7 | .1569 | 8.9 | .8431 | .9956 | .2 | 50′ |
| 20′ | .1612 | 8.5 | .1658 | 8.7 | .8342 | .9954 | .2 | 40′ |
| 30′ | .1697 | 8.4 | .1745 | 8.6 | .8255 | .9952 | .2 | 30′ |
| 40′ | .1781 | 8.2 | .1831 | 8.4 | .8169 | .9950 | .2 | 20′ |
| 50′ | .1863 | 8.0 | .1915 | 8.2 | .8085 | .9948 | .2 | 10′ |
| **9° 0′** | 9.1943 | 7.9 | 9.1997 | 8.1 | 10.8003 | 9.9946 | .2 | **81° 0′** |
| 10′ | .2022 | 7.8 | .2078 | 8.0 | .7922 | .9944 | .2 | 50′ |
| 20′ | .2100 | 7.6 | .2158 | 7.8 | .7842 | .9942 | .2 | 40′ |
| 30′ | .2176 | 7.5 | .2236 | 7.7 | .7764 | .9940 | .2 | 30′ |
| 40′ | .2251 | 7.3 | .2313 | 7.6 | .7687 | .9938 | .2 | 20′ |
| 50′ | .2324 | 7.3 | .2389 | 7.4 | .7611 | .9936 | .2 | 10′ |
| **10° 0′** | 9.2397 | 7.1 | 9.2463 | 7.3 | 10.7537 | 9.9934 | .3 | **80° 0′** |
| 10′ | .2468 | 7.0 | .2536 | 7.3 | .7464 | .9931 | .2 | 50′ |
| 20′ | .2538 | 6.8 | .2609 | 7.1 | .7391 | .9929 | .2 | 40′ |
| 30′ | .2606 | 6.8 | .2680 | 7.0 | .7320 | .9927 | .3 | 30′ |
| 40′ | .2674 | 6.6 | .2750 | 6.9 | .7250 | .9924 | .2 | 20′ |
| 50′ | .2740 | 6.6 | .2819 | 6.8 | .7181 | .9922 | .3 | 10′ |
| **11° 0′** | 9.2806 | 6.4 | 9.2887 | 6.6 | 10.7113 | 9.9919 | .2 | **79° 0′** |
| 10′ | .2870 | 6.4 | .2953 | 6.7 | .7047 | .9917 | .3 | 50′ |
| 20′ | .2934 | 6.3 | .3020 | 6.5 | .6980 | .9914 | .2 | 40′ |
| 30′ | .2997 | 6.1 | .3085 | 6.4 | .6915 | .9912 | .3 | 30′ |
| 40′ | .3058 | 6.1 | .3149 | 6.3 | .6851 | .9909 | .2 | 20′ |
| 50′ | .3119 | 6.0 | .3212 | 6.3 | .6788 | .9907 | .3 | 10′ |
| **12° 0′** | 9.3179 | 5.9 | 9.3275 | 6.1 | 10.6725 | 9.9904 | .3 | **78° 0′** |
| 10′ | .3238 | 5.8 | .3336 | 6.1 | .6664 | .9901 | .2 | 50′ |
| 20′ | .3296 | 5.7 | .3397 | 6.1 | .6603 | .9899 | .3 | 40′ |
| 30′ | .3353 | 5.7 | .3458 | 5.9 | .6542 | .9896 | .3 | 30′ |
| 40′ | .3410 | 5.6 | .3517 | 5.9 | .6483 | .9893 | .3 | 20′ |
| 50′ | .3466 | 5.5 | .3576 | 5.8 | .6424 | .9890 | .3 | 10′ |
| **13° 0′** | 9.3521 | | 9.3634 | | 10.6366 | 9.9887 | | **77° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **13° 0′** | 9.3521 | 5.4 | 9.3634 | 5.7 | 10.6366 | 9.9887 | .3 | **77° 0′** |
| 10′ | .3575 | 5.4 | .3691 | 5.7 | .6309 | .9884 | .3 | 50′ |
| 20′ | .3629 | 5.3 | .3748 | 5.6 | .6252 | .9881 | .3 | 40′ |
| 30′ | .3682 | 5.2 | .3804 | 5.5 | .6196 | .9878 | .3 | 30′ |
| 40′ | .3734 | 5.2 | .3859 | 5.5 | .6141 | .9875 | .3 | 20′ |
| 50′ | .3786 | 5.1 | .3914 | 5.4 | .6086 | .9872 | .3 | 10′ |
| **14° 0′** | 9.3837 | 5.0 | 9.3968 | 5.3 | 10.6032 | 9.9869 | .3 | **76° 0′** |
| 10′ | .3887 | 5.0 | .4021 | 5.3 | .5979 | .9866 | .3 | 50′ |
| 20′ | .3937 | 4.9 | .4074 | 5.3 | .5926 | .9863 | .4 | 40′ |
| 30′ | .3986 | 4.9 | .4127 | 5.1 | .5873 | .9859 | .3 | 30′ |
| 40′ | .4035 | 4.8 | .4178 | 5.2 | .5822 | .9856 | .3 | 20′ |
| 50′ | .4083 | 4.7 | .4230 | 5.1 | .5770 | .9853 | .4 | 10′ |
| **15° 0′** | 9.4130 | 4.7 | 9.4281 | 5.0 | 10.5719 | 9.9849 | .3 | **75° 0′** |
| 10′ | .4177 | 4.6 | .4331 | 5.0 | .5669 | .9846 | .3 | 50′ |
| 20′ | .4223 | 4.6 | .4381 | 4.9 | .5619 | .9843 | .4 | 40′ |
| 30′ | .4269 | 4.5 | .4430 | 4.9 | .5570 | .9839 | .3 | 30′ |
| 40′ | .4314 | 4.5 | .4479 | 4.8 | .5521 | .9836 | .4 | 20′ |
| 50′ | .4359 | 4.4 | .4527 | 4.8 | .5473 | .9832 | .4 | 10′ |
| **16° 0′** | 9.4403 | 4.4 | 9.4575 | 4.7 | 10.5425 | 9.9828 | .3 | **74° 0′** |
| 10′ | .4447 | 4.4 | .4622 | 4.7 | .5378 | .9825 | .4 | 50′ |
| 20′ | .4491 | 4.2 | .4669 | 4.7 | .5331 | .9821 | .4 | 40′ |
| 30′ | .4533 | 4.3 | .4716 | 4.6 | .5284 | .9817 | .3 | 30′ |
| 40 | .4576 | 4.2 | .4762 | 4.6 | .5238 | .9814 | .4 | 20′ |
| 50′ | .4618 | 4.1 | .4808 | 4.5 | .5192 | .9810 | .4 | 10′ |
| **17° 0′** | 9.4659 | 4.1 | 9.4853 | 4.5 | 10.5147 | 9.9806 | .4 | **73° 0′** |
| 10′ | .4700 | 4.1 | .4898 | 4.5 | .5102 | .9802 | .4 | 50′ |
| 20′ | .4741 | 4.0 | .4943 | 4.4 | .5057 | .9798 | .4 | 40′ |
| 30′ | .4781 | 4.0 | .4987 | 4.4 | .5013 | .9794 | .4 | 30′ |
| 40′ | .4821 | 4.0 | .5031 | 4.4 | .4969 | .9790 | .4 | 20′ |
| 50′ | .4861 | 3.9 | .5075 | 4.3 | .4925 | .9786 | .4 | 10′ |
| **18° 0′** | 9.4900 | 3.9 | 9.5118 | 4.3 | 10.4882 | 9.9782 | .4 | **72° 0′** |
| 10′ | .4939 | 3.8 | .5161 | 4.2 | .4839 | .9778 | .4 | 50′ |
| 20′ | .4977 | 3.8 | .5203 | 4.2 | .4797 | .9774 | .4 | 40′ |
| 30′ | .5015 | 3.7 | .5245 | 4.2 | .4755 | .9770 | .5 | 30′ |
| 40′ | .5052 | 3.8 | .5287 | 4.2 | .4713 | .9765 | .4 | 20′ |
| 50′ | .5090 | 3.6 | .5329 | 4.1 | .4671 | .9761 | .4 | 10′ |
| **19° 0′** | 9.5126 | 3.7 | 9.5370 | 4.1 | 10.4630 | 9.9757 | .5 | **71° 0′** |
| 10′ | .5163 | 3.6 | .5411 | 4.0 | .4589 | .9752 | .4 | 50′ |
| 20′ | .5199 | 3.6 | .5451 | 4.0 | .4549 | .9748 | .5 | 40′ |
| 30′ | .5235 | 3.5 | .5491 | 4.0 | .4509 | .9743 | .4 | 30′ |
| 40′ | .5270 | 3.6 | .5531 | 4.0 | .4469 | .9739 | .5 | 20′ |
| 50′ | .5306 | 3.5 | .5571 | 4.0 | .4429 | .9734 | .4 | 10′ |
| **20° 0′** | 9.5341 | 3.4 | 9.5611 | 3.9 | 10.4389 | 9.9730 | .5 | **70° 0′** |
| 10′ | .5375 | 3.4 | .5650 | 3.9 | .4350 | .9725 | .4 | 50′ |
| 20′ | .5409 | 3.4 | .5689 | 3.8 | .4311 | .9721 | .5 | 40′ |
| 30′ | .5443 | 3.4 | .5727 | 3.9 | .4273 | .9716 | .5 | 30′ |
| 40′ | .5477 | 3.3 | .5766 | 3.8 | .4234 | .9711 | .5 | 20′ |
| 50′ | .5510 | 3.3 | .5804 | 3.8 | .4196 | .9706 | .4 | 10′ |
| **21° 0′** | 9.5543 | | 9.5842 | | 10.4158 | 9.9702 | | **69° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **21° 0′** | 9.5543 | 3.3 | 9.5842 | 3.7 | 10.4158 | 9.9702 | .5 | **69° 0′** |
| 10′ | .5576 | 3.3 | .5879 | 3.8 | .4121 | .9697 | .5 | 50′ |
| 20′ | .5609 | 3.2 | .5917 | 3.7 | .4083 | .9692 | .5 | 40′ |
| 30′ | .5641 | 3.2 | .5954 | 3.7 | .4046 | .9687 | .5 | 30′ |
| 40′ | .5673 | 3.1 | .5991 | 3.7 | .4009 | .9682 | .5 | 20′ |
| 50′ | .5704 | 3.2 | .6028 | 3.6 | .3972 | .9677 | .5 | 10′ |
| **22° 0′** | 9.5736 | 3.1 | 9.6064 | 3.6 | 10.3936 | 9.9672 | .5 | **68° 0′** |
| 10′ | .5767 | 3.1 | .6100 | 3.6 | .3900 | .9667 | .6 | 50′ |
| 20′ | .5798 | 3.0 | .6136 | 3.6 | .3864 | .9661 | .5 | 40′ |
| 30′ | .5828 | 3.1 | .6172 | 3.6 | .3828 | .9656 | .5 | 30′ |
| 40′ | .5859 | 3.0 | .6208 | 3.5 | .3792 | .9651 | .5 | 20′ |
| 50′ | .5889 | 3.0 | .6243 | 3.6 | .3757 | .9646 | .6 | 10′ |
| **23° 0′** | 9.5919 | 2.9 | 9.6279 | 3.5 | 10.3721 | 9.9640 | .5 | **67° 0′** |
| 10′ | .5948 | 3.0 | .6314 | 3.4 | .3686 | .9635 | .6 | 50′ |
| 20′ | .5978 | 2.9 | .6348 | 3.5 | .3652 | .9629 | .5 | 40′ |
| 30′ | .6007 | 2.9 | .6383 | 3.4 | .3617 | .9624 | .6 | 30′ |
| 40′ | .6036 | 2.9 | .6417 | 3.5 | .3583 | .9618 | .5 | 20′ |
| 50′ | .6065 | 2.8 | .6452 | 3.4 | .3548 | .9613 | .6 | 10′ |
| **24° 0′** | 9.6093 | 2.8 | 9.6486 | 3.4 | 10.3514 | 9.9607 | .5 | **66° 0′** |
| 10′ | .6121 | 2.8 | .6520 | 3.3 | .3480 | .9602 | .6 | 50′ |
| 20′ | .6149 | 2.8 | .6553 | 3.4 | .3447 | .9596 | .6 | 40′ |
| 30′ | .6177 | 2.8 | .6587 | 3.3 | .3413 | .9590 | .6 | 30′ |
| 40′ | .6205 | 2.7 | .6620 | 3.4 | .3380 | .9584 | .5 | 20′ |
| 50′ | .6232 | 2.7 | .6654 | 3.3 | .3346 | .9579 | .6 | 10′ |
| **25° 0′** | 9.6259 | 2.7 | 9.6687 | 3.3 | 10.3313 | 9.9573 | .6 | **65° 0′** |
| 10′ | .6286 | 2.7 | .6720 | 3.2 | .3280 | .9567 | .6 | 50′ |
| 20′ | .6313 | 2.7 | .6752 | 3.3 | .3248 | .9561 | .6 | 40′ |
| 30′ | .6340 | 2.6 | .6785 | 3.2 | .3215 | .9555 | .6 | 30′ |
| 40′ | .6366 | 2.6 | .6817 | 3.3 | .3183 | .9549 | .6 | 20′ |
| 50′ | .6392 | 2.6 | .6850 | 3.2 | .3150 | .9543 | .6 | 10′ |
| **26° 0′** | 9.6418 | 2.6 | 9.6882 | 3.2 | 10.3118 | 9.9537 | .7 | **64° 0′** |
| 10′ | .6444 | 2.6 | .6914 | 3.2 | .3086 | .9530 | .6 | 50′ |
| 20′ | .6470 | 2.5 | .6946 | 3.1 | .3054 | .9524 | .6 | 40′ |
| 30′ | .6495 | 2.6 | .6977 | 3.2 | .3023 | .9518 | .6 | 30′ |
| 40′ | .6521 | 2.5 | .7009 | 3.1 | .2991 | .9512 | .7 | 20′ |
| 50′ | .6546 | 2.4 | .7040 | 3.2 | .2960 | .9505 | .6 | 10′ |
| **27° 0′** | 9.6570 | 2.5 | 9.7072 | 3.1 | 10.2928 | 9.9499 | .7 | **63° 0′** |
| 10′ | .6595 | 2.5 | .7103 | 3.1 | .2897 | .9492 | .6 | 50′ |
| 20′ | .6620 | 2.4 | .7134 | 3.1 | .2866 | .9486 | .7 | 40′ |
| 30′ | .6644 | 2.4 | .7165 | 3.1 | .2835 | .9479 | .6 | 30′ |
| 40′ | .6668 | 2.4 | .7196 | 3.0 | .2804 | .9473 | .7 | 20′ |
| 50′ | .6692 | 2.4 | .7226 | 3.1 | .2774 | .9466 | .7 | 10′ |
| **28° 0′** | 9.6716 | 2.4 | 9.7257 | 3.0 | 10.2743 | 9.9459 | .6 | **62° 0′** |
| 10′ | .6740 | 2.3 | .7287 | 3.0 | .2713 | .9453 | .7 | 50′ |
| 20′ | .6763 | 2.4 | .7317 | 3.1 | .2683 | .9446 | .7 | 40′ |
| 30′ | .6787 | 2.3 | .7348 | 3.0 | .2652 | .9439 | .7 | 30′ |
| 40′ | .6810 | 2.3 | .7378 | 3.0 | .2622 | .9432 | .7 | 20′ |
| 50′ | .6833 | 2.3 | .7408 | 3.0 | .2592 | .9425 | .7 | 10′ |
| **29° 0′** | 9.6856 | | 9.7438 | | 10.2562 | 9.9418 | | **61° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **29° 0′** | 9.6856 | 2.2 | 9.7438 | 2.9 | 10.2562 | 9.9418 | .7 | **61° 0′** |
| 10′ | .6878 | 2.3 | .7467 | 3.0 | .2533 | .9411 | .7 | 50′ |
| 20′ | .6901 | 2.2 | .7497 | 2.9 | .2503 | .9404 | .7 | 40′ |
| 30′ | .6923 | 2.3 | .7526 | 3.0 | .2474 | .9397 | .7 | 30′ |
| 40′ | .6946 | 2.2 | .7556 | 2.9 | .2444 | .9390 | .7 | 20′ |
| 50′ | .6968 | 2.2 | .7585 | 2.9 | .2415 | .9383 | .8 | 10′ |
| **30° 0′** | 9.6990 | 2.2 | 9.7614 | 3.0 | 10.2386 | 9.9375 | .7 | **60° 0′** |
| 10′ | .7012 | 2.1 | .7644 | 2.9 | .2356 | .9368 | .7 | 50′ |
| 20′ | .7033 | 2.2 | .7673 | 2.8 | .2327 | .9361 | .8 | 40′ |
| 30′ | .7055 | 2.1 | .7701 | 2.9 | .2299 | .9353 | .7 | 30′ |
| 40′ | .7076 | 2.1 | .7730 | 2.9 | .2270 | .9346 | .8 | 20′ |
| 50′ | .7097 | 2.1 | .7759 | 2.9 | .2241 | .9338 | .7 | 10′ |
| **31° 0′** | 9.7118 | 2.1 | 9.7788 | 2.8 | 10.2212 | 9.9331 | .8 | **59° 0′** |
| 10′ | .7139 | 2.1 | .7816 | 2.9 | .2184 | .9323 | .8 | 50′ |
| 20′ | .7160 | 2.1 | .7845 | 2.8 | .2155 | .9315 | .7 | 40′ |
| 30′ | .7181 | 2.0 | .7873 | 2.9 | .2127 | .9308 | .8 | 30′ |
| 40′ | .7201 | 2.1 | .7902 | 2.8 | .2098 | .9300 | .8 | 20′ |
| 50′ | .7222 | 2.0 | .7930 | 2.8 | .2070 | .9292 | .8 | 10′ |
| **32° 0′** | 9.7242 | 2.0 | 9.7958 | 2.8 | 10.2042 | 9.9284 | .8 | **58° 0′** |
| 10′ | .7262 | 2.0 | .7986 | 2.8 | .2014 | .9276 | .8 | 50′ |
| 20′ | .7282 | 2.0 | .8014 | 2.8 | .1986 | .9268 | .8 | 40′ |
| 30′ | .7302 | 2.0 | .8042 | 2.8 | .1958 | .9260 | .8 | 30′ |
| 40′ | .7322 | 2.0 | .8070 | 2.7 | .1930 | .9252 | .8 | 20′ |
| 50′ | .7342 | 1.9 | .8097 | 2.8 | .1903 | .9244 | .8 | 10′ |
| **33° 0′** | 9.7361 | 1.9 | 9.8125 | 2.8 | 10.1875 | 9.9236 | .8 | **57° 0′** |
| 10′ | .7380 | 2.0 | .8153 | 2.7 | .1847 | .9228 | .9 | 50′ |
| 20′ | .7400 | 1.9 | .8180 | 2.8 | .1820 | .9219 | .8 | 40′ |
| 30′ | .7419 | 1.9 | .8208 | 2.7 | .1792 | .9211 | .8 | 30′ |
| 40′ | .7438 | 1.9 | .8235 | 2.8 | .1765 | .9203 | .9 | 20′ |
| 50′ | .7457 | 1.9 | .8263 | 2.7 | .1737 | .9194 | .8 | 10′ |
| **34° 0′** | 9.7476 | 1.8 | 9.8290 | 2.7 | 10.1710 | 9.9186 | .9 | **56° 0′** |
| 10′ | .7494 | 1.9 | .8317 | 2.7 | .1683 | .9177 | .8 | 50′ |
| 20′ | .7513 | 1.8 | .8344 | 2.7 | .1656 | .9169 | .9 | 40′ |
| 30′ | .7531 | 1.9 | .8371 | 2.7 | .1629 | .9160 | .9 | 30′ |
| 40′ | .7550 | 1.8 | .8398 | 2.7 | .1602 | .9151 | .9 | 20′ |
| 50′ | .7568 | 1.8 | .8425 | 2.7 | .1575 | .9142 | .8 | 10′ |
| **35° 0′** | 9.7586 | 1.8 | 9.8452 | 2.7 | 10.1548 | 9.9134 | .9 | **55° 0′** |
| 10′ | .7604 | 1.8 | .8479 | 2.7 | .1521 | .9125 | .9 | 50′ |
| 20′ | .7622 | 1.8 | .8506 | 2.7 | .1494 | .9116 | .9 | 40′ |
| 30′ | .7640 | 1.7 | .8533 | 2.6 | .1467 | .9107 | .9 | 30′ |
| 40′ | .7657 | 1.8 | .8559 | 2.7 | .1441 | .9098 | .9 | 20′ |
| 50′ | .7675 | 1.7 | .8586 | 2.7 | .1414 | .9089 | .9 | 10′ |
| **36° 0′** | 9.7692 | 1.8 | 9.8613 | 2.6 | 10.1387 | 9.9080 | 1.0 | **54° 0′** |
| 10′ | .7710 | 1.7 | .8639 | 2.7 | .1361 | .9070 | .9 | 50′ |
| 20′ | .7727 | 1.7 | .8666 | 2.6 | .1334 | .9061 | .9 | 40′ |
| 30′ | .7744 | 1.7 | .8692 | 2.6 | .1308 | .9052 | 1.0 | 30′ |
| 40′ | .7761 | 1.7 | .8718 | 2.7 | .1282 | .9042 | .9 | 20′ |
| 50′ | .7778 | 1.7 | .8745 | 2.6 | .1255 | .9033 | 1.0 | 10′ |
| **37° 0′** | 9.7795 | | 9.8771 | | 10.1229 | 9.9023 | | **53° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

| Angle | L Sin | d 1′ | L Tan | c d 1′ | L Cot | L Cos | d 1′ | |
|---|---|---|---|---|---|---|---|---|
| **37° 0′** | 9.7795 | 1.6 | 9.8771 | 2.6 | 10.1229 | 9.9023 | .9 | **53° 0′** |
| 10′ | .7811 | 1.7 | .8797 | 2.7 | .1203 | .9014 | 1.0 | 50′ |
| 20′ | .7828 | 1.6 | .8824 | 2.6 | .1176 | .9004 | .9 | 40′ |
| 30′ | .7844 | 1.7 | .8850 | 2.6 | .1150 | .8995 | 1.0 | 30′ |
| 40′ | .7861 | 1.6 | .8876 | 2.6 | .1124 | .8985 | 1.0 | 20′ |
| 50′ | .7877 | 1.6 | .8902 | 2.6 | .1098 | .8975 | 1.0 | 10′ |
| **38° 0′** | 9.7893 | 1.7 | 9.8928 | 2.6 | 10.1072 | 9.8965 | 1.0 | **52° 0′** |
| 10′ | .7910 | 1.6 | .8954 | 2.6 | .1046 | .8955 | 1.0 | 50′ |
| 20′ | .7926 | 1.5 | .8980 | 2.6 | .1020 | .8945 | 1.0 | 40′ |
| 30′ | .7941 | 1.6 | .9006 | 2.6 | .0994 | .8935 | 1.0 | 30′ |
| 40′ | .7957 | 1.6 | .9032 | 2.6 | .0968 | .8925 | 1.0 | 20′ |
| 50′ | .7973 | 1.6 | .9058 | 2.6 | .0942 | .8915 | 1.0 | 10′ |
| **39° 0′** | 9.7989 | 1.5 | 9.9084 | 2.6 | 10.0916 | 9.8905 | 1.0 | **51° 0′** |
| 10′ | .8004 | 1.6 | .9110 | 2.5 | .0890 | .8895 | 1.1 | 50′ |
| 20′ | .8020 | 1.5 | .9135 | 2.6 | .0865 | .8884 | 1.0 | 40′ |
| 30′ | .8035 | 1.5 | .9161 | 2.6 | .0839 | .8874 | 1.0 | 30′ |
| 40′ | .8050 | 1.6 | .9187 | 2.5 | .0813 | .8864 | 1.1 | 20′ |
| 50′ | .8066 | 1.5 | .9212 | 2.6 | .0788 | .8853 | 1.0 | 10′ |
| **40° 0′** | 9.8081 | 1.5 | 9.9238 | 2.6 | 10.0762 | 9.8843 | 1.1 | **50° 0′** |
| 10′ | .8096 | 1.5 | .9264 | 2.5 | .0736 | .8832 | 1.1 | 50′ |
| 20′ | .8111 | 1.4 | .9289 | 2.6 | .0711 | .8821 | 1.1 | 40′ |
| 30′ | .8125 | 1.5 | .9315 | 2.6 | .0685 | .8810 | 1.0 | 30′ |
| 40′ | .8140 | 1.5 | .9341 | 2.5 | .0659 | .8800 | 1.1 | 20′ |
| 50′ | .8155 | 1.4 | .9366 | 2.6 | .0634 | .8789 | 1.1 | 10′ |
| **41° 0′** | 9.8169 | 1.5 | 9.9392 | 2.5 | 10.0608 | 9.8778 | 1.1 | **49° 0′** |
| 10′ | .8184 | 1.4 | .9417 | 2.6 | .0583 | .8767 | 1.1 | 50′ |
| 20′ | .8198 | 1.5 | .9443 | 2.5 | .0557 | .8756 | 1.1 | 40′ |
| 30′ | .8213 | 1.4 | .9468 | 2.6 | .0532 | .8745 | 1.2 | 30′ |
| 40′ | .8227 | 1.4 | .9494 | 2.5 | .0506 | .8733 | 1.1 | 20′ |
| 50′ | .8241 | 1.4 | .9519 | 2.5 | .0481 | .8722 | 1.1 | 10′ |
| **42° 0′** | 9.8255 | 1.4 | 9.9544 | 2.6 | 10.0456 | 9.8711 | 1.2 | **48° 0′** |
| 10′ | .8269 | 1.4 | .9570 | 2.5 | .0430 | .8699 | 1.1 | 50′ |
| 20′ | .8283 | 1.4 | .9595 | 2.6 | .0405 | .8688 | 1.2 | 40′ |
| 30′ | .8297 | 1.4 | .9621 | 2.5 | .0379 | .8676 | 1.1 | 30′ |
| 40′ | .8311 | 1.3 | .9646 | 2.5 | .0354 | .8665 | 1.2 | 20′ |
| 50′ | .8324 | 1.4 | .9671 | 2.6 | .0329 | .8653 | 1.2 | 10′ |
| **43° 0′** | 9.8338 | 1.3 | 9.9697 | 2.5 | 10.0303 | 9.8641 | 1.2 | **47° 0′** |
| 10′ | .8351 | 1.4 | .9722 | 2.5 | .0278 | .8629 | 1.1 | 50′ |
| 20′ | .8365 | 1.3 | .9747 | 2.5 | .0253 | .8618 | 1.2 | 40′ |
| 30′ | .8378 | 1.3 | .9772 | 2.6 | .0228 | .8606 | 1.2 | 30′ |
| 40′ | .8391 | 1.4 | .9798 | 2.5 | .0202 | .8594 | 1.2 | 20′ |
| 50′ | .8405 | 1.3 | .9823 | 2.5 | .0177 | .8582 | 1.3 | 10′ |
| **44° 0′** | 9.8418 | 1.3 | 9.9848 | 2.6 | 10.0152 | 9.8569 | 1.2 | **46° 0′** |
| 10′ | .8431 | 1.3 | .9874 | 2.5 | .0126 | .8557 | 1.2 | 50′ |
| 20′ | .8444 | 1.3 | .9899 | 2.5 | .0101 | .8545 | 1.3 | 40′ |
| 30′ | .8457 | 1.2 | .9924 | 2.5 | .0076 | .8532 | 1.2 | 30′ |
| 40′ | .8469 | 1.3 | .9949 | 2.6 | .0051 | .8520 | 1.3 | 20′ |
| 50′ | .8482 | 1.3 | .9975 | 2.5 | .0025 | .8507 | 1.2 | 10′ |
| **45° 0′** | 9.8495 | | 10.0000 | | 10.0000 | 9.8495 | | **45° 0′** |
| | L Cos | d 1′ | L Cot | c d 1′ | L Tan | L Sin | d 1′ | Angle |

# FIVE-PLACE TABLES

# TABLE V

## FIVE-PLACE LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS OF ANGLES BETWEEN 0° AND 3° AND BETWEEN 87° AND 90°

NOTE. — For angles between 0° and 3° and between 87° and 90° Table V*a* or Table V*b* may be used to avoid interpolation in Table IV or in ordinary five-place tables; the results thus obtained are more accurate. Errors of interpolation in Table V*b* correspond to differences of angle of less than 1″; Table V*a* gives still more accurate results.

## V*a*. AUXILIARY TABLE OF *S* AND *T* FOR *A* IN MINUTES

For angles near 0°: $\log \sin A = S + \log A'$ and $\log \tan A = T + \log A'$.

For angles near 90°: $\log \cos A = S_1 + \log A'_1$ and $\log \cot A = T_1 + \log A'_1$ where $A'_1$ is the number of minutes in $90° - A$ and $S_1$ and $T_1$ are corresponding values of $S$ and $T$.

| $A'$ | $S + 10$ |
|---|---|
| 0′ – 13′ | 6.46 373 |
| 14′ – 42′ | 372 |
| 43′ – 58′ | 371 |
| 59′ – 71′ | 6.46 370 |
| 72′ – 81′ | 369 |
| 82′ – 91′ | 368 |
| 92′ – 99′ | 6.46 367 |
| 100′ – 107′ | 366 |
| 108′ – 115′ | 365 |
| 116′ – 121′ | 6.46 364 |
| 122′ – 128′ | 363 |
| 129′ – 134′ | 362 |
| 135′ – 140′ | 6.46 361 |
| 141′ – 146′ | 360 |
| 147′ – 151′ | 359 |
| 152′ – 157′ | 6.46 358 |
| 158′ – 162′ | 357 |
| 163′ – 167′ | 356 |
| 168′ – 171′ | 6.46 355 |
| 172′ – 176′ | 354 |
| 177′ – 181′ | 353 |

| $A'$ | $T + 10$ | $A'$ | $T + 10$ |
|---|---|---|---|
| 0′ – 26′ | 6.46 373 | 131′ – 133′ | 6.46 394 |
| 27′ – 39′ | 374 | 134′ – 136′ | 395 |
| 40′ – 48′ | 375 | 137′ – 139′ | 396 |
| 49′ – 56′ | 6.46 376 | 140′ – 142′ | 6.46 397 |
| 57′ – 63′ | 377 | 143′ – 145′ | 398 |
| 64′ – 69′ | 378 | 146′ – 148′ | 399 |
| 70′ – 74′ | 6.46 379 | 149′ – 150′ | 6.46 400 |
| 75′ – 80′ | 380 | 151′ – 153′ | 401 |
| 81′ – 85′ | 381 | 154′ – 156′ | 402 |
| 86′ – 89′ | 6.46 382 | 157′ – 158′ | 6.46 403 |
| 90′ – 94′ | 383 | 159′ – 161′ | 404 |
| 95′ – 98′ | 384 | 162′ – 163′ | 405 |
| 99′ – 102′ | 6.46 385 | 164′ – 166′ | 6.46 406 |
| 103′ – 106′ | 386 | 167′ – 168′ | 407 |
| 107′ – 110′ | 387 | 169′ – 171′ | 408 |
| 111′ – 113′ | 6.46 388 | 172′ – 173′ | 6.46 409 |
| 114′ – 117′ | 389 | 174′ – 175′ | 410 |
| 118′ – 120′ | 390 | 176′ – 178′ | 411 |
| 121′ – 124′ | 6.46 391 | 179′ – 180′ | 6.46 412 |
| 125′ – 127′ | 392 | 181′ – 182′ | 413 |
| 128′ – 130′ | 393 | 183′ – 184′ | 414 |

## ANGLES NEAR 0° AND 90°

| Angle $A$ | Log Sin $A$ + 10 | | | | | | $C$ |
|---|---|---|---|---|---|---|---|
| | 0″ | 10″ | 20″ | 30″ | 40″ | 50″ | .0000 |
| **0° 0′** | | 5.68 557 | 5.98 660 | 6.16 270 | 6.28 763 | 6.38 454 | 0 |
| 1′ | 6.46 373 | 6.53 067 | 6.58 866 | 6.63 982 | 6.68 557 | 6.72 697 | 0 |
| 2′ | 6.76 476 | 6.79 952 | 6.83 170 | 6.86 167 | 6.88 969 | 6.91 602 | 0 |
| 3′ | 6.94 085 | 6.96 433 | 6.98 660 | 7.00 779 | 7.02 800 | 7.04 730 | 0 |
| 4′ | 7.06 579 | 7.08 351 | 7.10 055 | 7.11 694 | 7.13 273 | 7.14 797 | 0 |
| 5′ | 7.16 270 | 7.17 694 | 7.19 072 | 7.20 409 | 7.21 705 | 7.22 964 | 0 |
| 6′ | 7.24 188 | 7.25 378 | 7.26 536 | 7.27 664 | 7.28 763 | 7.29 836 | 0 |
| 7′ | 7.30 882 | 7.31 904 | 7.32 903 | 7.33 879 | 7.34 833 | 7.35 767 | 0 |
| 8′ | 7.36 682 | 7.37 577 | 7.38 454 | 7.39 314 | 7.40 158 | 7.40 985 | 0 |
| 9′ | 7.41 797 | 7.42 594 | 7.43 376 | 7.44 145 | 7.44 900 | 7.45 643 | 0 |
| **0° 10′** | 7.46 373 | 7.47 090 | 7.47 797 | 7.48 491 | 7.49 175 | 7.49 849 | 0 |
| 11′ | 7.50 512 | 7.51 165 | 7.51 808 | 7.52 442 | 7.53 067 | 7.53 683 | 0 |
| 12′ | 7.54 291 | 7.54 890 | 7.55 481 | 7.56 064 | 7.56 639 | 7.57 206 | 0 |
| 13′ | 7.57 767 | 7.58 320 | 7.58 866 | 7.59 406 | 7.59 939 | 7.60 465 | 0 |
| 14′ | 7.60 985 | 7.61 499 | 7.62 007 | 7.62 509 | 7.63 006 | 7.63 496 | 1 |
| 15′ | 7.63 982 | 7.64 461 | 7.64 936 | 7.65 406 | 7.65 870 | 7.66 330 | 1 |
| 16′ | 7.66 784 | 7.67 235 | 7.67 680 | 7.68 121 | 7.68 557 | 7.68 989 | 1 |
| 17′ | 7.69 417 | 7.69 841 | 7.70 261 | 7.70 676 | 7.71 088 | 7.71 496 | 1 |
| 18′ | 7.71 900 | 7.72 300 | 7.72 697 | 7.73 090 | 7.73 479 | 7.73 865 | 1 |
| 19′ | 7.74 248 | 7.74 627 | 7.75 003 | 7.75 376 | 7.75 745 | 7.76 112 | 1 |
| **0° 20′** | 7.76 475 | 7.76 836 | 7.77 193 | 7.77 548 | 7.77 899 | 7.78 248 | 1 |
| 21′ | 7.78 594 | 7.78 938 | 7.79 278 | 7.79 616 | 7.79 952 | 7.80 284 | 1 |
| 22′ | 7.80 615 | 7.80 942 | 7.81 268 | 7.81 591 | 7.81 911 | 7.82 229 | 1 |
| 23′ | 7.82 545 | 7.82 859 | 7.83 170 | 7.83 479 | 7.83 786 | 7.84 091 | 1 |
| 24′ | 7.84 393 | 7.84 694 | 7.84 992 | 7.85 289 | 7.85 583 | 7.85 876 | 1 |
| 25′ | 7.86 166 | 7.86 455 | 7.86 741 | 7.87 026 | 7.87 309 | 7.87 590 | 1 |
| 26′ | 7.87 870 | 7.88 147 | 7.88 423 | 7.88 697 | 7.88 969 | 7.89 240 | 1 |
| 27′ | 7.89 509 | 7.89 776 | 7.90 041 | 7.90 305 | 7.90 568 | 7.90 829 | 1 |
| 28′ | 7.91 088 | 7.91 346 | 7.91 602 | 7.91 857 | 7.92 110 | 7.92 362 | 1 |
| 29′ | 7.92 612 | 7.92 861 | 7.93 108 | 7.93 354 | 7.93-599 | 7.93 842 | 2 |
| **0° 30′** | 7.94 084 | 7.94 325 | 7.94 564 | 7.94 802 | 7.95 039 | 7.95 274 | 2 |
| 31′ | 7.95 508 | 7.95 741 | 7.95 973 | 7.96 203 | 7.96 432 | 7.96 660 | 2 |
| 32′ | 7.96 887 | 7.97 113 | 7.97 337 | 7.97 560 | 7.97 782 | 7.98 003 | 2 |
| 33′ | 7.98 223 | 7.98 442 | 7.98 660 | 7.98 876 | 7.99 092 | 7.99 306 | 2 |
| 34′ | 7.99 520 | 7.99 732 | 7.99 943 | 8.00 154 | 8.00 363 | 8.00 571 | 2 |
| 35′ | 8.00 779 | 8.00 985 | 8.01 190 | 8.01 395 | 8.01 598 | 8.01 801 | 2 |
| 36′ | 8.02 002 | 8.02 203 | 8.02 402 | 8.02 601 | 8.02 799 | 8.02 996 | 2 |
| 37′ | 8.03 192 | 8.03 387 | 8.03 581 | 8.03 775 | 8.03 967 | 8.04 159 | 3 |
| 38′ | 8.04 350 | 8.04 540 | 8.04 729 | 8.04 918 | 8.05 105 | 8.05 292 | 3 |
| 39′ | 8.05 478 | 8.05 663 | 8.05 848 | 8.06 031 | 8.06 214 | 8.06 396 | 3 |
| **0° 40′** | 8.06 578 | 8.06 758 | 8.06 938 | 8.07 117 | 8.07 295 | 8.07 473 | 3 |
| 41′ | 8.07 650 | 8.07 826 | 8.08 002 | 8.08 176 | 8.08 350 | 8.08 524 | 3 |
| 42′ | 8.08 696 | 8.08 868 | 8.09 040 | 8.09 210 | 8.09 380 | 8.09 550 | 4 |
| 43′ | 8.09 718 | 8.09 886 | 8.10 054 | 8.10 220 | 8.10 386 | 8.10 552 | 4 |
| 44′ | 8.10 717 | 8.10 881 | 8.11 044 | 8.11 207 | 8.11 370 | 8.11 531 | 4 |
| 45′ | 8.11 693 | 8.11 853 | 8.12 013 | 8.12 172 | 8.12 331 | 8.12 489 | 4 |
| 46′ | 8.12 647 | 8.12 804 | 8.12 961 | 8.13 117 | 8.13 272 | 8.13 427 | 4 |
| 47′ | 8.13 581 | 8.13 735 | 8.13 888 | 8.14 041 | 8.14 193 | 8.14 344 | 4 |
| 48′ | 8.14 495 | 8.14 646 | 8.14 796 | 8.14 945 | 8.15 094 | 8.15 243 | 4 |
| 49′ | 8.15 391 | 8.15 538 | 8.15 685 | 8.15 832 | 8.15 978 | 8.16 123 | 4 |
| **0° 50′** | 8.16 268 | 8.16 413 | 8.16 557 | 8.16 700 | 8.16 843 | 8.16 986 | 5 |
| 51′ | 8.17 128 | 8.17 270 | 8.17 411 | 8.17 552 | 8.17 692 | 8.17 832 | 5 |
| 52′ | 8.17 971 | 8.18 110 | 8.18 249 | 8.18 387 | 8.18 524 | 8.18 662 | 5 |
| 53′ | 8.18 798 | 8.18 935 | 8.19 071 | 8.19 206 | 8.19 341 | 8.19 476 | 5 |
| 54′ | 8.19 610 | 8.19 744 | 8.19 877 | 8.20 010 | 8.20 143 | 8.20 275 | 6 |
| 55′ | 8.20 407 | 8.20 538 | 8.20 669 | 8.20 800 | 8.20 930 | 8.21 060 | 6 |
| 56′ | 8.21 189 | 8.21 319 | 8.21 447 | 8.21 576 | 8.21 703 | 8.21 831 | 6 |
| 57′ | 8.21 958 | 8.22 085 | 8.22 211 | 8.22 337 | 8.22 463 | 8.22 588 | 6 |
| 58′ | 8.22 713 | 8.22 838 | 8.22 962 | 8.23 086 | 8.23 210 | 8.23 333 | 6 |
| 59′ | 8.23 456 | 8.23 578 | 8.23 700 | 8.23 822 | 8.23 944 | 8.24 065 | 6 |

See Note, p. 21. Log tan $A$ = Log sin $A$ + $C$; Log cot $A$ = 10 − Log tan $A$ − 10; Log cos $A$ = 10 − $C$ − 10; except for a possible error of 1 in the last place. For functions of angles between 87° and 90° use the relations: Sin $B$ = cos(90° − $B$); tan $B$ = cot (90° − $B$); cot $B$ = tan (90° − $B$); cos $B$ = sin (90° − $B$).

## ANGLES NEAR 0° AND 90°

| Angle A | Log Sin A + 10 | | | | | | C |
|---|---|---|---|---|---|---|---|
| | 0″ | 10″ | 20″ | 30″ | 40″ | 50″ | .0000 |
| 1° 0′ | 8.24 186 | 8.24 306 | 8.24 426 | 8.24 546 | 8.24 665 | 8.24 785 | 7 |
| 1′ | 8.24 903 | 8.25 022 | 8.25 140 | 8.25 258 | 8.25 375 | 8.25 493 | 7 |
| 2′ | 8.25 609 | 8.25 726 | 8.25 842 | 8.25 958 | 8.26 074 | 8.26 189 | 7 |
| 3′ | 8.26 304 | 8.26 419 | 8.26 533 | 8.26 648 | 8.26 761 | 8.26 875 | 7 |
| 4′ | 8.26 988 | 8.27 101 | 8.27 214 | 8.27 326 | 8.27 438 | 8.27 550 | 8 |
| 5′ | 8.27 661 | 8.27 773 | 8.27 883 | 8.27 994 | 8.28 104 | 8.28 215 | 8 |
| 6′ | 8.28 324 | 8.28 434 | 8.28 543 | 8.28 652 | 8.28 761 | 8.28 869 | 8 |
| 7′ | 8.28 977 | 8.29 085 | 8.29 193 | 8.29 300 | 8.29 407 | 8.29 514 | 9 |
| 8′ | 8.29 621 | 8.29 727 | 8.29 833 | 8.29 939 | 8.30 044 | 8.30 150 | 9 |
| 9′ | 8.30 255 | 8.30 359 | 8.30 464 | 8.30 568 | 8.30 672 | 8.30 776 | 9 |
| 1° 10′ | 8.30 879 | 8.30 983 | 8.31 086 | 8.31 188 | 8.31 291 | 8.31 393 | 9 |
| 11′ | 8.31 495 | 8.31 597 | 8.31 699 | 8.31 800 | 8.31 901 | 8.32 002 | 9 |
| 12′ | 8.32 103 | 8.32 203 | 8.32 303 | 8.32 403 | 8.32 503 | 8.32 602 | .00010 |
| 13′ | 8.32 702 | 8.32 801 | 8.32 899 | 8.32 998 | 8.33 096 | 8.33 195 | 10 |
| 14′ | 8.33 292 | 8.33 390 | 8.33 488 | 8.33 585 | 8.33 682 | 8.33 779 | 10 |
| 15′ | 8.33 875 | 8.33 972 | 8.34 068 | 8.34 164 | 8.34 260 | 8.34 355 | 10 |
| 16′ | 8.34 450 | 8.34 546 | 8.34 640 | 8.34 735 | 8.34 830 | 8.34 924 | 11 |
| 17′ | 8.35 018 | 8.35 112 | 8.35 206 | 8.35 299 | 8.35 392 | 8.35 485 | 11 |
| 18′ | 8.35 578 | 8.35 671 | 8.35 764 | 8.35 856 | 8.35 948 | 8.36 040 | 11 |
| 19′ | 8.36 131 | 8.36 223 | 8.36 314 | 8.36 405 | 8.36 496 | 8.36 587 | 12 |
| 1° 20′ | 8.36 678 | 8.36 768 | 8.36 858 | 8.36 948 | 8.37 038 | 8.37 128 | 12 |
| 21′ | 8.37 217 | 8.37 306 | 8.37 395 | 8.37 484 | 8.37 573 | 8.37 662 | 12 |
| 22′ | 8.37 750 | 8.37 838 | 8.37 926 | 8.38 014 | 8.38 101 | 8.38 189 | 12 |
| 23′ | 8.38 276 | 8.38 363 | 8.38 450 | 8.38 537 | 8.38 624 | 8.38 710 | 13 |
| 24′ | 8.38 796 | 8.38 882 | 8.38 968 | 8.39 054 | 8.39 139 | 8.39 225 | 13 |
| 25′ | 8.39 310 | 8.39 395 | 8.39 480 | 8.39 565 | 8.39 649 | 8.39 734 | 13 |
| 26′ | 8.39 818 | 8.39 902 | 8.39 986 | 8.40 070 | 8.40 153 | 8.40 237 | 14 |
| 27′ | 8.40 320 | 8.40 403 | 8.40 486 | 8.40 569 | 8.40 651 | 8.40 734 | 14 |
| 28′ | 8.40 816 | 8.40 898 | 8.40 980 | 8.41 062 | 8.41 144 | 8.41 225 | 15 |
| 29′ | 8.41 307 | 8.41 388 | 8.41 469 | 8.41 550 | 8.41 631 | 8.41 711 | 15 |
| 1° 30′ | 8.41 792 | 8.41 872 | 8.41 952 | 8.42 032 | 8.42 112 | 8.42 192 | 15 |
| 31′ | 8.42 272 | 8.42 351 | 8.42 430 | 8.42 510 | 8.42 589 | 8.42 667 | 15 |
| 32′ | 8.42 746 | 8.42 825 | 8.42 903 | 8.42 982 | 8.43 060 | 8.43 138 | 16 |
| 33′ | 8.43 216 | 8.43 293 | 8.43 371 | 8.43 448 | 8.43 526 | 8.43 603 | 16 |
| 34′ | 8.43 680 | 8.43 757 | 8.43 834 | 8.43 910 | 8.43 987 | 8.44 063 | 16 |
| 35′ | 8.44 139 | 8.44 216 | 8.44 292 | 8.44 367 | 8.44 443 | 8.44 519 | 17 |
| 36′ | 8.44 594 | 8.44 669 | 8.44 745 | 8.44 820 | 8.44 895 | 8.44 969 | 17 |
| 37′ | 8.45 044 | 8.45 119 | 8.45 193 | 8.45 267 | 8.45 341 | 8.45 415 | 17 |
| 38′ | 8.45 489 | 8.45 563 | 8.45 637 | 8.45 710 | 8.45 784 | 8.45 857 | 18 |
| 39′ | 8.45 930 | 8.46 003 | 8.46 076 | 8.46 149 | 8.46 222 | 8.46 294 | 18 |
| 1° 40′ | 8.46 366 | 8.46 439 | 8.46 511 | 8.46 583 | 8.46 655 | 8.46 727 | 18 |
| 41′ | 8.46 799 | 8.46 870 | 8.46 942 | 8.47 013 | 8.47 084 | 8.47 155 | 19 |
| 42′ | 8.47 226 | 8.47 297 | 8.47 368 | 8.47 439 | 8.47 509 | 8.47 580 | 19 |
| 43′ | 8.47 650 | 8.47 720 | 8.47 790 | 8.47 860 | 8.47 930 | 8.48 000 | 20 |
| 44′ | 8.48 069 | 8.48 139 | 8.48 208 | 8.48 278 | 8.48 347 | 8.48 416 | 20 |
| 45′ | 8.48 485 | 8.48 554 | 8.48 622 | 8.48 691 | 8.48 760 | 8.48 828 | 20 |
| 46′ | 8.48 896 | 8.48 965 | 8.49 033 | 8.49 101 | 8.49 169 | 8.49 236 | 20 |
| 47′ | 8.49 304 | 8.49 372 | 8.49 439 | 8.49 506 | 8.49 574 | 8.49 641 | 21 |
| 48′ | 8.49 708 | 8.49 775 | 8.49 842 | 8.49 908 | 8.49 975 | 8.50 042 | 21 |
| 49′ | 8.50 108 | 8.50 174 | 8.50 241 | 8.50 307 | 8.50 373 | 8.50 439 | 22 |
| 1° 50′ | 8.50 504 | 8.50 570 | 8.50 636 | 8.50 701 | 8.50 767 | 8.50 832 | 23 |
| 51′ | 8.50 897 | 8.50 963 | 8.51 028 | 8.51 092 | 8.51 157 | 8.51 222 | 23 |
| 52′ | 8.51 287 | 8.51 351 | 8.51 416 | 8.51 480 | 8.51 544 | 8.51 609 | 23 |
| 53′ | 8.51 673 | 8.51 737 | 8.51 801 | 8.51 864 | 8.51 928 | 8.51 992 | 23 |
| 54′ | 8.52 055 | 8.52 119 | 8.52 182 | 8.52 245 | 8.52 308 | 8.52 371 | 24 |
| 55′ | 8.52 434 | 8.52 497 | 8.52 560 | 8.52 623 | 8.52 685 | 8.52 748 | 24 |
| 56′ | 8.52 810 | 8.52 872 | 8.52 935 | 8.52 997 | 8.53 059 | 8.53 121 | 25 |
| 57′ | 8.53 183 | 8.53 245 | 8.53 306 | 8.53 368 | 8.53 429 | 8.53 491 | 25 |
| 58′ | 8.53 552 | 8.53 614 | 8.53 675 | 8.53 736 | 8.53 797 | 8.53 858 | 26 |
| 59′ | 8.53 919 | 8.53 979 | 8.54 040 | 8.54 101 | 8.54 161 | 8.54 222 | 26 |

See Note, p. 21. Log tan $A$ = Log sin $A + C$; Log cot $A = 10 -$ Log tan $A - 10$; Log cos $A = 10 - C - 10$; except for a possible error of 1 in the last place. For functions of angles between 87° and 90° use the relations: sin $B$ = cos(90° − $B$); tan $B$ = cot (90° − $B$); cot $B$ = tan (90° − $B$); cos $B$ = sin (90° − $B$).

## ANGLES NEAR 0° AND 90°

| Angle A | Log Sin A + 10 | | | | | | C |
|---|---|---|---|---|---|---|---|
| | 0″ | 10″ | 20″ | 30″ | 40″ | 50″ | .000 |
| 2° 0′ | 8.54 282 | 8.54 342 | 8.54 402 | 8.54 462 | 8.54 522 | 8.54 582 | 27 |
| 1′ | 8.54 642 | 8.54 702 | 8.54 762 | 8.54 821 | 8.54 881 | 8.54 940 | 27 |
| 2′ | 8.54 999 | 8.55 059 | 8.55 118 | 8.55 177 | 8.55 236 | 8.55 295 | 28 |
| 3′ | 8.55 354 | 8.55 413 | 8.55 471 | 8.55 530 | 8.55 589 | 8.55 647 | 28 |
| 4′ | 8.55 705 | 8.55 764 | 8.55 822 | 8.55 880 | 8.55 938 | 8.55 996 | 29 |
| 5′ | 8.56 054 | 8.56 112 | 8.56 170 | 8.56 227 | 8.56 285 | 8.56 342 | 29 |
| 6′ | 8.56 400 | 8.56 457 | 8.56 515 | 8.56 572 | 8.56 629 | 8.56 686 | 29 |
| 7′ | 8.56 743 | 8.56 800 | 8.56 857 | 8.56 914 | 8.56 970 | 8.57 027 | 30 |
| 8′ | 8.57 084 | 8.57 140 | 8.57 196 | 8.57 253 | 8.57 309 | 8.57 365 | 30 |
| 9′ | 8.57 421 | 8.57 477 | 8.57 533 | 8.57 589 | 8.57 645 | 8.57 701 | 31 |
| 2° 10′ | 8.57 757 | 8.57 812 | 8.57 868 | 8.57 927 | 8.57 979 | 8.58 034 | 31 |
| 11′ | 8.58 089 | 8.58 144 | 8.58 200 | 8.58 255 | 8.58 310 | 8.58 364 | 32 |
| 12′ | 8.58 419 | 8.58 474 | 8.58 529 | 8.58 583 | 8.58 638 | 8.58 693 | 32 |
| 13′ | 8.58 747 | 8.58 801 | 8.58 856 | 8.58 910 | 8.58 964 | 8.59 018 | 33 |
| 14′ | 8.59 072 | 8.59 126 | 8.59 180 | 8.59 234 | 8.59 288 | 8.59 341 | 33 |
| 15′ | 8.59 395 | 8.59 448 | 8.59 502 | 8.59 555 | 8.59 609 | 8.59 662 | 34 |
| 16′ | 8.59 715 | 8.59 768 | 8.59 821 | 8.59 874 | 8.59 927 | 8.59 980 | 35 |
| 17′ | 8.60 033 | 8.60 086 | 8.60 139 | 8.60 191 | 8.60 244 | 8.60 296 | 35 |
| 18′ | 8.60 349 | 8.60 401 | 8.60 454 | 8.60 506 | 8.60 558 | 8.60 610 | 35 |
| 19′ | 8.60 662 | 8.60 714 | 8.60 766 | 8.60 818 | 8.60 870 | 8.60 922 | 36 |
| 2° 20′ | 8.60 973 | 8.61 025 | 8.61 077 | 8.61 128 | 8.61 180 | 8.61 231 | 36 |
| 21′ | 8.61 282 | 8.61 334 | 8.61 385 | 8.61 436 | 8.61 487 | 8.61 538 | 37 |
| 22′ | 8.61 589 | 8.61 640 | 8.61 691 | 8.61 742 | 8.61 792 | 8.61 843 | 37 |
| 23′ | 8.61 894 | 8.61 944 | 8.61 995 | 8.62 045 | 8.62 096 | 8.62 146 | 38 |
| 24′ | 8.62 196 | 8.62 246 | 8.62 297 | 8.62 347 | 8.62 397 | 8.62 447 | 38 |
| 25′ | 8.62 497 | 8.62 546 | 8.62 596 | 8.62 646 | 8.62 696 | 8.62 745 | 39 |
| 26′ | 8.62 795 | 8.62 844 | 8.62 894 | 8.62 943 | 8.62 993 | 8.63 042 | 39 |
| 27′ | 8.63 091 | 8.63 140 | 8.63 189 | 8.63 238 | 8.63 288 | 8.63 336 | 40 |
| 28′ | 8.63 385 | 8.63 434 | 8.63 483 | 8.63 532 | 8.63 580 | 8.63 629 | 41 |
| 29′ | 8.63 678 | 8.63 726 | 8.63 775 | 8.63 823 | 8.63 871 | 8.63 920 | 41 |
| 2° 30′ | 8.63 968 | 8.64 016 | 8.64 064 | 8.64 112 | 8.64 160 | 8.64 208 | 42 |
| 31′ | 8.64 256 | 8.64 304 | 8.64 352 | 8.64 400 | 8.64 448 | 8.64 495 | 42 |
| 32′ | 8.64 543 | 8.64 590 | 8.64 638 | 8.64 685 | 8.64 733 | 8.64 780 | 43 |
| 33′ | 8.64 827 | 8.64 875 | 8.64 922 | 8.64 969 | 8.65 016 | 8.65 063 | 43 |
| 34′ | 8.65 110 | 8.65 157 | 8.65 204 | 8.65 251 | 8.65 298 | 8.65 344 | 44 |
| 35′ | 8.65 391 | 8.65 438 | 8.65 484 | 8.65 531 | 8.65 577 | 8.65 624 | 44 |
| 36′ | 8.65 670 | 8.65 717 | 8.65 763 | 8.65 809 | 8.65 855 | 8.65 901 | 45 |
| 37′ | 8.65 947 | 8.65 994 | 8.66 040 | 8.66 085 | 8.66 131 | 8.66 177 | 46 |
| 38′ | 8.66 223 | 8.66 269 | 8.66 314 | 8.66 360 | 8.66 406 | 8.66 451 | 46 |
| 39′ | 8.66 497 | 8.66 542 | 8.66 588 | 8.66 633 | 8.66 678 | 8.66 724 | 47 |
| 2° 40′ | 8.66 769 | 8.66 814 | 8.66 859 | 8.66 904 | 8.66 949 | 8.66 994 | 47 |
| 41′ | 8.67 039 | 8.67 084 | 8.67 129 | 8.67 174 | 8.67 219 | 8.67 263 | 48 |
| 42′ | 8.67 308 | 8.67 353 | 8.67 397 | 8.67 442 | 8.67 486 | 8.67 531 | 48 |
| 43′ | 8.67 575 | 8.67 619 | 8.67 664 | 8.67 708 | 8.67 752 | 8.67 796 | 49 |
| 44′ | 8.67 841 | 8.67 885 | 8.67 929 | 8.67 973 | 8.68 017 | 8.68 060 | 49 |
| 45′ | 8.68 104 | 8.68 148 | 8.68 192 | 8.68 236 | 8.68 279 | 8.68 323 | 50 |
| 46′ | 8.68 367 | 8.68 410 | 8.68 454 | 8.68 497 | 8.68 540 | 8.68 584 | 51 |
| 47′ | 8.68 627 | 8.68 670 | 8.68 714 | 8.68 757 | 8.68 800 | 8.68 843 | 51 |
| 48′ | 8.68 886 | 8.68 929 | 8.68 972 | 8.69 015 | 8.69 058 | 8.69 101 | 52 |
| 49′ | 8.69 144 | 8.69 187 | 8.69 229 | 8.69 272 | 8.69 315 | 8.69 357 | 53 |
| 2° 50′ | 8.69 400 | 8.69 442 | 8.69 485 | 8.69 527 | 8.69 570 | 8.69 612 | 53 |
| 51′ | 8.69 654 | 8.69 697 | 8.69 739 | 8.69 781 | 8.69 823 | 8.69 865 | 54 |
| 52′ | 8.69 907 | 8.69 949 | 8.69 991 | 8.70 033 | 8.70 075 | 8.70 117 | 55 |
| 53′ | 8.70 159 | 8.70 201 | 8.70 242 | 8.70 284 | 8.70 326 | 8.70 367 | 55 |
| 54′ | 8.70 409 | 8.70 451 | 8.70 492 | 8.70 534 | 8.70 575 | 8.70 616 | 56 |
| 55′ | 8.70 658 | 8.70 699 | 8.70 740 | 8.70 781 | 8.70 823 | 8.70 864 | 56 |
| 56′ | 8.70 905 | 8.70 946 | 8.70 987 | 8.71 028 | 8.71 069 | 8.71 110 | 57 |
| 57′ | 8.71 151 | 8.71 192 | 8.71 232 | 8.71 273 | 8.71 314 | 8.71 355 | 58 |
| 58′ | 8.71 395 | 8.71 436 | 8.71 476 | 8.71 517 | 8.71 557 | 8.71 598 | 58 |
| 59′ | 8.71 638 | 8.71 679 | 8.71 719 | 8.71 759 | 8.71 800 | 8.71 840 | 59 |

See Note, p. 21. Log tan $A$ = Log sin $A + C$; Log cot $A = 10 -$ Log tan $A - 10$; Log cos $A = 10 - C - 10$; except for a possible error of 1 in the last place. For functions of angles between 87° and 90° use the relations: sin $B$ = cos $(90° - B)$; tan $B$ = cot$(90° - B)$; cot $B$ = tan$(90° - B)$; cos $B$ = sin $(90° - B)$.

## 0°

| ′ | L Sin | L Tan | L Cot | L Cos | | Prop. Pts. |
|---|---|---|---|---|---|---|
| **0** | — | — | — | 10.00 000 | **60** | To avoid interpolation use Table Va or Vb. See Note page 21. Must subtract 10 from tabulated values of logarithms. |
| 1 | 6.46 373 | 6.46 373 | 13.53 627 | 10.00 000 | 59 | |
| 2 | 6.76 476 | 6.76 476 | 13.23 524 | 10.00 000 | 58 | |
| 3 | 6.94 085 | 6.94 085 | 13.05 915 | 10.00 000 | 57 | |
| 4 | 7.06 579 | 7.06 579 | 12.93 421 | 10.00 000 | 56 | |
| 5 | 7.16 270 | 7.16 270 | 12.83 730 | 10.00 000 | 55 | |
| 6 | 7.24 188 | 7.24 188 | 12.75 812 | 10.00 000 | 54 | |
| 7 | 7.30 882 | 7.30 882 | 12.69 118 | 10.00 000 | 53 | |
| 8 | 7.36 682 | 7.36 682 | 12.63 318 | 10.00 000 | 52 | |
| 9 | 7.41 797 | 7.41 797 | 12.58 203 | 10.00 000 | 51 | |
| **10** | 7.46 373 | 7.46 373 | 12.53 627 | 10.00 000 | **50** | |
| 11 | 7.50 512 | 7.50 512 | 12.49 488 | 10.00 000 | 49 | |
| 12 | 7.54 291 | 7.54 291 | 12.45 709 | 10.00 000 | 48 | |
| 13 | 7.57 767 | 7.57 767 | 12.42 233 | 10.00 000 | 47 | |
| 14 | 7.60 985 | 7.60 986 | 12.39 014 | 10.00 000 | 46 | |
| 15 | 7.63 982 | 7.63 982 | 12.36 018 | 10.00 000 | 45 | |
| 16 | 7.66 784 | 7.66 785 | 12.33 215 | 10.00 000 | 44 | |
| 17 | 7.69 417 | 7.69 418 | 12.30 582 | 9.99 999 | 43 | |
| 18 | 7.71 900 | 7.71 900 | 12.28 100 | 9.99 999 | 42 | |
| 19 | 7.74 248 | 7.74 248 | 12.25 752 | 9.99 999 | **41** | |
| **20** | 7.76 475 | 7.76 476 | 12.23 524 | 9.99 999 | **40** | |
| 21 | 7.78 594 | 7.78 595 | 12.21 405 | 9.99 999 | 39 | |
| 22 | 7.80 615 | 7.80 615 | 12.19 385 | 9.99 999 | 38 | |
| 23 | 7.82 545 | 7.82 546 | 12.17 454 | 9.99 999 | **37** | |
| 24 | 7.84 393 | 7.84 394 | 12.15 606 | 9.99 999 | **36** | |
| 25 | 7.86 166 | 7.86 167 | 12.13 833 | 9.99 999 | **35** | |
| 26 | 7.87 870 | 7.87 871 | 12.12 129 | 9.99 999 | **34** | |
| 27 | 7.89 509 | 7.89 510 | 12.10 490 | 9.99 999 | 33 | |
| 28 | 7.91 088 | 7.91 089 | 12.08 911 | 9.99 999 | 32 | |
| 29 | 7.92 612 | 7.92 613 | 12.07 387 | 9.99 998 | 31 | |
| **30** | 7.94 084 | 7.94 086 | 12.05 914 | 9.99 998 | **30** | |
| 31 | 7.95 508 | 7.95 510 | 12.04 490 | 9.99 998 | 29 | |
| 32 | 7.96 887 | 7.96 889 | 12.03 111 | 9.99 998 | 28 | |
| 33 | 7.98 223 | 7.98 225 | 12.01 775 | 9.99 998 | 27 | |
| 34 | 7.99 520 | 7.99 522 | 12.00 478 | 9.99 998 | 26 | |
| 35 | 8.00 779 | 8.00 781 | 11.99 219 | 9.99 998 | 25 | |
| 36 | 8.02 002 | 8.02 004 | 11.97 996 | 9.99 998 | 24 | |
| 37 | 8.03 192 | 8.03 194 | 11.96 806 | 9.99 997 | 23 | |
| 38 | 8.04 350 | 8.04 353 | 11.95 647 | 9.99 997 | 22 | |
| 39 | 8.05 478 | 8.05 481 | 11.94 519 | 9.99 997 | 21 | |
| **40** | 8.06 578 | 8.06 581 | 11.93 419 | 9.99 997 | **20** | |
| 41 | 8.07 650 | 8.07 653 | 11.92 347 | 9.99 997 | 19 | |
| 42 | 8.08 696 | 8.08 700 | 11.91 300 | 9.99 997 | 18 | |
| 43 | 8.09 718 | 8.09 722 | 11.90 278 | 9.99 997 | 17 | |
| 44 | 8.10 717 | 8.10 720 | 11.89 280 | 9.99 996 | 16 | |
| 45 | 8.11 693 | 8.11 696 | 11.88 304 | 9.99 996 | 15 | |
| 46 | 8.12 647 | 8.12 651 | 11.87 349 | 9.99 996 | 14 | |
| 47 | 8.13 581 | 8.13 585 | 11.86 415 | 9.99 996 | 13 | |
| 48 | 8.14 495 | 8.14 500 | 11.85 500 | 9.99 996 | 12 | |
| 49 | 8.15 391 | 8.15 395 | 11.84 605 | 9.99 996 | 11 | |
| **50** | 8.16 268 | 8.16 273 | 11.83 727 | 9.99 995 | **10** | |
| 51 | 8.17 128 | 8.17 133 | 11.82 867 | 9.99 995 | 9 | |
| 52 | 8.17 971 | 8.17 976 | 11.82 024 | 9.99 995 | 8 | |
| 53 | 8.18 798 | 8.18 804 | 11.81 196 | 9.99 995 | 7 | |
| 54 | 8.19 610 | 8.19 616 | 11.80 384 | 9.99 995 | 6 | |
| 55 | 8.20 407 | 8.20 413 | 11.79 587 | 9.99 994 | 5 | |
| 56 | 8.21 189 | 8.21 195 | 11.78 805 | 9.99 994 | 4 | |
| 57 | 8.21 958 | 8.21 964 | 11.78 036 | 9.99 994 | 3 | |
| 58 | 8.22 713 | 8.22 720 | 11.77 280 | 9.99 994 | 2 | |
| 59 | 8.23 456 | 8.23 462 | 11.76 538 | 9.99 994 | 1 | |
| **60** | 8.24 186 | 8.24 192 | 11.75 808 | 9.99 993 | **0** | |
| | L Cos | L Cot | L Tan | L Sin | ′ | Prop. Pts. |

## 89°

## 1°

| ′ | L Sin | L Tan | L Cot | L Cos. | | Prop. Pts. |
|---|---|---|---|---|---|---|
| **0** | 8.24 186 | 8.24 192 | 11.75 808 | 9.99 993 | **60** | To avoid interpolation use Table Va or Vb. See Note page 21. Must subtract 10 from tabulated values of logarithms. |
| 1 | 8.24 903 | 8.24 910 | 11.75 090 | 9.99 993 | 59 | |
| 2 | 8.25 609 | 8.25 616 | 11.74 384 | 9.99 993 | 58 | |
| 3 | 8.26 304 | 8.26 312 | 11.73 688 | 9.99 993 | 57 | |
| 4 | 8.26 988 | 8.26 996 | 11.73 004 | 9.99 992 | 56 | |
| 5 | 8.27 661 | 8.27 669 | 11.72 331 | 9.99 992 | 55 | |
| 6 | 8.28 324 | 8.28 332 | 11.71 668 | 9.99 992 | 54 | |
| 7 | 8.28 977 | 8.28 986 | 11.71 014 | 9.99 992 | 53 | |
| 8 | 8.29 621 | 8.29 629 | 11.70 371 | 9.99 992 | 52 | |
| 9 | 8.30 255 | 8.30 263 | 11.69 737 | 9.99 991 | 51 | |
| **10** | 8.30 879 | 8.30 888 | 11.69 112 | 9.99 991 | **50** | |
| 11 | 8.31 495 | 8.31 505 | 11.68 495 | 9.99 991 | 49 | |
| 12 | 8.32 103 | 8.32 112 | 11.67 888 | 9.99 990 | 48 | |
| 13 | 8.32 702 | 8.32 711 | 11.67 289 | 9.99 990 | 47 | |
| 14 | 8.33 292 | 8.33 302 | 11.66 698 | 9.99 990 | 46 | |
| 15 | 8.33 875 | 8.33 886 | 11.66 114 | 9.99 990 | 45 | |
| 16 | 8.34 450 | 8.34 461 | 11.65 539 | 9.99 989 | 44 | |
| 17 | 8.35 018 | 8.35 029 | 11.64 971 | 9.99 989 | 43 | |
| 18 | 8.35 578 | 8.35 590 | 11.64 410 | 9.99 989 | 42 | |
| 19 | 8.36 131 | 8.36 143 | 11.63 857 | 9.99 989 | 41 | |
| **20** | 8.36 678 | 8.36 689 | 11.63 311 | 9.99 988 | **40** | |
| 21 | 8.37 217 | 8.37 229 | 11.62 771 | 9.99 988 | 39 | |
| 22 | 8.37 750 | 8.37 762 | 11.62 238 | 9.99 988 | 38 | |
| 23 | 8.38 276 | 8.38 289 | 11.61 711 | 9.99 987 | 37 | |
| 24 | 8.38 796 | 8.38 809 | 11.61 191 | 9.99 987 | 36 | |
| 25 | 8.39 310 | 8.39 323 | 11.60 677 | 9.99 987 | 35 | |
| 26 | 8.39 818 | 8.39 832 | 11.60 168 | 9.99 986 | 34 | |
| 27 | 8.40 320 | 8.40 334 | 11.59 666 | 9.99 986 | 33 | |
| 28 | 8.40 816 | 8.40 830 | 11.59 170 | 9.99 986 | 32 | |
| 29 | 8.41 307 | 8.41 321 | 11.58 679 | 9.99 985 | 31 | |
| **30** | 8.41 792 | 8.41 807 | 11.58 193 | 9.99 985 | **30** | |
| 31 | 8.42 272 | 8.42 287 | 11.57 713 | 9.99 985 | 29 | |
| 32 | 8.42 746 | 8.42 762 | 11.57 238 | 9.99 984 | 28 | |
| 33 | 8.43 216 | 8.43 232 | 11.56 768 | 9.99 984 | 27 | |
| 34 | 8.43 680 | 8.43 696 | 11.56 304 | 9.99 984 | 26 | |
| 35 | 8.44 139 | 8.44 156 | 11.55 844 | 9.99 983 | 25 | |
| 36 | 8.44 594 | 8.44 611 | 11.55 389 | 9.99 983 | 24 | |
| 37 | 8.45 044 | 8.45 061 | 11.54 939 | 9.99 983 | 23 | |
| 38 | 8.45 489 | 8.45 507 | 11.54 493 | 9.99 982 | 22 | |
| 39 | 8.45 930 | 8.45 948 | 11.54 052 | 9.99 982 | 21 | |
| **40** | 8.46 366 | 8.46 385 | 11.53 615 | 9.99 982 | **20** | |
| 41 | 8.46 799 | 8.46 817 | 11.53 183 | 9.99 981 | 19 | |
| 42 | 8.47 226 | 8.47 245 | 11.52 755 | 9.99 981 | 18 | |
| 43 | 8.47 650 | 8.47 669 | 11.52 331 | 9.99 981 | 17 | |
| 44 | 8.48 069 | 8.48 089 | 11.51 911 | 9.99 980 | 16 | |
| 45 | 8.48 485 | 8.48 505 | 11.51 495 | 9.99 980 | 15 | |
| 46 | 8.48 896 | 8.48 917 | 11.51 083 | 9.99 979 | 14 | |
| 47 | 8.49 304 | 8.49 325 | 11.50 675 | 9.99 979 | 13 | |
| 48 | 8.49 708 | 8.49 729 | 11.50 271 | 9.99 979 | 12 | |
| 49 | 8.50 108 | 8.50 130 | 11.49 870 | 9.99 978 | 11 | |
| **50** | 8.50 504 | 8.50 527 | 11.49 473 | 9.99 978 | **10** | |
| 51 | 8.50 897 | 8.50 920 | 11.49 080 | 9.99 977 | 9 | |
| 52 | 8.51 287 | 8.51 310 | 11.48 690 | 9.99 977 | 8 | |
| 53 | 8.51 673 | 8.51 696 | 11.48 304 | 9.99 977 | 7 | |
| 54 | 8.52 055 | 8.52 079 | 11.47 921 | 9.99 976 | 6 | |
| 55 | 8.52 434 | 8.52 459 | 11.47 541 | 9.99 976 | 5 | |
| 56 | 8.52 810 | 8.52 835 | 11.47 165 | 9.99 975 | 4 | |
| 57 | 8.53 183 | 8.53 208 | 11.46 792 | 9.99 975 | 3 | |
| 58 | 8.53 552 | 8.53 578 | 11.46 422 | 9.99 974 | 2 | |
| 59 | 8.53 919 | 8.53 945 | 11.46 055 | 9.99 974 | 1 | |
| **60** | 8.54 282 | 8.54 308 | 11.45 692 | 9.99 974 | **0** | |
| | L Cos | L Cot | L Tan | L Sin | ′ | Prop. Pts. |

## 88°

## 2°

| ′ | L Sin | L Tan | L Cot | L Cos | | Prop. Pts. |
|---|---|---|---|---|---|---|
| **0** | 8.54 282 | 8.54 308 | 11.45 692 | 9.99 974 | **60** | To avoid interpolation use Table Va or Vb. See Note page 21. Must subtract 10 from tabulated values of logarithms. |
| 1 | 8.54 642 | 8.54 669 | 11.45 331 | 9.99 973 | 59 | |
| 2 | 8.54 999 | 8.55 027 | 11.44 973 | 9.99 973 | 58 | |
| 3 | 8.55 354 | 8.55 382 | 11.44 618 | 9.99 972 | 57 | |
| 4 | 8.55 705 | 8.55 734 | 11.44 266 | 9.99 972 | 56 | |
| 5 | 8.56 054 | 8.56 083 | 11.43 917 | 9.99 971 | 55 | |
| 6 | 8.56 400 | 8.56 429 | 11.43 571 | 9.99 971 | 54 | |
| 7 | 8.56 743 | 8.56 773 | 11.43 227 | 9.99 970 | 53 | |
| 8 | 8.57 084 | 8.57 114 | 11.42 886 | 9.99 970 | 52 | |
| 9 | 8.57 421 | 8.57 452 | 11.42 548 | 9.99 969 | 51 | |
| **10** | 8.57 757 | 8.57 788 | 11.42 212 | 9.99 969 | **50** | |
| 11 | 8.58 089 | 8.58 121 | 11.41 879 | 9.99 968 | 49 | |
| 12 | 8.58 419 | 8.58 451 | 11.41 549 | 9.99 968 | 48 | |
| 13 | 8.58 747 | 8.58 779 | 11.41 221 | 9.99 967 | 47 | |
| 14 | 8.59 072 | 8.59 105 | 11.40 895 | 9.99 967 | 46 | |
| 15 | 8.59 395 | 8.59 428 | 11.40 572 | 9.99 967 | 45 | |
| 16 | 8.59 715 | 8.59 749 | 11.40 251 | 9.99 966 | 44 | |
| 17 | 8.60 033 | 8.60 068 | 11.39 932 | 9.99 966 | 43 | |
| 18 | 8.60 349 | 8.60 384 | 11.39 616 | 9.99 965 | 42 | |
| 19 | 8.60 662 | 8.60 698 | 11.39 302 | 9.99 964 | 41 | |
| **20** | 8.60 973 | 8.61 009 | 11.38 991 | 9.99 964 | **40** | |
| 21 | 8.61 282 | 8.61 319 | 11.38 681 | 9.99 963 | 39 | |
| 22 | 8.61 589 | 8.61 626 | 11.38 374 | 9.99 963 | 38 | |
| 23 | 8.61 894 | 8.61 931 | 11.38 069 | 9.99 962 | 37 | |
| 24 | 8.62 196 | 8.62 234 | 11.37 766 | 9.99 962 | 36 | |
| 25 | 8.62 497 | 8.62 535 | 11.37 465 | 9.99 961 | 35 | |
| 26 | 8.62 795 | 8.62 834 | 11.37 166 | 9.99 961 | 34 | |
| 27 | 8.63 091 | 8.63 131 | 11.36 869 | 9.99 960 | 33 | |
| 28 | 8.63 385 | 8.63 426 | 11.36 574 | 9.99 960 | 32 | |
| 29 | 8.63 678 | 8.63 718 | 11.36 282 | 9.99 959 | 31 | |
| **30** | 8.63 968 | 8.64 009 | 11.35 991 | 9.99 959 | **30** | |
| 31 | 8.64 256 | 8.64 298 | 11.35 702 | 9.99 958 | 29 | |
| 32 | 8.64 543 | 8.64 585 | 11.35 415 | 9.99 958 | 28 | |
| 33 | 8.64 827 | 8.64 870 | 11.35 130 | 9.99 957 | 27 | |
| 34 | 8.65 110 | 8.65 154 | 11.34 846 | 9.99 956 | 26 | |
| 35 | 8.65 391 | 8.65 435 | 11.34 565 | 9.99 956 | 25 | |
| 36 | 8.65 670 | 8.65 715 | 11.34 285 | 9.99 955 | 24 | |
| 37 | 8.65 947 | 8.65 993 | 11.34 007 | 9.99 955 | 23 | |
| 38 | 8.66 223 | 8.66 269 | 11.33 731 | 9.99 954 | 22 | |
| 39 | 8.66 497 | 8.66 543 | 11.33 457 | 9.99 954 | 21 | |
| **40** | 8.66 769 | 8.66 816 | 11.33 184 | 9.99 953 | **20** | |
| 41 | 8.67 039 | 8.67 087 | 11.32 913 | 9.99 952 | 19 | |
| 42 | 8.67 308 | 8.67 356 | 11.32 644 | 9.99 952 | 18 | |
| 43 | 8.67 575 | 8.67 624 | 11.32 376 | 9.99 951 | 17 | |
| 44 | 8.67 841 | 8.67 890 | 11.32 110 | 9.99 951 | 16 | |
| 45 | 8.68 104 | 8.68 154 | 11.31 846 | 9.99 950 | 15 | |
| 46 | 8.68 367 | 8.68 417 | 11.31 583 | 9.99 949 | 14 | |
| 47 | 8.68 627 | 8.68 678 | 11.31 322 | 9.99 949 | 13 | |
| 48 | 8.68 886 | 8.68 938 | 11.31 062 | 9.99 948 | 12 | |
| 49 | 8.69 144 | 8.69 196 | 11.30 804 | 9.99 948 | 11 | |
| **50** | 8.69 400 | 8.69 453 | 11.30 547 | 9.99 947 | **10** | |
| 51 | 8.69 654 | 8.69 708 | 11.30 292 | 9.99 946 | 9 | |
| 52 | 8.69 907 | 8.69 962 | 11.30 038 | 9.99 946 | 8 | |
| 53 | 8.70 159 | 8.70 214 | 11.29 786 | 9.99 945 | 7 | |
| 54 | 8.70 409 | 8.70 465 | 11.29 535 | 9.99 944 | 6 | |
| 55 | 8.70 658 | 8.70 714 | 11.29 286 | 9.99 944 | 5 | |
| 56 | 8.70 905 | 8.70 962 | 11.29 038 | 9.99 943 | 4 | |
| 57 | 8.71 151 | 8.71 208 | 11.28 792 | 9.99 942 | 3 | |
| 58 | 8.71 395 | 8.71 453 | 11.28 547 | 9.99 942 | 2 | |
| 59 | 8 71 638 | 8.71 697 | 11.28 303 | 9.99 941 | 1 | |
| **60** | 8.71 880 | 8.71 940 | 11.28 060 | 9.99 940 | **0** | |
| | L Cos | L Cot | L Tan | L Sin | ′ | Prop. Pts. |

## 3°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|
| **0** | 8.71 880 | | 8.71 940 | | 11.28 060 | 9.99 940 | **60** | See opposite page for Proportional Parts. |
| 1 | 8.72 120 | 240 | 8.72 181 | 241 | 11.27 819 | 9.99 940 | 59 | |
| 2 | 8.72 359 | 239 | 8.72 420 | 239 | 11.27 580 | 9.99 939 | 58 | |
| 3 | 8.72.597 | 238 | 8.72 659 | 239 | 11.27 341 | 9.99 938 | 57 | |
| 4 | 8.72 834 | 237 | 8.72 896 | 237 | 11.27 104 | 9.99 938 | 56 | |
| 5 | 8.73 069 | 235 | 8.73 132 | 236 | 11.26 868 | 9.99 937 | 55 | |
| 6 | 8.73 303 | 234 | 8.73 366 | 234 | 11.26 634 | 9.99 936 | 54 | |
| 7 | 8.73 535 | 232 | 8.73 600 | 234 | 11.26 400 | 9.99 936 | 53 | |
| 8 | 8.73 767 | 232 | 8.73 832 | 232 | 11.26 168 | 9.99 935 | 52 | |
| 9 | 8.73 997 | 230 | 8.74 063 | 231 | 11.25 937 | 9.99 934 | 51 | |
| **10** | 8.74 226 | 229 | 8.74 292 | 229 | 11.25 708 | 9.99 934 | **50** | |
| 11 | 8.74 454 | 228 | 8.74 521 | 229 | 11.25 479 | 9.99 933 | 49 | |
| 12 | 8.74 680 | 226 | 8.74 748 | 227 | 11.25 252 | 9.99 932 | 48 | |
| 13 | 8.74 906 | 226 | 8.74 974 | 226 | 11.25 026 | 9.99 932 | 47 | |
| 14 | 8.75 130 | 224 | 8.75 199 | 225 | 11.24 801 | 9.99 931 | 46 | |
| 15 | 8.75 353 | 223 | 8.75 423 | 224 | 11.24 577 | 9.99 930 | 45 | |
| 16 | 8.75 575 | 222 | 8.75 645 | 222 | 11.24 355 | 9.99 929 | 44 | |
| 17 | 8.75 795 | 220 | 8.75 867 | 222 | 11.24 133 | 9.99 929 | 43 | |
| 18 | 8.76 015 | 220 | 8.76 087 | 220 | 11.23 913 | 9.99 928 | 42 | |
| 19 | 8.76 234 | 219 | 8.76 306 | 219 | 11.23 694 | 9.99 927 | 41 | |
| **20** | 8.76 451 | 217 | 8.76 525 | 219 | 11.23 475 | 9.99 926 | **40** | |
| 21 | 8.76 667 | 216 | 8.76 742 | 217 | 11.23 258 | 9.99 926 | 39 | |
| 22 | 8.76 883 | 216 | 8.76 958 | 216 | 11.23 042 | 9.99 925 | 38 | |
| 23 | 8.77 097 | 214 | 8.77 173 | 215 | 11.22 827 | 9.99 924 | 37 | |
| 24 | 8.77 310 | 213 | 8.77 387 | 214 | 11.22 613 | 9.99 923 | 36 | |
| 25 | 8.77 522 | 212 | 8.77 600 | 213 | 11.22 400 | 9.99 923 | 35 | |
| 26 | 8.77 733 | 211 | 8.77 811 | 211 | 11.22 189 | 9.99 922 | 34 | |
| 27 | 8.77 943 | 210 | 8.78 022 | 211 | 11.21 978 | 9.99 921 | 33 | |
| 28 | 8.78 152 | 209 | 8.78 232 | 210 | 11.21 768 | 9.99 920 | 32 | |
| 29 | 8.78 360 | 208 | 8.78 441 | 209 | 11.21 559 | 9.99 920 | 31 | |
| **30** | 8.78 568 | 208 | 8.78 649 | 208 | 11.21 351 | 9.99 919 | **30** | |
| 31 | 8.78 774 | 206 | 8.78 855 | 206 | 11.21 145 | 9.99 918 | 29 | |
| 32 | 8.78 979 | 205 | 8.79 061 | 206 | 11.20 939 | 9.99 917 | 28 | |
| 33 | 8.79 183 | 204 | 8.79 266 | 205 | 11.20 734 | 9.99 917 | 27 | |
| 34 | 8.79 386 | 203 | 8.79 470 | 204 | 11.20 530 | 9.99 916 | 26 | |
| 35 | 8.79 588 | 202 | 8.79 673 | 203 | 11.20 327 | 9.99 915 | 25 | |
| 36 | 8.79 789 | 201 | 8.79 875 | 202 | 11.20 125 | 9.99 914 | 24 | |
| 37 | 8.79 990 | 201 | 8.80 076 | 201 | 11.19 924 | 9.99 913 | 23 | |
| 38 | 8.80 189 | 199 | 8.80 277 | 201 | 11.19 723 | 9.99 913 | 22 | |
| 39 | 8.80 388 | 199 | 8.80 476 | 199 | 11.19 524 | 9.99 912 | 21 | |
| **40** | 8.80 585 | 197 | 8.80 674 | 198 | 11.19 326 | 9.99 911 | **20** | |
| 41 | 8.80 782 | 197 | 8.80 872 | 198 | 11.19 128 | 9.99 910 | 19 | |
| 42 | 8.80 978 | 196 | 8.81 068 | 196 | 11.18 932 | 9.99 909 | 18 | |
| 43 | 8.81 173 | 195 | 8.81 264 | 196 | 11.18 736 | 9.99 909 | 17 | |
| 44 | 8.81 367 | 194 | 8.81 459 | 195 | 11.18 541 | 9.99 908 | 16 | |
| 45 | 8.81 560 | 193 | 8.81 653 | 194 | 11.18 347 | 9.99 907 | 15 | |
| 46 | 8.81 752 | 192 | 8.81 846 | 193 | 11.18 154 | 9.99 906 | 14 | |
| 47 | 8.81 944 | 192 | 8.82 038 | 192 | 11.17 962 | 9.99 905 | 13 | |
| 48 | 8.82 134 | 190 | 8.82 230 | 192 | 11.17 770 | 9.99 904 | 12 | |
| 49 | 8.82 324 | 190 | 8.82 420 | 190 | 11.17 580 | 9.99 904 | 11 | |
| **50** | 8.82 513 | 189 | 8.82 610 | 190 | 11.17 390 | 9.99 903 | **10** | |
| 51 | 8.82 701 | 188 | 8.82 799 | 189 | 11.17 201 | 9.99 902 | 9 | |
| 52 | 8.82 888 | 187 | 8.82 987 | 188 | 11.17 013 | 9.99 901 | 8 | |
| 53 | 8.83 075 | 187 | 8.83 175 | 188 | 11.16 825 | 9.99 900 | 7 | |
| 54 | 8.83 261 | 186 | 8.83 361 | 186 | 11.16 639 | 9.99 899 | 6 | |
| 55 | 8.83 446 | 185 | 8.83 547 | 186 | 11.16 453 | 9.99 898 | 5 | |
| 56 | 8.83 630 | 184 | 8.83 732 | 185 | 11.16 268 | 9.99 898 | 4 | |
| 57 | 8.83 813 | 183 | 8.83 916 | 184 | 11.16 084 | 9.99 897 | 3 | |
| 58 | 8.83 996 | 183 | 8.84 100 | 184 | 11.15 900 | 9.99 896 | 2 | |
| 59 | 8.84 177 | 181 | 8.84 282 | 182 | 11.15 718 | 9.99 895 | 1 | |
| **60** | 8.84 358 | 181 | 8.84 464 | 182 | 11.15 536 | 9.99 894 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ | Prop. Pts. |

## 86°

## Proportional Parts for 3°

| " | 241 | 240 | 239 | 238 | 237 | 236 | 235 | 234 | 232 | 231 | 230 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 24.1 | 24.0 | 23.9 | 23.8 | 23.7 | 23.6 | 23.5 | 23.4 | 23.2 | 23.1 | 23.0 |
| 7 | 28.1 | 28.0 | 27.9 | 27.8 | 27.6 | 27.5 | 27.4 | 27.3 | 27.1 | 27.0 | 26.8 |
| 8 | 32.1 | 32.0 | 31.9 | 31.7 | 31.6 | 31.5 | 31.3 | 31.2 | 30.9 | 30.8 | 30.7 |
| 9 | 36.2 | 36.0 | 35.8 | 35.7 | 35.6 | 35.4 | 35.2 | 35.1 | 34.8 | 34.7 | 34.5 |
| 10 | 40.2 | 40.0 | 39.8 | 39.7 | 39.5 | 39.3 | 39.2 | 39.0 | 38.7 | 38.5 | 38.3 |
| 20 | 80.3 | 80.0 | 79.7 | 79.3 | 79.0 | 78.7 | 78.3 | 78.0 | 77.3 | 77.0 | 76.7 |
| 30 | 120.5 | 120.0 | 119.5 | 119.0 | 118.5 | 118.0 | 117.5 | 117.0 | 116.0 | 115.5 | 115.0 |
| 40 | 160.7 | 160.0 | 159.3 | 158.7 | 158.0 | 157.3 | 156.7 | 156.0 | 154.7 | 154.0 | 153.3 |
| 50 | 200.8 | 200.0 | 199.2 | 198.3 | 197.5 | 196.7 | 195.8 | 195.0 | 193.3 | 192.5 | 191.7 |

| " | 229 | 228 | 227 | 226 | 225 | 224 | 223 | 222 | 220 | 219 | 217 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 22.9 | 22.8 | 22.7 | 22.6 | 22.5 | 22.4 | 22.3 | 22.2 | 22.0 | 21.9 | 21.7 |
| 7 | 26.7 | 26.6 | 26.5 | 26.4 | 26.2 | 26.1 | 26.0 | 25.9 | 25.7 | 25.6 | 25.3 |
| 8 | 30.5 | 30.4 | 30.3 | 30.1 | 30.0 | 29.9 | 29.7 | 29.6 | 29.3 | 29.2 | 28.9 |
| 9 | 34.4 | 34.2 | 34.0 | 33.9 | 33.8 | 33.6 | 33.4 | 33.3 | 33.0 | 32.9 | 32.6 |
| 10 | 38.2 | 38.0 | 37.8 | 37.7 | 37.5 | 37.3 | 37.2 | 37.0 | 36.7 | 36.5 | 36.2 |
| 20 | 76.3 | 76.0 | 75.7 | 75.3 | 75.0 | 74.7 | 74.3 | 74.0 | 73.3 | 73.0 | 72.3 |
| 30 | 114.5 | 114.0 | 113.5 | 113.0 | 112.5 | 112.0 | 111.5 | 111.0 | 110.0 | 109.5 | 108.5 |
| 40 | 152.7 | 152.0 | 151.3 | 150.7 | 150.0 | 149.3 | 148.7 | 148.0 | 146.7 | 146.0 | 144.7 |
| 50 | 190.8 | 190.0 | 189.2 | 188.3 | 187.5 | 186.7 | 185.8 | 185.0 | 183.3 | 182.5 | 180.8 |

| " | 216 | 215 | 214 | 213 | 212 | 211 | 210 | 209 | 208 | 206 | 205 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 21.6 | 21.5 | 21.4 | 21.3 | 21.2 | 21.1 | 21.0 | 20.9 | 20.8 | 20.6 | 20.5 |
| 7 | 25.2 | 25.1 | 25.0 | 24.9 | 24.7 | 24.6 | 24.5 | 24.4 | 24.3 | 24.0 | 23.9 |
| 8 | 28.8 | 28.7 | 28.5 | 28.4 | 28.3 | 28.1 | 28.0 | 27.9 | 27.7 | 27.5 | 27.3 |
| 9 | 32.4 | 32.2 | 32.1 | 32.0 | 31.8 | 31.6 | 31.5 | 31.4 | 31.2 | 30.9 | 30.8 |
| 10 | 36.0 | 35.8 | 35.7 | 35.5 | 35.3 | 35.2 | 35.0 | 34.8 | 34.7 | 34.3 | 34.2 |
| 20 | 72.0 | 71.7 | 71.3 | 71.0 | 70.7 | 70.3 | 70.0 | 69.7 | 69.3 | 68.7 | 68.3 |
| 30 | 108.0 | 107.5 | 107.0 | 106.5 | 106.0 | 105.5 | 105.0 | 104.5 | 104.0 | 103.0 | 102.5 |
| 40 | 144.0 | 143.3 | 142.7 | 142.0 | 141.3 | 140.7 | 140.0 | 139.3 | 138.7 | 137.3 | 136.7 |
| 50 | 180.0 | 179.2 | 178.3 | 177.5 | 176.7 | 175.8 | 175.0 | 174.2 | 173.3 | 171.7 | 170.8 |

| " | 204 | 203 | 202 | 201 | 199 | 198 | 197 | 196 | 195 | 194 | 193 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 20.4 | 20.3 | 20.2 | 20.1 | 19.9 | 19.8 | 19.7 | 19.6 | 19.5 | 19.4 | 19.3 |
| 7 | 23.8 | 23.7 | 23.6 | 23.4 | 23.2 | 23.1 | 23.0 | 22.9 | 22.8 | 22.6 | 22.5 |
| 8 | 27.2 | 27.1 | 26.9 | 26.8 | 26.5 | 26.4 | 26.3 | 26.1 | 26.0 | 25.9 | 25.7 |
| 9 | 30.6 | 30.4 | 30.3 | 30.2 | 29.8 | 29.7 | 29.6 | 29.4 | 29.2 | 29.1 | 29.0 |
| 10 | 34.0 | 33.8 | 33.7 | 33.5 | 33.2 | 33.0 | 32.8 | 32.7 | 32.5 | 32.3 | 32.2 |
| 20 | 68.0 | 67.7 | 67.3 | 67.0 | 66.3 | 66.0 | 65.7 | 65.3 | 65.0 | 64.7 | 64.3 |
| 30 | 102.0 | 101.5 | 101.0 | 100.5 | 99.5 | 99.0 | 98.5 | 98.0 | 97.5 | 97.0 | 96.5 |
| 40 | 136.0 | 135.3 | 134.7 | 134.0 | 132.7 | 132.0 | 131.3 | 130.7 | 130.0 | 129.3 | 128.7 |
| 50 | 170.0 | 169.2 | 168.3 | 167.5 | 165.8 | 165.0 | 164.2 | 163.3 | 162.5 | 161.7 | 160.8 |

| " | 192 | 190 | 189 | 188 | 187 | 186 | 185 | 184 | 183 | 182 | 181 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 19.2 | 19.0 | 18.9 | 18.8 | 18.7 | 18.6 | 18.5 | 18.4 | 18.3 | 18.2 | 18.1 |
| 7 | 22.4 | 22.2 | 22.1 | 21.9 | 21.8 | 21.7 | 21.6 | 21.5 | 21.4 | 21.2 | 21.1 |
| 8 | 25.6 | 25.3 | 25.2 | 25.1 | 24.9 | 24.8 | 24.7 | 24.5 | 24.4 | 24.3 | 24.1 |
| 9 | 28.8 | 28.5 | 28.4 | 28.2 | 28.1 | 27.9 | 27.8 | 27.6 | 27.4 | 27.3 | 27.2 |
| 10 | 32.0 | 31.7 | 31.5 | 31.3 | 31.2 | 31.0 | 30.8 | 30.7 | 30.5 | 30.3 | 30.2 |
| 20 | 64.0 | 63.3 | 63.0 | 62.7 | 62.3 | 62.0 | 61.7 | 61.3 | 61.0 | 60.7 | 60.3 |
| 30 | 96.0 | 95.0 | 94.5 | 94.0 | 93.5 | 93.0 | 92.5 | 92.0 | 91.5 | 91.0 | 90.5 |
| 40 | 128.0 | 126.7 | 126.0 | 125.3 | 124.7 | 124.0 | 123.3 | 122.7 | 122.0 | 121.3 | 120.7 |
| 50 | 160.0 | 158.3 | 157.5 | 156.7 | 155.8 | 155.0 | 154.2 | 153.3 | 152.5 | 151.7 | 150.8 |

## 4°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|
| **0** | 8.84 358 | | 8.84 464 | | 11.15 536 | 9.99 894 | **60** | |
| 1 | 8.84 539 | 181 | 8.84 646 | 182 | 11.15 354 | 9.99 893 | 59 | |
| 2 | 8.84 718 | 179 | 8.84 826 | 180 | 11.15 174 | 9.99 892 | 58 | |
| 3 | 8.84 897 | 179 | 8.85 006 | 180 | 11.14 994 | 9.99 891 | 57 | |
| 4 | 8.85 075 | 178 | 8.85 185 | 179 | 11.14 815 | 9.99 891 | 56 | |
| 5 | 8.85 252 | 177 | 8.85 363 | 178 | 11.14 637 | 9.99 890 | 55 | |
| 6 | 8.85 429 | 177 | 8.85 540 | 177 | 11.14 460 | 9.99 889 | 54 | |
| 7 | 8.85 605 | 176 | 8.85 717 | 177 | 11.14 283 | 9.99 888 | 53 | |
| 8 | 8.85 780 | 175 | 8.85 893 | 176 | 11.14 107 | 9.99 887 | 52 | |
| 9 | 8.85 955 | 175 | 8.86 069 | 176 | 11.13 931 | 9.99 886 | 51 | |
| **10** | 8.86 128 | 173 | 8.86 243 | 174 | 11.13 757 | 9.99 885 | **50** | |
| 11 | 8.86 301 | 173 | 8.86 417 | 174 | 11.13 583 | 9.99 884 | 49 | |
| 12 | 8.86 474 | 173 | 8.86 591 | 174 | 11.13 409 | 9.99 883 | 48 | |
| 13 | 8.86 645 | 171 | 8.86 763 | 172 | 11.13 237 | 9.99 882 | 47 | |
| 14 | 8.86 816 | 171 | 8.86 935 | 172 | 11.13 065 | 9.99 881 | 46 | |
| 15 | 8.86 987 | 171 | 8.87 106 | 171 | 11.12 894 | 9.99 880 | 45 | |
| 16 | 8.87 156 | 169 | 8.87 277 | 171 | 11.12 723 | 9.99 879 | 44 | |
| 17 | 8.87 325 | 169 | 8.87 447 | 170 | 11.12 553 | 9.99 879 | 43 | |
| 18 | 8.87 494 | 169 | 8.87 616 | 169 | 11.12 384 | 9.99 878 | 42 | |
| 19 | 8.87 661 | 167 | 8.87 785 | 169 | 11.12 215 | 9.99 877 | 41 | |
| **20** | 8.87 829 | 168 | 8.87 953 | 168 | 11.12 047 | 9.99 876 | **40** | |
| 21 | 8.87 995 | 166 | 8.88 120 | 167 | 11.11 880 | 9.99 875 | 39 | |
| 22 | 8.88 161 | 166 | 8.88 287 | 167 | 11.11 713 | 9.99 874 | 38 | |
| 23 | 8.88 326 | 165 | 8.88 453 | 166 | 11.11 547 | 9.99 873 | 37 | |
| 24 | 8.88 490 | 164 | 8.88 618 | 165 | 11.11 382 | 9.99 872 | 36 | |
| 25 | 8.88 654 | 164 | 8.88 783 | 165 | 11.11 217 | 9.99 871 | 35 | |
| 26 | 8.88 817 | 163 | 8.88 948 | 165 | 11.11 052 | 9.99 870 | 34 | |
| 27 | 8.88 980 | 163 | 8.89 111 | 163 | 11.10 889 | 9.99 869 | 33 | |
| 28 | 8.89 142 | 162 | 8.89 274 | 163 | 11.10 726 | 9.99 868 | 32 | |
| 29 | 8.89 304 | 162 | 8.89 437 | 163 | 11.10 563 | 9.99 867 | 31 | |
| **30** | 8.89 464 | 160 | 8.89 598 | 161 | 11.10 402 | 9.99 866 | **30** | |
| 31 | 8.89 625 | 161 | 8.89 760 | 162 | 11.10 240 | 9.99 865 | 29 | |
| 32 | 8.89 784 | 159 | 8.89 920 | 160 | 11.10 080 | 9.99 864 | 28 | |
| 33 | 8.89 943 | 159 | 8.90 080 | 160 | 11.09 920 | 9.99 863 | 27 | |
| 34 | 8.90 102 | 159 | 8.90 240 | 160 | 11.09 760 | 9.99 862 | 26 | |
| 35 | 8.90 260 | 158 | 8.90 399 | 159 | 11.09 601 | 9.99 861 | 25 | |
| 36 | 8.90 417 | 157 | 8.90 557 | 158 | 11.09 443 | 9.99 860 | 24 | |
| 37 | 8.90 574 | 157 | 8.90 715 | 158 | 11.09 285 | 9.99 859 | 23 | |
| 38 | 8.90 730 | 156 | 8.90 872 | 157 | 11.09 128 | 9.99 858 | 22 | |
| 39 | 8.90 885 | 155 | 8.91 029 | 157 | 11.08 971 | 9.99 857 | 21 | |
| **40** | 8.91 040 | 155 | 8.91 185 | 156 | 11.08 815 | 9.99 856 | **20** | |
| 41 | 8.91 195 | 155 | 8.91 340 | 155 | 11.08 660 | 9.99 855 | 19 | |
| 42 | 8.91 349 | 154 | 8.91 495 | 155 | 11.08 505 | 9.99 854 | 18 | |
| 43 | 8.91 502 | 153 | 8.91 650 | 155 | 11.08 350 | 9.99 853 | 17 | |
| 44 | 8.91 655 | 153 | 8.91 803 | 153 | 11.08 197 | 9.99 852 | 16 | |
| 45 | 8.91 807 | 152 | 8.91 957 | 154 | 11.08 043 | 9.99 851 | 15 | |
| 46 | 8.91 959 | 152 | 8.92 110 | 153 | 11.07 890 | 9.99 850 | 14 | |
| 47 | 8.92 110 | 151 | 8.92 262 | 152 | 11.07 738 | 9.99 848 | 13 | |
| 48 | 8.92 261 | 151 | 8.92 414 | 152 | 11.07 586 | 9.99 847 | 12 | |
| 49 | 8.92 411 | 150 | 8.92 565 | 151 | 11.07 435 | 9.99 846 | 11 | |
| **50** | 8.92 561 | 150 | 8.92 716 | 151 | 11.07 284 | 9.99 845 | **10** | |
| 51 | 8.92 710 | 149 | 8.92 866 | 150 | 11.07 134 | 9.99 844 | 9 | |
| 52 | 8.92 859 | 149 | 8.93 016 | 150 | 11.06 984 | 9.99 843 | 8 | |
| 53 | 8.93 007 | 148 | 8.93 165 | 149 | 11.06 835 | 9.99 842 | 7 | |
| 54 | 8.93 154 | 147 | 8.93 313 | 148 | 11.06 687 | 9.99 841 | 6 | |
| 55 | 8.93 301 | 147 | 8.93 462 | 149 | 11.06 538 | 9.99 840 | 5 | |
| 56 | 8.93 448 | 147 | 8.93 609 | 147 | 11.06 391 | 9.99 839 | 4 | |
| 57 | 8.93 594 | 146 | 8.93 756 | 147 | 11.06 244 | 9.99 838 | 3 | |
| 58 | 8.93 740 | 146 | 8.93 903 | 147 | 11.06 097 | 9.99 837 | 2 | |
| 59 | 8.93 885 | 145 | 8.94 049 | 146 | 11.05 951 | 9.99 836 | 1 | |
| **60** | 8.94 030 | 145 | 8.94 195 | 146 | 11.05 805 | 9.99 834 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ | Prop. Pts. |

See opposite page for Proportional Parts.

## 85°

## Proportional Parts for 4°

| ″ | 182 | 181 | 180 | 179 | 178 | 177 | 176 |
|---|---|---|---|---|---|---|---|
| 6 | 18.2 | 18.1 | 18.0 | 17.9 | 17.8 | 17.7 | 17.6 |
| 7 | 21.2 | 21.1 | 21.0 | 20.9 | 20.8 | 20.6 | 20.5 |
| 8 | 24.3 | 24.1 | 24.0 | 23.9 | 23.7 | 23.6 | 23.5 |
| 9 | 27.3 | 27.2 | 27.0 | 26.8 | 26.7 | 26.6 | 26.4 |
| 10 | 30.3 | 30.2 | 30.0 | 29.8 | 29.7 | 29.5 | 29.3 |
| 20 | 60.7 | 60.3 | 60.0 | 59.7 | 59.3 | 59.0 | 58.7 |
| 30 | 91.0 | 90.5 | 90.0 | 89.5 | 89.0 | 88.5 | 88.0 |
| 40 | 121.3 | 120.7 | 120.0 | 119.3 | 118.7 | 118.0 | 117.3 |
| 50 | 151.7 | 150.8 | 150.0 | 149.2 | 148.3 | 147.5 | 146.7 |

| ″ | 175 | 174 | 173 | 172 | 171 | 170 | 169 |
|---|---|---|---|---|---|---|---|
| 6 | 17.5 | 17.4 | 17.3 | 17.2 | 17.1 | 17.0 | 16.9 |
| 7 | 20.4 | 20.3 | 20.2 | 20.1 | 20.0 | 19.8 | 19.7 |
| 8 | 23.3 | 23.2 | 23.1 | 22.9 | 22.8 | 22.7 | 22.5 |
| 9 | 26.2 | 26.1 | 26.0 | 25.8 | 25.6 | 25.5 | 25.4 |
| 10 | 29.2 | 29.0 | 28.8 | 28.7 | 28.5 | 28.3 | 28.2 |
| 20 | 58.3 | 58.0 | 57.7 | 57.3 | 57.0 | 56.7 | 56.3 |
| 30 | 87.5 | 87.0 | 86.5 | 86.0 | 85.5 | 85.0 | 84.5 |
| 40 | 116.7 | 116.0 | 115.3 | 114.7 | 114.0 | 113.3 | 112.7 |
| 50 | 145.8 | 145.0 | 144.2 | 143.3 | 142.5 | 141.7 | 140.8 |

| ″ | 168 | 167 | 166 | 165 | 164 | 163 | 162 |
|---|---|---|---|---|---|---|---|
| 6 | 16.8 | 16.7 | 16.6 | 16.5 | 16.4 | 16.3 | 16.2 |
| 7 | 19.6 | 19.5 | 19.4 | 19.2 | 19.1 | 19.0 | 18.9 |
| 8 | 22.4 | 22.3 | 22.1 | 22.0 | 21.9 | 21.7 | 21.6 |
| 9 | 25.2 | 25.0 | 24.9 | 24.8 | 24.6 | 24.4 | 24.3 |
| 10 | 28.0 | 27.8 | 27.7 | 27.5 | 27.3 | 27.2 | 27.0 |
| 20 | 56.0 | 55.7 | 55.3 | 55.0 | 54.7 | 54.3 | 54.0 |
| 30 | 84.0 | 83.5 | 83.0 | 82.5 | 82.0 | 81.5 | 81.0 |
| 40 | 112.0 | 111.3 | 110.7 | 110.0 | 109.3 | 108.7 | 108.0 |
| 50 | 140.0 | 139.2 | 138.3 | 137.5 | 136.7 | 135.8 | 135.0 |

| ″ | 161 | 160 | 159 | 158 | 157 | 156 | 155 |
|---|---|---|---|---|---|---|---|
| 6 | 16.1 | 16.0 | 15.9 | 15.8 | 15.7 | 15.6 | 15.5 |
| 7 | 18.8 | 18.7 | 18.6 | 18.4 | 18.3 | 18.2 | 18.1 |
| 8 | 21.5 | 21.3 | 21.2 | 21.1 | 20.9 | 20.8 | 20.7 |
| 9 | 24.2 | 24.0 | 23.8 | 23.7 | 23.6 | 23.4 | 23.2 |
| 10 | 26.8 | 26.7 | 26.5 | 26.3 | 26.2 | 26.0 | 25.8 |
| 20 | 53.7 | 53.3 | 53.0 | 52.7 | 52.3 | 52.0 | 51.7 |
| 30 | 80.5 | 80.0 | 79.5 | 79.0 | 78.5 | 78.0 | 77.5 |
| 40 | 107.3 | 106.7 | 106.0 | 105.3 | 104.7 | 104.0 | 103.3 |
| 50 | 134.2 | 133.3 | 132.5 | 131.7 | 130.8 | 130.0 | 129.2 |

| ″ | 154 | 153 | 152 | 151 | 150 | 149 | 148 |
|---|---|---|---|---|---|---|---|
| 6 | 15.4 | 15.3 | 15.2 | 15.1 | 15.0 | 14.9 | 14.8 |
| 7 | 18.0 | 17.8 | 17.7 | 17.6 | 17.5 | 17.4 | 17.3 |
| 8 | 20.5 | 20.4 | 20.3 | 20.1 | 20.0 | 19.9 | 19.7 |
| 9 | 23.1 | 23.0 | 22.8 | 22.6 | 22.5 | 22.4 | 22.2 |
| 10 | 25.7 | 25.5 | 25.3 | 25.2 | 25.0 | 24.8 | 24.7 |
| 20 | 51.3 | 51.0 | 50.7 | 50.3 | 50.0 | 49.7 | 49.3 |
| 30 | 77.0 | 76.5 | 76.0 | 75.5 | 75.0 | 74.5 | 74.0 |
| 40 | 102.7 | 102.0 | 101.3 | 100.7 | 100.0 | 99.3 | 98.7 |
| 50 | 128.3 | 127.5 | 126.7 | 125.8 | 125.0 | 124.2 | 123.3 |

For **147**, **146**, and **145** see page 32.

## 5°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | | Prop. Pts. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8.94 030 | | 8.94 195 | | 11.05 805 | 9.99 834 | 60 | ″ | 147 | 146 | 145 | 144 |
| 1 | 8.94 174 | 144 | 8.94 340 | 145 | 11.05 660 | 9.99 833 | 59 | 6 | 14.7 | 14.6 | 14.5 | 14.4 |
| 2 | 8.94 317 | 143 | 8.94 485 | 145 | 11.05 515 | 9.99 832 | 58 | 7 | 17.2 | 17.0 | 16.9 | 16.8 |
| 3 | 8.94 461 | 144 | 8.94 630 | 145 | 11.05 370 | 9.99 831 | 57 | 8 | 19.6 | 19.5 | 19.3 | 19.2 |
| 4 | 8.94 603 | 142 | 8.94 773 | 143 | 11.05 227 | 9.99 830 | 56 | 9 | 22.0 | 21.9 | 21.8 | 21.6 |
| 5 | 8.94 746 | 143 | 8.94 917 | 144 | 11.05 083 | 9.99 829 | 55 | 10 | 24.5 | 24.3 | 24.2 | 24.0 |
| 6 | 8.94 887 | 141 | 8.95 060 | 143 | 11.04 940 | 9.99 828 | 54 | 20 | 49.0 | 48.7 | 48.3 | 48.0 |
| 7 | 8.95 029 | 142 | 8.95 202 | 142 | 11.04 798 | 9.99 827 | 53 | 30 | 73.5 | 73.0 | 72.5 | 72.0 |
| 8 | 8.95 170 | 141 | 8.95 344 | 142 | 11.04 656 | 9.99 825 | 52 | 40 | 98.0 | 97.3 | 96.7 | 96.0 |
| 9 | 8.95 310 | 140 | 8.95 486 | 142 | 11.04 514 | 9.99 824 | 51 | 50 | 122.5 | 121.7 | 120.8 | 120.0 |
| 10 | 8.95 450 | 140 | 8.95 627 | 141 | 11.04 373 | 9.99 823 | 50 | ″ | 143 | 142 | 141 | 140 |
| 11 | 8.95 589 | 139 | 8.95 767 | 140 | 11.04 233 | 9.99 822 | 49 | 6 | 14.3 | 14.2 | 14.1 | 14.0 |
| 12 | 8.95 728 | 139 | 8.95 908 | 141 | 11.04 092 | 9.99 821 | 48 | 7 | 16.7 | 16.6 | 16.4 | 16.3 |
| 13 | 8.95 867 | 139 | 8.96 047 | 139 | 11.03 953 | 9.99 820 | 47 | 8 | 19.1 | 18.9 | 18.8 | 18.7 |
| 14 | 8.96 005 | 138 | 8.96 187 | 140 | 11.03 813 | 9.99 819 | 46 | 9 | 21.4 | 21.3 | 21.2 | 21.0 |
| 15 | 8.96 143 | 138 | 8.96 325 | 138 | 11.03 675 | 9.99 817 | 45 | 10 | 23.8 | 23.7 | 23.5 | 23.3 |
| 16 | 8.96 280 | 137 | 8.96 464 | 139 | 11.03 536 | 9.99 816 | 44 | 20 | 47.7 | 47.3 | 47.0 | 46.7 |
| 17 | 8.96 417 | 137 | 8.96 602 | 138 | 11.03 398 | 9.99 815 | 43 | 30 | 71.5 | 71.0 | 70.5 | 70.0 |
| 18 | 8.96 553 | 136 | 8.96 739 | 137 | 11.03 261 | 9.99 814 | 42 | 40 | 95.3 | 94.7 | 94.0 | 93.3 |
| 19 | 8.96 689 | 136 | 8.96 877 | 138 | 11.03 123 | 9.99 813 | 41 | 50 | 119.2 | 118.3 | 117.5 | 116.7 |
| 20 | 8.96 825 | 136 | 8.97 013 | 136 | 11.02 987 | 9.99 812 | 40 | ″ | 139 | 138 | 137 | 136 |
| 21 | 8.96 960 | 135 | 8.97 150 | 137 | 11.02 850 | 9.99 810 | 39 | 6 | 13.9 | 13.8 | 13.7 | 13.6 |
| 22 | 8.97 095 | 135 | 8.97 285 | 135 | 11.02 715 | 9.99 809 | 38 | 7 | 16.2 | 16.1 | 16.0 | 15.9 |
| 23 | 8.97 229 | 134 | 8.97 421 | 136 | 11.02 579 | 9.99 808 | 37 | 8 | 18.5 | 18.4 | 18.3 | 18.1 |
| 24 | 8.97 363 | 134 | 8.97 556 | 135 | 11.02 444 | 9.99 807 | 36 | 9 | 20.9 | 20.7 | 20.6 | 20.4 |
| 25 | 8.97 496 | 133 | 8.97 691 | 135 | 11.02 309 | 9.99 806 | 35 | 10 | 23.2 | 23.0 | 22.8 | 22.7 |
| 26 | 8.97 629 | 133 | 8.97 825 | 134 | 11.02 175 | 9.99 804 | 34 | 20 | 46.3 | 46.0 | 45.7 | 45.3 |
| 27 | 8.97 762 | 133 | 8.97 959 | 134 | 11.02 041 | 9.99 803 | 33 | 30 | 69.5 | 69.0 | 68.5 | 68.0 |
| 28 | 8.97 894 | 132 | 8.98 092 | 133 | 11.01 908 | 9.99 802 | 32 | 40 | 92.7 | 92.0 | 91.3 | 90.7 |
| 29 | 8.98 026 | 132 | 8.98 225 | 133 | 11.01 775 | 9.99 801 | 31 | 50 | 115.8 | 115.0 | 114.2 | 113.3 |
| 30 | 8.98 157 | 131 | 8.98 358 | 133 | 11.01 642 | 9.99 800 | 30 | ″ | 135 | 134 | 133 | 132 |
| 31 | 8.98 288 | 131 | 8.98 490 | 132 | 11.01 510 | 9.99 798 | 29 | 6 | 13.5 | 13.4 | 13.3 | 13.2 |
| 32 | 8.98 419 | 131 | 8.98 622 | 132 | 11.01 378 | 9.99 797 | 28 | 7 | 15.8 | 15.6 | 15.5 | 15.4 |
| 33 | 8.98 549 | 130 | 8.98 753 | 131 | 11.01 247 | 9.99 796 | 27 | 8 | 18.0 | 17.9 | 17.7 | 17.6 |
| 34 | 8.98 679 | 130 | 8.98 884 | 131 | 11.01 116 | 9.99 795 | 26 | 9 | 20.2 | 20.1 | 20.0 | 19.8 |
| 35 | 8.98 808 | 129 | 8.99 015 | 131 | 11.00 985 | 9.99 793 | 25 | 10 | 22.5 | 22.3 | 22.2 | 22.0 |
| 36 | 8.98 937 | 129 | 8.99 145 | 130 | 11.00 855 | 9.99 792 | 24 | 20 | 45.0 | 44.7 | 44.3 | 44.0 |
| 37 | 8.99 066 | 129 | 8.99 275 | 130 | 11.00 725 | 9.99 791 | 23 | 30 | 67.5 | 67.0 | 66.5 | 66.0 |
| 38 | 8.99 194 | 128 | 8.99 405 | 130 | 11.00 595 | 9.99 790 | 22 | 40 | 90.0 | 89.3 | 88.7 | 88.0 |
| 39 | 8.99 322 | 128 | 8.99 534 | 129 | 11.00 466 | 9.99 788 | 21 | 50 | 112.5 | 111.7 | 110.8 | 110.0 |
| 40 | 8.99 450 | 128 | 8.99 662 | 128 | 11.00 338 | 9.99 787 | 20 | ″ | 131 | 130 | 129 | 128 |
| 41 | 8.99 577 | 127 | 8.99 791 | 129 | 11.00 209 | 9.99 786 | 19 | 6 | 13.1 | 13.0 | 12.9 | 12.8 |
| 42 | 8.99 704 | 127 | 8.99 919 | 128 | 11.00 081 | 9.99 785 | 18 | 7 | 15.3 | 15.2 | 15.0 | 14.9 |
| 43 | 8.99 830 | 126 | 9.00 046 | 127 | 10.99 954 | 9.99 783 | 17 | 8 | 17.5 | 17.3 | 17.2 | 17.1 |
| 44 | 8.99 956 | 126 | 9.00 174 | 128 | 10.99 826 | 9.99 782 | 16 | 9 | 19.6 | 19.5 | 19.4 | 19.2 |
| 45 | 9.00 082 | 126 | 9.00 301 | 127 | 10.99 699 | 9.99 781 | 15 | 10 | 21.8 | 21.7 | 21.5 | 21.3 |
| 46 | 9.00 207 | 125 | 9.00 427 | 126 | 10.99 573 | 9.99 780 | 14 | 20 | 43.7 | 43.3 | 43.0 | 42.7 |
| 47 | 9.00 332 | 125 | 9.00 553 | 126 | 10.99 447 | 9.99 778 | 13 | 30 | 65.5 | 65.0 | 64.5 | 64.0 |
| 48 | 9.00 456 | 124 | 9.00 679 | 126 | 10.99 321 | 9.99 777 | 12 | 40 | 87.3 | 86.7 | 86.0 | 85.3 |
| 49 | 9.00 581 | 125 | 9.00 805 | 125 | 10.99 195 | 9.99 776 | 11 | 50 | 109.2 | 108.3 | 107.5 | 106.7 |
| 50 | 9.00 704 | 123 | 9.00 930 | 125 | 10.99 070 | 9.99 775 | 10 | ″ | 127 | 126 | 125 | 124 |
| 51 | 9.00 828 | 124 | 9.01 055 | 124 | 10.98 945 | 9.99 773 | 9 | 6 | 12.7 | 12.6 | 12.5 | 12.4 |
| 52 | 9.00 951 | 123 | 9.01 179 | 124 | 10.98 821 | 9.99 772 | 8 | 7 | 14.8 | 14.7 | 14.6 | 14.5 |
| 53 | 9.01 074 | 123 | 9.01 303 | 124 | 10.98 697 | 9.99 771 | 7 | 8 | 16.9 | 16.8 | 16.7 | 16.5 |
| 54 | 9.01 196 | 122 | 9.01 427 | 123 | 10.98 573 | 9.99 769 | 6 | 9 | 19.0 | 18.9 | 18.8 | 18.6 |
| 55 | 9.01 318 | 122 | 9.01 550 | 123 | 10.98 450 | 9.99 768 | 5 | 10 | 21.2 | 21.0 | 20.8 | 20.7 |
| 56 | 9.01 440 | 121 | 9.01 673 | 123 | 10.98 327 | 9.99 767 | 4 | 20 | 42.3 | 42.0 | 41.7 | 41.3 |
| 57 | 9.01 561 | 121 | 9.01 796 | 122 | 10.98 204 | 9.99 765 | 3 | 30 | 63.5 | 63.0 | 62.5 | 62.0 |
| 58 | 9.01 682 | 121 | 9.01 918 | 122 | 10.98 082 | 9.99 764 | 2 | 40 | 84.7 | 84.0 | 83.3 | 82.7 |
| 59 | 9.01 803 | 120 | 9.02 040 | 122 | 10.97 960 | 9.99 763 | 1 | 50 | 105.8 | 105.0 | 104.2 | 103.3 |
| 60 | 9.01 923 | | 9.02 162 | | 10.97 838 | 9.99 761 | 0 | ″ | 123 | 122 | 121 | 120 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ | 6 | 12.3 | 12.2 | 12.1 | 12.0 |
| | | | | | | | | 7 | 14.4 | 14.2 | 14.1 | 14.0 |
| | | | | | | | | 8 | 16.4 | 16.3 | 16.1 | 16.0 |
| | | | | | | | | 9 | 18.4 | 18.3 | 18.2 | 18.0 |
| | | | | | | | | 10 | 20.5 | 20.3 | 20.2 | 20.0 |
| | | | | | | | | 20 | 41.0 | 40.7 | 40.3 | 40.0 |
| | | | | | | | | 30 | 61.5 | 61.0 | 60.5 | 60.0 |
| | | | | | | | | 40 | 82.0 | 81.3 | 80.7 | 80.0 |
| | | | | | | | | 50 | 102.5 | 101.7 | 100.8 | 100.0 |
| | | | | | | | | | Prop. Pts. | | | |

## 84°

## 6°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| 0 | 9.01 923 | 120 | 9.02 162 | 121 | 10.97 838 | 9.99 761 | 60 |
| 1 | 9.02 043 | 120 | 9.02 283 | 121 | 10.97 717 | 9.99 760 | 59 |
| 2 | 9.02 163 | 120 | 9.02 404 | 121 | 10.97 596 | 9.99 759 | 58 |
| 3 | 9.02 283 | 119 | 9.02 525 | 120 | 10.97 475 | 9.99 757 | 57 |
| 4 | 9.02 402 | 118 | 9.02 645 | 121 | 10.97 355 | 9.99 756 | 56 |
| 5 | 9.02 520 | 119 | 9.02 766 | 119 | 10.97 234 | 9.99 755 | 55 |
| 6 | 9.02 639 | 118 | 9.02 885 | 120 | 10.97 115 | 9.99 753 | 54 |
| 7 | 9.02 757 | 117 | 9.03 005 | 119 | 10.96 995 | 9.99 752 | 53 |
| 8 | 9.02 874 | 118 | 9.03 124 | 118 | 10.96 876 | 9.99 751 | 52 |
| 9 | 9.02 992 | 117 | 9.03 242 | 119 | 10.96 758 | 9.99 749 | 51 |
| 10 | 9.03 109 | 117 | 9.03 361 | 118 | 10.96 639 | 9.99 748 | 50 |
| 11 | 9.03 226 | 116 | 9.03 479 | 118 | 10.96 521 | 9.99 747 | 49 |
| 12 | 9.03 342 | 116 | 9.03 597 | 117 | 10.96 403 | 9.99 745 | 48 |
| 13 | 9.03 458 | 116 | 9.03 714 | 118 | 10.96 286 | 9.99 744 | 47 |
| 14 | 9.03 574 | 116 | 9.03 832 | 116 | 10.96 168 | 9.99 742 | 46 |
| 15 | 9.03 690 | 115 | 9.03 948 | 117 | 10.96 052 | 9.99 741 | 45 |
| 16 | 9.03 805 | 115 | 9.04 065 | 116 | 10.95 935 | 9.99 740 | 44 |
| 17 | 9.03 920 | 114 | 9.04 181 | 116 | 10.95 819 | 9.99 738 | 43 |
| 18 | 9.04 034 | 115 | 9.04 297 | 116 | 10.95 703 | 9.99 737 | 42 |
| 19 | 9.04 149 | 113 | 9.04 413 | 115 | 10.95 587 | 9.99 736 | 41 |
| 20 | 9.04 262 | 114 | 9.04 528 | 115 | 10.95 472 | 9.99 734 | 40 |
| 21 | 9.04 376 | 114 | 9.04 643 | 115 | 10.95 357 | 9.99 733 | 39 |
| 22 | 9.04 490 | 113 | 9.04 758 | 115 | 10.95 242 | 9.99 731 | 38 |
| 23 | 9.04 603 | 112 | 9.04 873 | 114 | 10.95 127 | 9.99 730 | 37 |
| 24 | 9.04 715 | 113 | 9.04 987 | 114 | 10.95 013 | 9.99 728 | 36 |
| 25 | 9.04 828 | 112 | 9.05 101 | 113 | 10.94 899 | 9.99 727 | 35 |
| 26 | 9.04 940 | 112 | 9.05 214 | 114 | 10.94 786 | 9.99 726 | 34 |
| 27 | 9.05 052 | 112 | 9.05 328 | 113 | 10.94 672 | 9.99 724 | 33 |
| 28 | 9.05 164 | 111 | 9.05 441 | 112 | 10.94 559 | 9.99 723 | 32 |
| 29 | 9.05 275 | 111 | 9.05 553 | 113 | 10.94 447 | 9.99 721 | 31 |
| 30 | 9.05 386 | 111 | 9.05 666 | 112 | 10.94 334 | 9.99 720 | 30 |
| 31 | 9.05 497 | 110 | 9.05 778 | 112 | 10.94 222 | 9.99 718 | 29 |
| 32 | 9.05 607 | 110 | 9.05 890 | 112 | 10.94 110 | 9.99 717 | 28 |
| 33 | 9.05 717 | 110 | 9.06 002 | 111 | 10.93 998 | 9.99 716 | 27 |
| 34 | 9.05 827 | 110 | 9.06 113 | 111 | 10.93 887 | 9.99 714 | 26 |
| 35 | 9.05 937 | 109 | 9.06 224 | 111 | 10.93 776 | 9.99 713 | 25 |
| 36 | 9.06 046 | 109 | 9.06 335 | 110 | 10.93 665 | 9.99 711 | 24 |
| 37 | 9.06 155 | 109 | 9.06 445 | 111 | 10.93 555 | 9.99 710 | 23 |
| 38 | 9.06 264 | 108 | 9.06 556 | 110 | 10.93 444 | 9.99 708 | 22 |
| 39 | 9.06 372 | 109 | 9.06 666 | 109 | 10.93 334 | 9.99 707 | 21 |
| 40 | 9.06 481 | 108 | 9.06 775 | 110 | 10.93 225 | 9.99 705 | 20 |
| 41 | 9.06 589 | 107 | 9.06 885 | 109 | 10.93 115 | 9.99 704 | 19 |
| 42 | 9.06 696 | 108 | 9.06 994 | 109 | 10.93 006 | 9.99 702 | 18 |
| 43 | 9.06 804 | 107 | 9.07 103 | 108 | 10.92 897 | 9.99 701 | 17 |
| 44 | 9.06 911 | 107 | 9.07 211 | 109 | 10.92 789 | 9.99 699 | 16 |
| 45 | 9.07 018 | 106 | 9.07 320 | 108 | 10.92 680 | 9.99 698 | 15 |
| 46 | 9.07 124 | 107 | 9.07 428 | 108 | 10.92 572 | 9.99 696 | 14 |
| 47 | 9.07 231 | 106 | 9.07 536 | 107 | 10.92 464 | 9.99 695 | 13 |
| 48 | 9.07 337 | 105 | 9.07 643 | 108 | 10.92 357 | 9.99 693 | 12 |
| 49 | 9.07 442 | 106 | 9.07 751 | 107 | 10.92 249 | 9.99 692 | 11 |
| 50 | 9.07 548 | 105 | 9.07 858 | 106 | 10.92 142 | 9.99 690 | 10 |
| 51 | 9.07 653 | 105 | 9.07 964 | 107 | 10.92 036 | 9.99 689 | 9 |
| 52 | 9.07 758 | 105 | 9.08 071 | 106 | 10.91 929 | 9.99 687 | 8 |
| 53 | 9.07 863 | 105 | 9.08 177 | 106 | 10.91 823 | 9.99 686 | 7 |
| 54 | 9.07 968 | 104 | 9.08 283 | 106 | 10.91 717 | 9.99 684 | 6 |
| 55 | 9.08 072 | 104 | 9.08 389 | 105 | 10.91 611 | 9.99 683 | 5 |
| 56 | 9.08 176 | 104 | 9.08 495 | 105 | 10.91 505 | 9.99 681 | 4 |
| 57 | 9.08 280 | 103 | 9.08 600 | 105 | 10.91 400 | 9.99 680 | 3 |
| 58 | 9.08 383 | 103 | 9.08 705 | 104 | 10.91 295 | 9.99 678 | 2 |
| 59 | 9.08 486 | 103 | 9.08 810 | | 10.91 190 | 9.99 677 | 1 |
| 60 | 9.08 589 | | 9.08 914 | | 10.91 086 | 9.99 675 | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

**Prop. Pts.**

| ″ | 121 | 120 | 119 |
|---|---|---|---|
| 6 | 12.1 | 12.0 | 11.9 |
| 7 | 14.1 | 14.0 | 13.9 |
| 8 | 16.1 | 16.0 | 15.9 |
| 9 | 18.2 | 18.0 | 17.8 |
| 10 | 20.2 | 20.0 | 19.8 |
| 20 | 40.3 | 40.0 | 39.7 |
| 30 | 60.5 | 60.0 | 59.5 |
| 40 | 80.7 | 80.0 | 79.3 |
| 50 | 100.8 | 100.0 | 99.2 |

| ″ | 118 | 117 | 116 |
|---|---|---|---|
| 6 | 11.8 | 11.7 | 11.6 |
| 7 | 13.8 | 13.6 | 13.5 |
| 8 | 15.7 | 15.6 | 15.5 |
| 9 | 17.7 | 17.6 | 17.4 |
| 10 | 19.7 | 19.5 | 19.3 |
| 20 | 39.3 | 39.0 | 38.7 |
| 30 | 59.0 | 58.5 | 58.0 |
| 40 | 78.7 | 78.0 | 77.3 |
| 50 | 98.3 | 97.5 | 96.7 |

| ″ | 115 | 114 | 113 |
|---|---|---|---|
| 6 | 11.5 | 11.4 | 11.3 |
| 7 | 13.4 | 13.3 | 13.2 |
| 8 | 15.3 | 15.2 | 15.1 |
| 9 | 17.2 | 17.1 | 17.0 |
| 10 | 19.2 | 19.0 | 18.8 |
| 20 | 38.3 | 38.0 | 37.7 |
| 30 | 57.5 | 57.0 | 56.5 |
| 40 | 76.7 | 76.0 | 75.3 |
| 50 | 95.8 | 95.0 | 94.2 |

| ″ | 112 | 111 | 110 |
|---|---|---|---|
| 6 | 11.2 | 11.1 | 11.0 |
| 7 | 13.1 | 13.0 | 12.8 |
| 8 | 14.9 | 14.8 | 14.7 |
| 9 | 16.8 | 16.6 | 16.5 |
| 10 | 18.7 | 18.5 | 18.3 |
| 20 | 37.3 | 37.0 | 36.7 |
| 30 | 56.0 | 55.5 | 55.0 |
| 40 | 74.7 | 74.0 | 73.3 |
| 50 | 93.3 | 92.5 | 91.7 |

| ″ | 109 | 108 | 107 |
|---|---|---|---|
| 6 | 10.9 | 10.8 | 10.7 |
| 7 | 12.7 | 12.6 | 12.5 |
| 8 | 14.5 | 14.4 | 14.3 |
| 9 | 16.4 | 16.2 | 16.0 |
| 10 | 18.2 | 18.0 | 17.8 |
| 20 | 36.3 | 36.0 | 35.7 |
| 30 | 54.5 | 54.0 | 53.5 |
| 40 | 72.7 | 72.0 | 71.3 |
| 50 | 90.8 | 90.0 | 89.2 |

| ″ | 106 | 105 | 104 |
|---|---|---|---|
| 6 | 10.6 | 10.5 | 10.4 |
| 7 | 12.4 | 12.2 | 12.1 |
| 8 | 14.1 | 14.0 | 13.9 |
| 9 | 15.9 | 15.8 | 15.6 |
| 10 | 17.7 | 17.5 | 17.3 |
| 20 | 35.3 | 35.0 | 34.7 |
| 30 | 53.0 | 52.5 | 52.0 |
| 40 | 70.7 | 70.0 | 69.3 |
| 50 | 88.3 | 87.5 | 86.7 |

**Prop. Pts.**

## 83°

## 7°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| **0** | 9.08 589 | | 9.08 914 | | 10.91 086 | 9.99 675 | **60** |
| 1 | 9.08 692 | 103 | 9.09 019 | 105 | 10.90 981 | 9.99 674 | 59 |
| 2 | 9.08 795 | 103 | 9.09 123 | 104 | 10.90 877 | 9.99 672 | 58 |
| 3 | 9.08 897 | 102 | 9.09 227 | 104 | 10.90 773 | 9.99 670 | 57 |
| 4 | 9.08 999 | 102 | 9.09 330 | 103 | 10.90 670 | 9.99 669 | 56 |
| 5 | 9.09 101 | 102 | 9.09 434 | 104 | 10.90 566 | 9.99 667 | 55 |
| 6 | 9.09 202 | 101 | 9.09 537 | 103 | 10.90 463 | 9.99 666 | 54 |
| 7 | 9.09 304 | 102 | 9.09 640 | 103 | 10.90 360 | 9.99 664 | 53 |
| 8 | 9.09 405 | 101 | 9.09 742 | 102 | 10.90 258 | 9.99 663 | 52 |
| 9 | 9.09 506 | 101 | 9.09 845 | 103 | 10.90 155 | 9.99 661 | 51 |
| **10** | 9.09 606 | 100 | 9.09 947 | 102 | 10.90 053 | 9.99 659 | **50** |
| 11 | 9.09 707 | 101 | 9.10 049 | 102 | 10.89 951 | 9.99 658 | 49 |
| 12 | 9.09 807 | 100 | 9.10 150 | 101 | 10.89 850 | 9.99 656 | 48 |
| 13 | 9.09 907 | 100 | 9.10 252 | 102 | 10.89 748 | 9.99 655 | 47 |
| 14 | 9.10 006 | 99 | 9.10 353 | 101 | 10.89 647 | 9.99 653 | 46 |
| 15 | 9.10 106 | 100 | 9.10 454 | 101 | 10.89 546 | 9.99 651 | 45 |
| 16 | 9.10 205 | 99 | 9.10 555 | 101 | 10.89 445 | 9.99 650 | 44 |
| 17 | 9.10 304 | 99 | 9.10 656 | 101 | 10.89 344 | 9.99 648 | 43 |
| 18 | 9.10 402 | 98 | 9.10 756 | 100 | 10.89 244 | 9.99 647 | 42 |
| 19 | 9.10 501 | 99 | 9.10 856 | 100 | 10.89 144 | 9.99 645 | 41 |
| **20** | 9.10 599 | 98 | 9.10 956 | 100 | 10.89 044 | 9.99 643 | **40** |
| 21 | 9.10 697 | 98 | 9.11 056 | 100 | 10.88 944 | 9.99 642 | 39 |
| 22 | 9.10 795 | 98 | 9.11 155 | 99 | 10.88 845 | 9.99 640 | 38 |
| 23 | 9.10 893 | 97 | 9.11 254 | 99 | 10.88 746 | 9.99 638 | 37 |
| 24 | 9.10 990 | 97 | 9.11 353 | 99 | 10.88 647 | 9.99 637 | 36 |
| 25 | 9.11 087 | 97 | 9.11 452 | 99 | 10.88 548 | 9.99 635 | 35 |
| 26 | 9.11 184 | 97 | 9.11 551 | 99 | 10.88 449 | 9.99 633 | 34 |
| 27 | 9.11 281 | 96 | 9.11 649 | 98 | 10.88 351 | 9.99 632 | 33 |
| 28 | 9.11 377 | 97 | 9.11 747 | 98 | 10.88 253 | 9.99 630 | 32 |
| 29 | 9.11 474 | 96 | 9.11 845 | 98 | 10.88 155 | 9.99 629 | 31 |
| **30** | 9.11 570 | 96 | 9.11 943 | 98 | 10.88 057 | 9.99 627 | **30** |
| 31 | 9.11 666 | 95 | 9.12 040 | 97 | 10.87 960 | 9.99 625 | 29 |
| 32 | 9.11 761 | 95 | 9.12 138 | 98 | 10.87 862 | 9.99 624 | 28 |
| 33 | 9.11 857 | 96 | 9.12 235 | 97 | 10.87 765 | 9.99 622 | 27 |
| 34 | 9.11 952 | 95 | 9.12 332 | 97 | 10.87 668 | 9.99 620 | 26 |
| 35 | 9.12 047 | 95 | 9.12 428 | 96 | 10.87 572 | 9.99 618 | 25 |
| 36 | 9.12 142 | 95 | 9.12 525 | 97 | 10.87 475 | 9.99 617 | 24 |
| 37 | 9.12 236 | 94 | 9.12 621 | 96 | 10.87 379 | 9.99 615 | 23 |
| 38 | 9.12 331 | 95 | 9.12 717 | 96 | 10.87 283 | 9.99 613 | 22 |
| 39 | 9.12 425 | 94 | 9.12 813 | 96 | 10.87 187 | 9.99 612 | 21 |
| **40** | 9.12 519 | 94 | 9.12 909 | 96 | 10.87 091 | 9.99 610 | **20** |
| 41 | 9.12 612 | 93 | 9.13 004 | 95 | 10.86 996 | 9.99 608 | 19 |
| 42 | 9.12 706 | 94 | 9.13 099 | 95 | 10.86 901 | 9.99 607 | 18 |
| 43 | 9.12 799 | 93 | 9.13 194 | 95 | 10.86 806 | 9.99 605 | 17 |
| 44 | 9.12 892 | 93 | 9.13 289 | 95 | 10.86 711 | 9.99 603 | 16 |
| 45 | 9.12 985 | 93 | 9.13 384 | 95 | 10.86 616 | 9.99 601 | 15 |
| 46 | 9.13 078 | 93 | 9.13 478 | 94 | 10.86 522 | 9.99 600 | 14 |
| 47 | 9.13 171 | 93 | 9.13 573 | 95 | 10.86 427 | 9.99 598 | 13 |
| 48 | 9.13 263 | 92 | 9.13 667 | 94 | 10.86 333 | 9.99 596 | 12 |
| 49 | 9.13 355 | 92 | 9.13 761 | 94 | 10.86 239 | 9.99 595 | 11 |
| **50** | 9.13 447 | 92 | 9.13 854 | 93 | 10.86 146 | 9.99 593 | **10** |
| 51 | 9.13 539 | 92 | 9.13 948 | 94 | 10.86 052 | 9.99 591 | 9 |
| 52 | 9.13 630 | 91 | 9.14 041 | 93 | 10.85 959 | 9.99 589 | 8 |
| 53 | 9.13 722 | 92 | 9.14 134 | 93 | 10.85 866 | 9.99 588 | 7 |
| 54 | 9.13 813 | 91 | 9.14 227 | 93 | 10.85 773 | 9.99 586 | 6 |
| 55 | 9.13 904 | 91 | 9.14 320 | 93 | 10.85 680 | 9.99 584 | 5 |
| 56 | 9.13 994 | 90 | 9.14 412 | 92 | 10.85 588 | 9.99 582 | 4 |
| 57 | 9.14 085 | 91 | 9.14 504 | 92 | 10.85 496 | 9.99 581 | 3 |
| 58 | 9.14 175 | 90 | 9.14 597 | 93 | 10.85 403 | 9.99 579 | 2 |
| 59 | 9.14 266 | 91 | 9.14 688 | 91 | 10.85 312 | 9.99 577 | 1 |
| **60** | 9.14 356 | 90 | 9.14 780 | 92 | 10.85 220 | 9.99 575 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

**Prop. Pts.**

| ″ | 105 | 104 | 103 |
|---|---|---|---|
| 6 | 10.5 | 10.4 | 10.3 |
| 7 | 12.3 | 12.1 | 12.0 |
| 8 | 14.0 | 13.9 | 13.7 |
| 9 | 15.8 | 15.6 | 15.4 |
| 10 | 17.5 | 17.3 | 17.2 |
| 20 | 35.0 | 34.7 | 34.3 |
| 30 | 52.5 | 52.0 | 51.5 |
| 40 | 70.0 | 69.3 | 68.7 |
| 50 | 87.5 | 86.7 | 85.8 |

| ″ | 102 | 101 | 100 |
|---|---|---|---|
| 6 | 10.2 | 10.1 | 10.0 |
| 7 | 11.9 | 11.8 | 11.7 |
| 8 | 13.6 | 13.5 | 13.3 |
| 9 | 15.3 | 15.2 | 15.0 |
| 10 | 17.0 | 16.8 | 16.7 |
| 20 | 34.0 | 33.7 | 33.3 |
| 30 | 51.0 | 50.5 | 50.0 |
| 40 | 68.0 | 67.3 | 66.7 |
| 50 | 85.0 | 84.2 | 83.3 |

| ″ | 99 | 98 | 97 |
|---|---|---|---|
| 6 | 9.9 | 9.8 | 9.7 |
| 7 | 11.6 | 11.4 | 11.3 |
| 8 | 13.2 | 13.1 | 12.9 |
| 9 | 14.8 | 14.7 | 14.6 |
| 10 | 16.5 | 16.3 | 16.2 |
| 20 | 33.0 | 32.7 | 32.3 |
| 30 | 49.5 | 49.0 | 48.5 |
| 40 | 66.0 | 65.3 | 64.7 |
| 50 | 82.5 | 81.7 | 80.8 |

| ″ | 96 | 95 | 94 |
|---|---|---|---|
| 6 | 9.6 | 9.5 | 9.4 |
| 7 | 11.2 | 11.1 | 11.0 |
| 8 | 12.8 | 12.7 | 12.5 |
| 9 | 14.4 | 14.2 | 14.1 |
| 10 | 16.0 | 15.8 | 15.7 |
| 20 | 32.0 | 31.7 | 31.3 |
| 30 | 48.0 | 47.5 | 47.0 |
| 40 | 64.0 | 63.3 | 62.7 |
| 50 | 80.0 | 79.2 | 78.3 |

| ″ | 93 | 92 | 91 |
|---|---|---|---|
| 6 | 9.3 | 9.2 | 9.1 |
| 7 | 10.9 | 10.7 | 10.6 |
| 8 | 12.4 | 12.3 | 12.1 |
| 9 | 14.0 | 13.8 | 13.6 |
| 10 | 15.5 | 15.3 | 15.2 |
| 20 | 31.0 | 30.7 | 30.3 |
| 30 | 46.5 | 46.0 | 45.5 |
| 40 | 62.0 | 61.3 | 60.7 |
| 50 | 77.5 | 76.7 | 75.8 |

| ″ | 90 | 2 | 1 |
|---|---|---|---|
| 6 | 9.0 | 0.2 | 0.1 |
| 7 | 10.5 | 0.2 | 0.1 |
| 8 | 12.0 | 0.3 | 0.1 |
| 9 | 13.5 | 0.3 | 0.2 |
| 10 | 15.0 | 0.3 | 0.2 |
| 20 | 30.0 | 0.7 | 0.3 |
| 30 | 45.0 | 1.0 | 0.5 |
| 40 | 60.0 | 1.3 | 0.7 |
| 50 | 75.0 | 1.7 | 0.8 |

## 82°

## 8°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| 0 | 9.14 356 | | 9.14 780 | | 10.85 220 | 9.99 575 | 60 |
| 1 | 9.14 445 | 89 | 9.14 872 | 92 | 10.85 128 | 9.99 574 | 59 |
| 2 | 9.14 535 | 90 | 9.14 963 | 91 | 10.85 037 | 9.99 572 | 58 |
| 3 | 9.14 624 | 89 | 9.15 054 | 91 | 10.84 946 | 9.99 570 | 57 |
| 4 | 9.14 714 | 90 | 9.15 145 | 91 | 10.84 855 | 9.99 568 | 56 |
| 5 | 9.14 803 | 89 | 9.15 236 | 91 | 10.84 764 | 9.99 566 | 55 |
| 6 | 9.14 891 | 88 | 9.15 327 | 90 | 10.84 673 | 9.99 565 | 54 |
| 7 | 9.14 980 | 89 | 9.15 417 | 91 | 10.84 583 | 9.99 563 | 53 |
| 8 | 9.15 069 | 89 | 9.15 508 | 90 | 10.84 492 | 9.99 561 | 52 |
| 9 | 9.15 157 | 88 | 9.15 598 | 90 | 10.84 402 | 9.99 559 | 51 |
| 10 | 9.15 245 | 88 | 9.15 688 | 89 | 10.84 312 | 9.99 557 | 50 |
| 11 | 9.15 333 | 88 | 9.15 777 | 90 | 10.84 223 | 9.99 556 | 49 |
| 12 | 9.15 421 | 87 | 9.15 867 | 89 | 10.84 133 | 9.99 554 | 48 |
| 13 | 9.15 508 | 88 | 9.15 956 | 90 | 10.84 044 | 9.99 552 | 47 |
| 14 | 9.15 596 | 87 | 9.16 046 | 89 | 10.83 954 | 9.99 550 | 46 |
| 15 | 9.15 683 | 87 | 9.16 135 | 89 | 10.83 865 | 9.99 548 | 45 |
| 16 | 9.15 770 | 87 | 9.16 224 | 88 | 10.83 776 | 9.99 546 | 44 |
| 17 | 9.15 857 | 86 | 9.16 312 | 89 | 10.83 688 | 9.99 545 | 43 |
| 18 | 9.15 944 | 86 | 9.16 401 | 88 | 10.83 599 | 9.99 543 | 42 |
| 19 | 9.16 030 | 86 | 9.16 489 | 88 | 10.83 511 | 9.99 541 | 41 |
| 20 | 9.16 116 | 87 | 9.16 577 | 88 | 10.83 423 | 9.99 539 | 40 |
| 21 | 9.16 203 | 86 | 9.16 665 | 88 | 10.83 335 | 9.99 537 | 39 |
| 22 | 9.16 289 | 85 | 9.16 753 | 87 | 10.83 247 | 9.99 535 | 38 |
| 23 | 9.16 374 | 86 | 9.16 841 | 88 | 10.83 159 | 9.99 533 | 37 |
| 24 | 9.16 460 | 85 | 9.16 928 | 87 | 10.83 072 | 9.99 532 | 36 |
| 25 | 9.16 545 | 86 | 9.17 016 | 87 | 10.82 984 | 9.99 530 | 35 |
| 26 | 9.16 631 | 85 | 9.17 103 | 87 | 10.82 897 | 9.99 528 | 34 |
| 27 | 9.16 716 | 85 | 9.17 190 | 86 | 10.82 810 | 9.99 526 | 33 |
| 28 | 9.16 801 | 85 | 9.17 277 | 87 | 10.82 723 | 9.99 524 | 32 |
| 29 | 9.16 886 | 84 | 9.17 363 | 86 | 10.82 637 | 9.99 522 | 31 |
| 30 | 9.16 970 | 85 | 9.17 450 | 86 | 10.82 550 | 9.99 520 | 30 |
| 31 | 9.17 055 | 84 | 9.17 536 | 86 | 10.82 464 | 9.99 518 | 29 |
| 32 | 9.17 139 | 84 | 9.17 622 | 86 | 10.82 378 | 9.99 517 | 28 |
| 33 | 9.17 223 | 84 | 9.17 708 | 86 | 10.82 292 | 9.99 515 | 27 |
| 34 | 9.17 307 | 84 | 9.17 794 | 86 | 10.82 206 | 9.99 513 | 26 |
| 35 | 9.17 391 | 83 | 9.17 880 | 85 | 10.82 120 | 9.99 511 | 25 |
| 36 | 9.17 474 | 84 | 9.17 965 | 86 | 10.82 035 | 9.99 509 | 24 |
| 37 | 9.17 558 | 83 | 9.18 051 | 85 | 10.81 949 | 9.99 507 | 23 |
| 38 | 9.17 641 | 83 | 9.18 136 | 85 | 10.81 864 | 9.99 505 | 22 |
| 39 | 9.17 724 | 83 | 9.18 221 | 85 | 10.81 779 | 9.99 503 | 21 |
| 40 | 9.17 807 | 83 | 9.18 306 | 85 | 10.81 694 | 9.99 501 | 20 |
| 41 | 9.17 890 | 83 | 9.18 391 | 84 | 10.81 609 | 9.99 499 | 19 |
| 42 | 9.17 973 | 82 | 9.18 475 | 85 | 10.81 525 | 9.99 497 | 18 |
| 43 | 9.18 055 | 82 | 9.18 560 | 84 | 10.81 440 | 9.99 495 | 17 |
| 44 | 9.18 137 | 83 | 9.18 644 | 84 | 10.81 356 | 9.99 494 | 16 |
| 45 | 9.18 220 | 82 | 9.18 728 | 84 | 10.81 272 | 9.99 492 | 15 |
| 46 | 9.18 302 | 81 | 9.18 812 | 84 | 10.81 188 | 9.99 490 | 14 |
| 47 | 9.18 383 | 82 | 9.18 896 | 83 | 10.81 104 | 9.99 488 | 13 |
| 48 | 9.18 465 | 82 | 9.18 979 | 84 | 10.81 021 | 9.99 486 | 12 |
| 49 | 9.18 547 | 81 | 9.19 063 | 83 | 10.80 937 | 9.99 484 | 11 |
| 50 | 9.18 628 | 81 | 9.19 146 | 83 | 10.80 854 | 9.99 482 | 10 |
| 51 | 9.18 709 | 81 | 9.19 229 | 83 | 10.80 771 | 9.99 480 | 9 |
| 52 | 9.18 790 | 81 | 9.19 312 | 83 | 10.80 688 | 9.99 478 | 8 |
| 53 | 9.18 871 | 81 | 9.19 395 | 83 | 10.80 605 | 9.99 476 | 7 |
| 54 | 9.18 952 | 81 | 9.19 478 | 83 | 10.80 522 | 9.99 474 | 6 |
| 55 | 9.19 033 | 80 | 9.19 561 | 82 | 10.80 439 | 9.99 472 | 5 |
| 56 | 9.19 113 | 80 | 9.19 643 | 82 | 10.80 357 | 9.99 470 | 4 |
| 57 | 9.19 193 | 80 | 9.19 725 | 82 | 10.80 275 | 9.99 468 | 3 |
| 58 | 9.19 273 | 80 | 9.19 807 | 82 | 10.80 193 | 9.99 466 | 2 |
| 59 | 9.19 353 | 80 | 9.19 889 | 82 | 10.80 111 | 9.99 464 | 1 |
| 60 | 9.19 433 | | 9.19 971 | | 10.80 029 | 9.99 462 | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

Prop. Pts.

| ″ | 92 | 91 | 90 |
|---|---|---|---|
| 6 | 9.2 | 9.1 | 9.0 |
| 7 | 10.7 | 10.6 | 10.5 |
| 8 | 12.3 | 12.1 | 12.0 |
| 9 | 13.8 | 13.6 | 13.5 |
| 10 | 15.3 | 15.2 | 15.0 |
| 20 | 30.7 | 30.3 | 30.0 |
| 30 | 46.0 | 45.5 | 45.0 |
| 40 | 61.3 | 60.7 | 60.0 |
| 50 | 76.7 | 75.8 | 75.0 |

| ″ | 89 | 88 | 87 |
|---|---|---|---|
| 6 | 8.9 | 8.8 | 8.7 |
| 7 | 10.4 | 10.3 | 10.2 |
| 8 | 11.9 | 11.7 | 11.6 |
| 9 | 13.4 | 13.2 | 13.0 |
| 10 | 14.8 | 14.7 | 14.5 |
| 20 | 29.7 | 29.3 | 29.0 |
| 30 | 44.5 | 44.0 | 43.5 |
| 40 | 59.3 | 58.7 | 58.0 |
| 50 | 74.2 | 73.3 | 72.5 |

| ″ | 86 | 85 | 84 |
|---|---|---|---|
| 6 | 8.6 | 8.5 | 8.4 |
| 7 | 10.0 | 9.9 | 9.8 |
| 8 | 11.5 | 11.3 | 11.2 |
| 9 | 12.9 | 12.8 | 12.6 |
| 10 | 14.3 | 14.2 | 14.0 |
| 20 | 28.7 | 28.3 | 28.0 |
| 30 | 43.0 | 42.5 | 42.0 |
| 40 | 57.3 | 56.7 | 56.0 |
| 50 | 71.7 | 70.8 | 70.0 |

| ″ | 83 | 82 | 81 |
|---|---|---|---|
| 6 | 8.3 | 8.2 | 8.1 |
| 7 | 9.7 | 9.6 | 9.4 |
| 8 | 11.1 | 10.9 | 10.8 |
| 9 | 12.4 | 12.3 | 12.2 |
| 10 | 13.8 | 13.7 | 13.5 |
| 20 | 27.7 | 27.3 | 27.0 |
| 30 | 41.5 | 41.0 | 40.5 |
| 40 | 55.3 | 54.7 | 54.0 |
| 50 | 69.2 | 68.3 | 67.5 |

| ″ | 80 | 2 | 1 |
|---|---|---|---|
| 6 | 8.0 | 0.2 | 0.1 |
| 7 | 9.3 | 0.2 | 0.1 |
| 8 | 10.7 | 0.3 | 0.1 |
| 9 | 12.0 | 0.3 | 0.2 |
| 10 | 13.3 | 0.3 | 0.2 |
| 20 | 26.7 | 0.7 | 0.3 |
| 30 | 40.0 | 1.0 | 0.5 |
| 40 | 53.3 | 1.3 | 0.7 |
| 50 | 66.7 | 1.7 | 0.8 |

## 81°

## 9°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| 0 | 9.19 433 | | 9.19 971 | | 10.80 029 | 9.99 462 | 60 |
| 1 | 9.19 513 | 80 | 9.20 053 | 82 | 10.79 947 | 9.99 460 | 59 |
| 2 | 9.19 592 | 79 | 9.20 134 | 81 | 10.79 866 | 9.99 458 | 58 |
| 3 | 9.19 672 | 80 | 9.20 216 | 82 | 10.79 784 | 9.99 456 | 57 |
| 4 | 9.19 751 | 79 | 9.20 297 | 81 | 10.79 703 | 9.99 454 | 56 |
| 5 | 9.19 830 | 79 | 9.20 378 | 81 | 10.79 622 | 9.99 452 | 55 |
| 6 | 9.19 909 | 79 | 9.20 459 | 81 | 10.79 541 | 9.99 450 | 54 |
| 7 | 9.19 988 | 79 | 9.20 540 | 81 | 10.79 460 | 9.99 448 | 53 |
| 8 | 9.20 067 | 78 | 9.20 621 | 80 | 10.79 379 | 9.99 446 | 52 |
| 9 | 9.20 145 | 78 | 9.20 701 | 81 | 10.79 299 | 9.99 444 | 51 |
| 10 | 9.20 223 | 79 | 9.20 782 | 80 | 10.79 218 | 9.99 442 | 50 |
| 11 | 9.20 302 | 78 | 9.20 862 | 80 | 10.79 138 | 9.99 440 | 49 |
| 12 | 9.20 380 | 78 | 9.20 942 | 80 | 10.79 058 | 9.99 438 | 48 |
| 13 | 9.20 458 | 77 | 9.21 022 | 80 | 10.78 978 | 9.99 436 | 47 |
| 14 | 9.20 535 | 78 | 9.21 102 | 80 | 10.78 898 | 9.99 434 | 46 |
| 15 | 9.20 613 | 78 | 9.21 182 | 79 | 10.78 818 | 9.99 432 | 45 |
| 16 | 9.20 691 | 77 | 9.21 261 | 80 | 10.78 739 | 9.99 429 | 44 |
| 17 | 9.20 768 | 77 | 9.21 341 | 79 | 10.78 659 | 9.99 427 | 43 |
| 18 | 9.20 845 | 77 | 9.21 420 | 79 | 10.78 580 | 9.99 425 | 42 |
| 19 | 9.20 922 | 77 | 9.21 499 | 79 | 10.78 501 | 9.99 423 | 41 |
| 20 | 9.20 999 | 77 | 9.21 578 | 79 | 10.78 422 | 9.99 421 | 40 |
| 21 | 9.21 076 | 77 | 9.21 657 | 79 | 10.78 343 | 9.99 419 | 39 |
| 22 | 9.21 153 | 76 | 9.21 736 | 78 | 10.78 264 | 9.99 417 | 38 |
| 23 | 9.21 229 | 77 | 9.21 814 | 79 | 10.78 186 | 9.99 415 | 37 |
| 24 | 9.21 306 | 76 | 9.21 893 | 78 | 10.78 107 | 9.99 413 | 36 |
| 25 | 9.21 382 | 76 | 9.21 971 | 78 | 10.78 029 | 9.99 411 | 35 |
| 26 | 9.21 458 | 76 | 9.22 049 | 78 | 10.77 951 | 9.99 409 | 34 |
| 27 | 9.21 534 | 76 | 9.22 127 | 78 | 10.77 873 | 9.99 407 | 33 |
| 28 | 9.21 610 | 75 | 9.22 205 | 78 | 10.77 795 | 9.99 404 | 32 |
| 29 | 9.21 685 | 76 | 9.22 283 | 78 | 10.77 717 | 9.99 402 | 31 |
| 30 | 9.21 761 | 75 | 9.22 361 | 77 | 10.77 639 | 9.99 400 | 30 |
| 31 | 9.21 836 | 76 | 9.22 438 | 78 | 10.77 562 | 9.99 398 | 29 |
| 32 | 9.21 912 | 75 | 9.22 516 | 77 | 10.77 484 | 9.99 396 | 28 |
| 33 | 9.21 987 | 75 | 9.22 593 | 77 | 10.77 407 | 9.99 394 | 27 |
| 34 | 9.22 062 | 75 | 9.22 670 | 77 | 10.77 330 | 9.99 392 | 26 |
| 35 | 9.22 137 | 74 | 9.22 747 | 77 | 10.77 253 | 9.99 390 | 25 |
| 36 | 9.22 211 | 75 | 9.22 824 | 77 | 10.77 176 | 9.99 388 | 24 |
| 37 | 9.22 286 | 75 | 9.22 901 | 76 | 10.77 099 | 9.99 385 | 23 |
| 38 | 9.22 361 | 74 | 9.22 977 | 77 | 10.77 023 | 9.99 383 | 22 |
| 39 | 9.22 435 | 74 | 9.23 054 | 76 | 10.76 946 | 9.99 381 | 21 |
| 40 | 9.22 509 | 74 | 9.23 130 | 76 | 10.76 870 | 9.99 379 | 20 |
| 41 | 9.22 583 | 74 | 9.23 206 | 77 | 10.76 794 | 9.99 377 | 19 |
| 42 | 9.22 657 | 74 | 9.23 283 | 76 | 10.76 717 | 9.99 375 | 18 |
| 43 | 9.22 731 | 74 | 9.23 359 | 76 | 10.76 641 | 9.99 372 | 17 |
| 44 | 9.22 805 | 73 | 9.23 435 | 75 | 10.76 565 | 9.99 370 | 16 |
| 45 | 9.22 878 | 74 | 9.23 510 | 76 | 10.76 490 | 9.99 368 | 15 |
| 46 | 9.22 952 | 73 | 9.23 586 | 75 | 10.76 414 | 9.99 366 | 14 |
| 47 | 9.23 025 | 73 | 9.23 661 | 76 | 10.76 339 | 9.99 364 | 13 |
| 48 | 9.23 098 | 73 | 9.23 737 | 75 | 10.76 263 | 9.99 362 | 12 |
| 49 | 9.23 171 | 73 | 9.23 812 | 75 | 10.76 188 | 9.99 359 | 11 |
| 50 | 9.23 244 | 73 | 9.23 887 | 75 | 10.76 113 | 9.99 357 | 10 |
| 51 | 9.23 317 | 73 | 9.23 962 | 75 | 10.76 038 | 9.99 355 | 9 |
| 52 | 9.23 390 | 72 | 9.24 037 | 75 | 10.75 963 | 9.99 353 | 8 |
| 53 | 9.23 462 | 73 | 9.24 112 | 74 | 10.75 888 | 9.99 351 | 7 |
| 54 | 9.23 535 | 72 | 9.24 186 | 75 | 10.75 814 | 9.99 348 | 6 |
| 55 | 9.23 607 | 72 | 9.24 261 | 74 | 10.75 739 | 9.99 346 | 5 |
| 56 | 9.23 679 | 73 | 9.24 335 | 75 | 10.75 665 | 9.99 344 | 4 |
| 57 | 9.23 752 | 71 | 9.24 410 | 74 | 10.75 590 | 9.99 342 | 3 |
| 58 | 9.23 823 | 72 | 9.24 484 | 74 | 10.75 516 | 9.99 340 | 2 |
| 59 | 9.23 895 | 72 | 9.24 558 | 74 | 10.75 442 | 9.99 337 | 1 |
| 60 | 9.23 967 | | 9.24 632 | | 10.75 368 | 9.99 335 | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

Prop. Pts.

| ″ | 80 | 79 | 78 |
|---|---|---|---|
| 6 | 8.0 | 7.9 | 7.8 |
| 7 | 9.3 | 9.2 | 9.1 |
| 8 | 10.7 | 10.5 | 10.4 |
| 9 | 12.0 | 11.8 | 11.7 |
| 10 | 13.3 | 13.2 | 13.0 |
| 20 | 26.7 | 26.3 | 26.0 |
| 30 | 40.0 | 39.5 | 39.0 |
| 40 | 53.3 | 52.7 | 52.0 |
| 50 | 66.7 | 65.8 | 65.0 |

| ″ | 77 | 76 | 75 |
|---|---|---|---|
| 6 | 7.7 | 7.6 | 7.5 |
| 7 | 9.0 | 8.9 | 8.8 |
| 8 | 10.3 | 10.1 | 10.0 |
| 9 | 11.6 | 11.4 | 11.2 |
| 10 | 12.8 | 12.7 | 12.5 |
| 20 | 25.7 | 25.3 | 25.0 |
| 30 | 38.5 | 38.0 | 37.5 |
| 40 | 51.3 | 50.7 | 50.0 |
| 50 | 64.2 | 63.3 | 62.5 |

| ″ | 74 | 73 | 72 |
|---|---|---|---|
| 6 | 7.4 | 7.3 | 7.2 |
| 7 | 8.6 | 8.5 | 8.4 |
| 8 | 9.9 | 9.7 | 9.6 |
| 9 | 11.1 | 11.0 | 10.8 |
| 10 | 12.3 | 12.2 | 12.0 |
| 20 | 24.7 | 24.3 | 24.0 |
| 30 | 37.0 | 36.5 | 36.0 |
| 40 | 49.3 | 48.7 | 48.0 |
| 50 | 61.7 | 60.8 | 60.0 |

| ″ | 71 | 3 | 2 |
|---|---|---|---|
| 6 | 7.1 | 0.3 | 0.2 |
| 7 | 8.3 | 0.4 | 0.2 |
| 8 | 9.5 | 0.4 | 0.3 |
| 9 | 10.6 | 0.4 | 0.3 |
| 10 | 11.8 | 0.5 | 0.3 |
| 20 | 23.7 | 1.0 | 0.7 |
| 30 | 35.5 | 1.5 | 1.0 |
| 40 | 47.3 | 2.0 | 1.3 |
| 50 | 59.2 | 2.5 | 1.7 |

Prop. Pts.

## 80°

## 10°

| | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| **0** | 9.23 967 | 72 | 9.24 632 | 74 | 10.75 368 | 9.99 335 | **60** |
| 1 | 9.24 039 | 71 | 9.24 706 | 73 | 10.75 294 | 9.99 333 | 59 |
| 2 | 9.24 110 | 71 | 9.24 779 | 74 | 10.75 221 | 9.99 331 | 58 |
| 3 | 9.24 181 | 72 | 9.24 853 | 73 | 10.75 147 | 9.99 328 | 57 |
| 4 | 9.24 253 | 71 | 9.24 926 | 74 | 10.75 074 | 9.99 326 | 56 |
| 5 | 9.24 324 | 71 | 9.25 000 | 73 | 10.75 000 | 9.99 324 | 55 |
| 6 | 9.24 395 | 71 | 9.25 073 | 73 | 10.74 927 | 9.99 322 | 54 |
| 7 | 9.24 466 | 70 | 9.25 146 | 73 | 10.74 854 | 9.99 319 | 53 |
| 8 | 9.24 536 | 71 | 9.25 219 | 73 | 10.74 781 | 9.99 317 | 52 |
| 9 | 9.24 607 | 70 | 9.25 292 | 73 | 10.74 708 | 99.9 315 | 51 |
| **10** | 9.24 677 | 71 | 9.25 365 | 72 | 10.74 635 | 9.99 313 | **50** |
| 11 | 9.24 748 | 70 | 9.25 437 | 73 | 10.74 563 | 9.99 310 | 49 |
| 12 | 9.24 818 | 70 | 9.25 510 | 72 | 10.74 490 | 9.99 308 | 48 |
| 13 | 9.24 888 | 70 | 9.25 582 | 73 | 10.74 418 | 9.99 306 | 47 |
| 14 | 9.24 958 | 70 | 9.25 655 | 72 | 10.74 345 | 9.99 304 | 46 |
| 15 | 9.25 028 | 70 | 9.25 727 | 72 | 10.74 273 | 9.99 301 | 45 |
| 16 | 9.25 098 | 70 | 9.25 799 | 72 | 10.74 201 | 9.99 299 | 44 |
| 17 | 9.25 168 | 69 | 9.25 871 | 72 | 10.74 129 | 9.99 297 | 43 |
| 18 | 8.25 237 | 70 | 9.25 943 | 72 | 10.74 057 | 9.99 294 | 42 |
| 19 | 9.25 307 | 69 | 9.26 015 | 71 | 10.73 985 | 9.99 292 | 41 |
| **20** | 9.25 376 | 69 | 9.26 086 | 72 | 10.73 914 | 9.99 290 | **40** |
| 21 | 9.25 445 | 69 | 9.26 158 | 71 | 10.73 842 | 9.99 288 | 39 |
| 22 | 9.25 514 | 69 | 9.26 229 | 71 | 10.73 771 | 9.99 285 | 38 |
| 23 | 9.25 583 | 69 | 9.26 301 | 71 | 10.73 699 | 9.99 283 | 37 |
| 24 | 9.25 652 | 69 | 9.26 372 | 71 | 10.73 628 | 9.99 281 | 36 |
| 25 | 9.25 721 | 69 | 9.26 443 | 71 | 10.73 557 | 9.99 278 | 35 |
| 26 | 9.25 790 | 68 | 9.26 514 | 71 | 10.73 486 | 9.99 276 | 34 |
| 27 | 9.25 858 | 69 | 9.26 585 | 70 | 10.73 415 | 9.99 274 | 33 |
| 28 | 9.25 927 | 68 | 9.26 655 | 71 | 10.73 345 | 9.99 271 | 32 |
| 29 | 9.25 995 | 68 | 9.26 726 | 71 | 10.73 274 | 9.99 269 | 31 |
| **30** | 9.26 063 | 68 | 9.26 797 | 70 | 10.73 203 | 9.99 267 | **30** |
| 31 | 9.26 131 | 68 | 9.26 867 | 70 | 10.73 133 | 9.99 264 | 29 |
| 32 | 9.26 199 | 68 | 9.26 937 | 70 | 10.73 063 | 9.99 262 | 28 |
| 33 | 9.26 267 | 68 | 9.27 008 | 70 | 10.72 992 | 9.99 260 | 27 |
| 34 | 9.26 335 | 68 | 9.27 078 | 70 | 10.72 922 | 9.99 257 | 26 |
| 35 | 9.26 403 | 67 | 9.27 148 | 70 | 10.72 852 | 9.99 255 | 25 |
| 36 | 9.26 470 | 68 | 9.27 218 | 70 | 10.72 782 | 9.99 252 | 24 |
| 37 | 9.26 538 | 67 | 9.27 288 | 69 | 10.72 712 | 9.99 250 | 23 |
| 38 | 9.26 605 | 67 | 9.27 357 | 70 | 10.72 643 | 9.99 248 | 22 |
| 39 | 9.26 672 | 67 | 9.27 427 | 69 | 10.72 573 | 9.99 245 | 21 |
| **40** | 9.26 739 | 67 | 9.27 496 | 70 | 10.72 504 | 9.99 243 | **20** |
| 41 | 9.26 806 | 67 | 9.27 566 | 69 | 10.72 434 | 9.99 241 | 19 |
| 42 | 9.26 873 | 67 | 9.27 635 | 69 | 10.72 365 | 9.99 238 | 18 |
| 43 | 9.26 940 | 67 | 9.27 704 | 69 | 10.72 296 | 9.99 236 | 17 |
| 44 | 9.27 007 | 66 | 9.27 773 | 69 | 10.72 227 | 9.99 233 | 16 |
| 45 | 9.27 073 | 67 | 9.27 842 | 69 | 10.72 158 | 9.99 231 | 15 |
| 46 | 9.27 140 | 66 | 9.27 911 | 69 | 10.72 089 | 9.99 229 | 14 |
| 47 | 9.27 206 | 67 | 9.27 980 | 69 | 10.72 020 | 9.99 226 | 13 |
| 48 | 9.27 273 | 66 | 9.28 049 | 68 | 10.71 951 | 9.99 224 | 12 |
| 49 | 9.27 339 | 66 | 9.28 117 | 69 | 10.71 883 | 9.99 221 | 11 |
| **50** | 9.27 405 | 66 | 9.28 186 | 68 | 10.71 814 | 9.99 219 | **10** |
| 51 | 9.27 471 | 66 | 9.28 254 | 69 | 10.71 746 | 9.99 217 | 9 |
| 52 | 9.27 537 | 65 | 9.28 323 | 68 | 10.71 677 | 9.99 214 | 8 |
| 53 | 9.27 602 | 66 | 9.28 391 | 68 | 10.71 609 | 9.99 212 | 7 |
| 54 | 9.27 668 | 66 | 9.28 459 | 68 | 10.71 541 | 9.99 209 | 6 |
| 55 | 9.27 734 | 65 | 9.28 527 | 68 | 10.71 473 | 9.99 207 | 5 |
| 56 | 9.27 799 | 65 | 9.28 595 | 67 | 10.71 405 | 9.99 204 | 4 |
| 57 | 9.27 864 | 66 | 9.28 662 | 68 | 10.71 338 | 9.99 202 | 3 |
| 58 | 9.27 930 | 65 | 9.28 730 | 68 | 10.71 270 | 9.99 200 | 2 |
| 59 | 9.27 995 | 65 | 9.28 798 | 67 | 10.71 202 | 9.99 197 | 1 |
| **60** | 9.28 060 | | 9.28 865 | | 10.71 135 | 9.99 195 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

Prop. Pts.

| ″ | 74 | 73 | 72 |
|---|---|---|---|
| 6 | 7.4 | 7.3 | 7.2 |
| 7 | 8.6 | 8.5 | 8.4 |
| 8 | 9.9 | 9.7 | 9.6 |
| 9 | 11.1 | 11.0 | 10.8 |
| 10 | 12.3 | 12.2 | 12.0 |
| 20 | 24.7 | 24.3 | 24.0 |
| 30 | 37.0 | 36.5 | 36.0 |
| 40 | 49.3 | 48.7 | 48.0 |
| 50 | 61.7 | 60.8 | 60.0 |

| ″ | 71 | 70 | 69 |
|---|---|---|---|
| 6 | 7.1 | 7.0 | 6.9 |
| 7 | 8.3 | 8.2 | 8.0 |
| 8 | 9.5 | 9.3 | 9.2 |
| 9 | 10.6 | 10.5 | 10.4 |
| 10 | 11.8 | 11.7 | 11.5 |
| 20 | 23.7 | 23.3 | 23.0 |
| 30 | 35.5 | 35.0 | 34.5 |
| 40 | 47.3 | 46.7 | 46.0 |
| 50 | 59.2 | 58.3 | 57.5 |

| ″ | 68 | 67 | 66 |
|---|---|---|---|
| 6 | 6.8 | 6.7 | 6.6 |
| 7 | 7.9 | 7.8 | 7.7 |
| 8 | 9.1 | 8.9 | 8.8 |
| 9 | 10.2 | 10.0 | 9.9 |
| 10 | 11.3 | 11.2 | 11.0 |
| 20 | 22.7 | 22.3 | 22.0 |
| 30 | 34.0 | 33.5 | 33.0 |
| 40 | 45.3 | 44.7 | 44.0 |
| 50 | 56.7 | 55.8 | 55.0 |

| ″ | 65 | 3 | 2 |
|---|---|---|---|
| 6 | 6.5 | 0.3 | 0.2 |
| 7 | 7.6 | 0.4 | 0.2 |
| 8 | 8.7 | 0.4 | 0.3 |
| 9 | 9.8 | 0.4 | 0.3 |
| 10 | 10.8 | 0.5 | 0.3 |
| 20 | 21.7 | 1.0 | 0.7 |
| 30 | 32.5 | 1.5 | 1.0 |
| 40 | 43.3 | 2.0 | 1.3 |
| 50 | 54.2 | 2.5 | 1.7 |

## 79°

## 11°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| **0** | 9.28 060 | | 9.28 865 | | 10.71 135 | 9.99 195 | **60** |
| 1 | 9.28 125 | 65 | 9.28 933 | 68 | 10.71 067 | 9.99 192 | 59 |
| 2 | 9.28 190 | 65 | 9.29 000 | 67 | 10.71 000 | 9.99 190 | 58 |
| 3 | 9.28 254 | 64 | 9.29 067 | 67 | 10.70 933 | 9.99 187 | 57 |
| 4 | 9.28 319 | 65 | 9.29 134 | 67 | 10.70 866 | 9.99 185 | 56 |
| 5 | 9.28 384 | 65 | 9.29 201 | 67 | 10.70 799 | 9.99 182 | 55 |
| 6 | 9.28 448 | 64 | 9.29 268 | 67 | 10.70 732 | 9.99 180 | 54 |
| 7 | 9.28 512 | 64 | 9.29 335 | 67 | 10.70 665 | 9.99 177 | 53 |
| 8 | 9.28 577 | 65 | 9.29 402 | 67 | 10.70 598 | 9.99 175 | 52 |
| 9 | 9.28 641 | 64 | 9.29 468 | 66 | 10.70 532 | 9.99 172 | 51 |
| **10** | 9.28 705 | 64 | 9.29 535 | 67 | 10.70 465 | 9.99 170 | **50** |
| 11 | 9.28 769 | 64 | 9.29 601 | 66 | 10.70 399 | 9.99 167 | 49 |
| 12 | 9.28 833 | 64 | 9.29 668 | 67 | 10.70 332 | 9.99 165 | 48 |
| 13 | 9.28 896 | 63 | 9.29 734 | 66 | 10.70 266 | 9.99 162 | 47 |
| 14 | 9.28 960 | 64 | 9.29 800 | 66 | 10.70 200 | 9.99 160 | 46 |
| 15 | 9.29 024 | 64 | 9.29 866 | 66 | 10.70 134 | 9.99 157 | 45 |
| 16 | 9.29 087 | 63 | 9.29 932 | 66 | 10.70 068 | 9.99 155 | 44 |
| 17 | 9.29 150 | 63 | 9.29 998 | 66 | 10.70 002 | 9.99 152 | 43 |
| 18 | 9.29 214 | 64 | 9.30 064 | 66 | 10.69 936 | 9.99 150 | 42 |
| 19 | 9.29 277 | 63 | 9.30 130 | 66 | 10.69 870 | 9.99 147 | 41 |
| **20** | 9.29 340 | 63 | 9.30 195 | 65 | 10.69 805 | 9.99 145 | **40** |
| 21 | 9.29 403 | 63 | 9.30 261 | 66 | 10.69 739 | 9.99 142 | 39 |
| 22 | 9.29 466 | 63 | 9.30 326 | 65 | 10.69 674 | 9.99 140 | 38 |
| 23 | 9.29 529 | 63 | 9.30 391 | 65 | 10.69 609 | 9.99 137 | 37 |
| 24 | 9.29 591 | 62 | 9.30 457 | 66 | 10.69 543 | 9.99 135 | 36 |
| 25 | 9.29 654 | 63 | 9.30 522 | 65 | 10.69 478 | 9.99 132 | 35 |
| 26 | 9.29 716 | 62 | 9.30 587 | 65 | 10.69 413 | 9.99 130 | 34 |
| 27 | 9.29 779 | 63 | 9.30 652 | 65 | 10.69 348 | 9.99 127 | 33 |
| 28 | 9.29 841 | 62 | 9.30 717 | 65 | 10.69 283 | 9.99 124 | 32 |
| 29 | 9.29 903 | 62 | 9.30 782 | 65 | 10.69 218 | 9.99 122 | 31 |
| **30** | 9.29 966 | 63 | 9.30 846 | 64 | 10.69 154 | 9.99 119 | **30** |
| 31 | 9.30 028 | 62 | 9.30 911 | 65 | 10.69 089 | 9.99 117 | 29 |
| 32 | 9.30 090 | 62 | 9.30 975 | 64 | 10.69 025 | 9.99 114 | 28 |
| 33 | 9.30 151 | 61 | 9.31 040 | 65 | 10.68 960 | 9.99 112 | 27 |
| 34 | 9.30 213 | 62 | 9.31 104 | 64 | 10.68 896 | 9.99 109 | 26 |
| 35 | 9.30 275 | 62 | 9.31 168 | 64 | 10.68 832 | 9.99 106 | 25 |
| 36 | 9.30 336 | 61 | 9.31 233 | 65 | 10.68 767 | 9.99 104 | 24 |
| 37 | 9.30 398 | 62 | 9.31 297 | 64 | 10.68 703 | 9.99 101 | 23 |
| 38 | 9.30 459 | 61 | 9.31 361 | 64 | 10.68 639 | 9.99 099 | 22 |
| 39 | 9.30 521 | 62 | 9.31 425 | 64 | 10.68 575 | 9.99 096 | 21 |
| **40** | 9.30 582 | 61 | 9.31 489 | 64 | 10.68 511 | 9.99 093 | **20** |
| 41 | 9.30 643 | 61 | 9.31 552 | 63 | 10.68 448 | 9.99 091 | 19 |
| 42 | 9.30 704 | 61 | 9.31 616 | 64 | 10.68 384 | 9.99 088 | 18 |
| 43 | 9.30 765 | 61 | 9.31 679 | 63 | 10.68 321 | 9.99 086 | 17 |
| 44 | 9.30 826 | 61 | 9.31 743 | 64 | 10.68 257 | 9.99 083 | 16 |
| 45 | 9.30 887 | 61 | 9.31 806 | 63 | 10.68 194 | 9.99 080 | 15 |
| 46 | 9.30 947 | 60 | 9.31 870 | 64 | 10.68 130 | 9.99 078 | 14 |
| 47 | 9.31 008 | 61 | 9.31 933 | 63 | 10.68 067 | 9.99 075 | 13 |
| 48 | 9.31 068 | 60 | 9.31 996 | 63 | 10.68 004 | 9.99 072 | 12 |
| 49 | 9.31 129 | 61 | 9.32 059 | 63 | 10.67 941 | 9.99 070 | 11 |
| **50** | 9.31 189 | 60 | 9.32 122 | 63 | 10.67 878 | 9.99 067 | **10** |
| 51 | 9.31 250 | 61 | 9.32 185 | 63 | 10.67 815 | 9.99 064 | 9 |
| 52 | 9.31 310 | 60 | 9.32 248 | 63 | 10.67 752 | 9.99 062 | 8 |
| 53 | 9.31 370 | 60 | 9.32 311 | 63 | 10.67 689 | 9.99 059 | 7 |
| 54 | 9.31 430 | 60 | 9.32 373 | 62 | 10.67 627 | 9.99 056 | 6 |
| 55 | 9.31 490 | 60 | 9.32 436 | 63 | 10.67 564 | 9.99 054 | 5 |
| 56 | 9.31 549 | 59 | 9.32 498 | 62 | 10.67 502 | 9.99 051 | 4 |
| 57 | 9.31 609 | 60 | 9.32 561 | 63 | 10.67 439 | 9.99 048 | 3 |
| 58 | 9.31 669 | 60 | 9.32 623 | 62 | 10.67 377 | 9.99 046 | 2 |
| 59 | 9.31 728 | 59 | 9.32 685 | 62 | 10.67 315 | 9.99 043 | 1 |
| **60** | 9.31 788 | 60 | 9.32 747 | 62 | 10.67 253 | 9.99 040 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

Prop. Pts.

| ″ | 68 | 67 | 66 |
|---|---|---|---|
| 6 | 6.8 | 6.7 | 6.6 |
| 7 | 7.9 | 7.8 | 7.7 |
| 8 | 9.1 | 8.9 | 8.8 |
| 9 | 10.2 | 10.0 | 9.9 |
| 10 | 11.3 | 11.2 | 11.0 |
| 20 | 22.7 | 22.3 | 22.0 |
| 30 | 34.0 | 33.5 | 33.0 |
| 40 | 45.3 | 44.7 | 44.0 |
| 50 | 56.7 | 55.8 | 55.0 |

| ″ | 65 | 64 | 63 |
|---|---|---|---|
| 6 | 6.5 | 6.4 | 6.3 |
| 7 | 7.6 | 7.5 | 7.4 |
| 8 | 8.7 | 8.5 | 8.4 |
| 9 | 9.8 | 9.6 | 9.4 |
| 10 | 10.8 | 10.7 | 10.5 |
| 20 | 21.7 | 21.3 | 21.0 |
| 30 | 32.5 | 32.0 | 31.5 |
| 40 | 43.3 | 42.7 | 42.0 |
| 50 | 54.2 | 53.3 | 52.5 |

| ″ | 62 | 61 | 60 |
|---|---|---|---|
| 6 | 6.2 | 6.1 | 6.0 |
| 7 | 7.2 | 7.1 | 7.0 |
| 8 | 8.3 | 8.1 | 8.0 |
| 9 | 9.3 | 9.2 | 9.0 |
| 10 | 10.3 | 10.2 | 10.0 |
| 20 | 20.7 | 20.3 | 20.0 |
| 30 | 31.0 | 30.5 | 30.0 |
| 40 | 41.3 | 40.7 | 40.0 |
| 50 | 51.7 | 50.8 | 50.0 |

| ″ | 59 | 3 | 2 |
|---|---|---|---|
| 6 | 5.9 | 0.3 | 0.2 |
| 7 | 6.9 | 0.4 | 0.2 |
| 8 | 7.9 | 0.4 | 0.3 |
| 9 | 8.8 | 0.5 | 0.3 |
| 10 | 9.8 | 0.5 | 0.3 |
| 20 | 19.7 | 1.0 | 0.7 |
| 30 | 29.5 | 1.5 | 1.0 |
| 40 | 39.3 | 2.0 | 1.3 |
| 50 | 49.2 | 2.5 | 1.7 |

## 78°

## 12°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| **0** | 9.31 788 | | 9.32 747 | | 10.67 253 | 9.99 040 | **60** |
| 1 | 9.31 847 | 59 | 9.32 810 | 63 | 10.67 190 | 9.99 038 | 59 |
| 2 | 9.31 907 | 60 | 9.32 872 | 62 | 10.67 128 | 9.99 035 | 58 |
| 3 | 9.31 966 | 59 | 9.32 933 | 61 | 10.67 067 | 9.99 032 | 57 |
| 4 | 9.32 025 | 59 | 9.32 995 | 62 | 10.67 005 | 9.99 030 | 56 |
| 5 | 9.32 084 | 59 | 9.33 057 | 62 | 10.66 943 | 9.99 027 | 55 |
| 6 | 9.32 143 | 59 | 9.33 119 | 62 | 10.66 881 | 9.99 024 | 54 |
| 7 | 9.32 202 | 59 | 9.33 180 | 61 | 10.66 820 | 9.99 022 | 53 |
| 8 | 9.32 261 | 59 | 9.33 242 | 62 | 10.66 758 | 9.99 019 | 52 |
| 9 | 9.32 319 | 58 | 9.33 303 | 61 | 10.66 697 | 9.99 016 | 51 |
| **10** | 9.32 378 | 59 | 9.33 365 | 62 | 10.66 635 | 9.99 013 | **50** |
| 11 | 9.32 437 | 59 | 9.33 426 | 61 | 10.66 574 | 9.99 011 | 49 |
| 12 | 9.32 495 | 58 | 9.33 487 | 61 | 10.66 513 | 9.99 008 | 48 |
| 13 | 9.32 553 | 58 | 9.33 548 | 61 | 10.66 452 | 9.99 005 | 47 |
| 14 | 9.32 612 | 59 | 9.33 609 | 61 | 10.66 391 | 9.99 002 | 46 |
| 15 | 9.32 670 | 58 | 9.33 670 | 61 | 10.66 330 | 9.99 000 | 45 |
| 16 | 9.32 728 | 58 | 9.33 731 | 61 | 10.66 269 | 9.98 997 | 44 |
| 17 | 9.32 786 | 58 | 9.33 792 | 61 | 10.66 208 | 9.98 994 | 43 |
| 18 | 9.32 844 | 58 | 9.33 853 | 61 | 10.66 147 | 9.98 991 | 42 |
| 19 | 9.32 902 | 58 | 9.33 913 | 60 | 10.66 087 | 9.98 989 | 41 |
| **20** | 9.32 960 | 58 | 9.33 974 | 61 | 10.66 026 | 9.98 986 | **40** |
| 21 | 9.33 018 | 58 | 9.34 034 | 60 | 10.65 966 | 9.98 983 | 39 |
| 22 | 9.33 075 | 57 | 9.34 095 | 61 | 10.65 905 | 9.98 980 | 38 |
| 23 | 9.33 133 | 58 | 9.34 155 | 60 | 10.65 845 | 9.98 978 | 37 |
| 24 | 9.33 190 | 57 | 9.34 215 | 60 | 10.65 785 | 9.98 975 | 36 |
| 25 | 9.33 248 | 58 | 9.34 276 | 61 | 10.65 724 | 9.98 972 | 35 |
| 26 | 9.33 305 | 57 | 9.34 336 | 60 | 10.65 664 | 9.98 969 | 34 |
| 27 | 9.33 362 | 57 | 9.34 396 | 60 | 10.65 604 | 9.98 967 | 33 |
| 28 | 9.33 420 | 58 | 9.34 456 | 60 | 10.65 544 | 9.98 964 | 32 |
| 29 | 9.33 477 | 57 | 9.34 516 | 60 | 10.65 484 | 9.98 961 | 31 |
| **30** | 9.33 534 | 57 | 9.34 576 | 60 | 10.65 424 | 9.98 958 | **30** |
| 31 | 9.33 591 | 57 | 9.34 635 | 59 | 10.65 365 | 9.98 955 | 29 |
| 32 | 9.33 647 | 56 | 9.34 695 | 60 | 10.65 305 | 9.98 953 | 28 |
| 33 | 9.33 704 | 57 | 9.34 755 | 60 | 10.65 245 | 9.98 950 | 27 |
| 34 | 9.33 761 | 57 | 9.34 814 | 59 | 10.65 186 | 9.98 947 | 26 |
| 35 | 9.33 818 | 57 | 9.34 874 | 60 | 10.65 126 | 9.98 944 | 25 |
| 36 | 9.33 874 | 56 | 9.34 933 | 59 | 10.65 067 | 9.98 941 | 24 |
| 37 | 9.33 931 | 57 | 9.34 992 | 59 | 10.65 008 | 9.98 938 | 23 |
| 38 | 9.33 987 | 56 | 9.35 051 | 59 | 10.64 949 | 9.98 936 | 22 |
| 39 | 9.34 043 | 56 | 9.35 111 | 60 | 10.64 889 | 9.98 933 | 21 |
| **40** | 9.34 100 | 57 | 9.35 170 | 59 | 10.64 830 | 9.98 930 | **20** |
| 41 | 9.34 156 | 56 | 9.35 229 | 59 | 10.64 771 | 9.98 927 | 19 |
| 42 | 9.34 212 | 56 | 9.35 288 | 59 | 10.64 712 | 9.98 924 | 18 |
| 43 | 9.34 268 | 56 | 9.35 347 | 59 | 10.64 653 | 9.98 921 | 17 |
| 44 | 9.34 324 | 56 | 9.35 405 | 58 | 10.64 595 | 9.98 919 | 16 |
| 45 | 9.34 380 | 56 | 9.35 464 | 59 | 10.64 536 | 9.98 916 | 15 |
| 46 | 9.34 436 | 56 | 9.35 523 | 59 | 10.64 477 | 9.98 913 | 14 |
| 47 | 9.34 491 | 55 | 9.35 581 | 58 | 10.64 419 | 9.98 910 | 13 |
| 48 | 9.34 547 | 56 | 9.35 640 | 59 | 10.64 360 | 9.98 907 | 12 |
| 49 | 9.34 602 | 55 | 9.35 698 | 58 | 10.64 302 | 9.98 904 | 11 |
| **50** | 9.34 658 | 56 | 9.35 757 | 59 | 10.64 243 | 9.98 901 | **10** |
| 51 | 9.34 713 | 55 | 9.35 815 | 58 | 10.64 185 | 9.98 898 | 9 |
| 52 | 9.34 769 | 56 | 9.35 873 | 58 | 10.64 127 | 9.98 896 | 8 |
| 53 | 9.34 824 | 55 | 9.35 931 | 58 | 10.64 069 | 9.98 893 | 7 |
| 54 | 9.34 879 | 55 | 9.35 989 | 58 | 10.64 011 | 9.98 890 | 6 |
| 55 | 9.34 934 | 55 | 9.36 047 | 58 | 10.63 953 | 9.98 887 | 5 |
| 56 | 9.34 989 | 55 | 9.36 105 | 58 | 10.63 895 | 9.98 884 | 4 |
| 57 | 9.35 044 | 55 | 9.36 163 | 58 | 10.63 837 | 9.98 881 | 3 |
| 58 | 9.35 099 | 55 | 9.36 221 | 58 | 10.63 779 | 9.98 878 | 2 |
| 59 | 9.35 154 | 55 | 9.36 279 | 58 | 10.63 721 | 9.98 875 | 1 |
| **60** | 9.35 209 | | 9.36 336 | 57 | 10.63 664 | 9.98 872 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

Prop. Pts.

| ″ | 63 | 62 | 61 |
|---|---|---|---|
| 6 | 6.3 | 6.2 | 6.1 |
| 7 | 7.4 | 7.2 | 7.1 |
| 8 | 8.4 | 8.3 | 8.1 |
| 9 | 9.4 | 9.3 | 9.2 |
| 10 | 10.5 | 10.3 | 10.2 |
| 20 | 21.0 | 20.7 | 20.3 |
| 30 | 31.5 | 31.0 | 30.5 |
| 40 | 42.0 | 41.3 | 40.7 |
| 50 | 52.5 | 51.7 | 50.8 |

| ″ | 60 | 59 | 58 |
|---|---|---|---|
| 6 | 6.0 | 5.9 | 5.8 |
| 7 | 7.0 | 6.9 | 6.8 |
| 8 | 8.0 | 7.9 | 7.7 |
| 9 | 9.0 | 8.8 | 8.7 |
| 10 | 10.0 | 9.8 | 9.7 |
| 20 | 20.0 | 19.7 | 19.3 |
| 30 | 30.0 | 29.5 | 29.0 |
| 40 | 40.0 | 39.3 | 38.7 |
| 50 | 50.0 | 49.2 | 48.3 |

| ″ | 57 | 56 | 55 |
|---|---|---|---|
| 6 | 5.7 | 5.6 | 5.5 |
| 7 | 6.6 | 6.5 | 6.4 |
| 8 | 7.6 | 7.5 | 7.3 |
| 9 | 8.6 | 8.4 | 8.2 |
| 10 | 9.5 | 9.3 | 9.2 |
| 20 | 19.0 | 18.7 | 18.3 |
| 30 | 28.5 | 28.0 | 27.5 |
| 40 | 38.0 | 37.3 | 36.7 |
| 50 | 47.5 | 46.7 | 45.8 |

| ″ | 3 | 2 |
|---|---|---|
| 6 | 0.3 | 0.2 |
| 7 | 0.4 | 0.2 |
| 8 | 0.4 | 0.3 |
| 9 | 0.5 | 0.3 |
| 10 | 0.5 | 0.3 |
| 20 | 1.0 | 0.7 |
| 30 | 1.5 | 1.0 |
| 40 | 2.0 | 1.3 |
| 50 | 2.5 | 1.7 |

## 77°

## 13°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| 0 | 9.35 209 | 54 | 9.36 336 | 58 | 10.63 664 | 9.98 872 | 60 |
| 1 | 9.35 263 | 55 | 9.36 394 | 58 | 10.63 606 | 9.98 869 | 59 |
| 2 | 9.35 318 | 55 | 9.36 452 | 57 | 10.63 548 | 9.98 867 | 58 |
| 3 | 9.35 373 | 54 | 9.36 509 | 57 | 10.63 491 | 9.98 864 | 57 |
| 4 | 9.35 427 | 54 | 9.36 566 | 58 | 10.63 434 | 9.98 861 | 56 |
| 5 | 9.35 481 | 55 | 9.36 624 | 57 | 10.63 376 | 9.98 858 | 55 |
| 6 | 9.35 536 | 54 | 9.36 681 | 57 | 10.63 319 | 9.98 855 | 54 |
| 7 | 9.35 590 | 54 | 9.36 738 | 57 | 10.63 262 | 9.98 852 | 53 |
| 8 | 9.35 644 | 54 | 9.36 795 | 57 | 10.63 205 | 9.98 849 | 52 |
| 9 | 9.35 698 | 54 | 9.36 852 | 57 | 10.63 148 | 9.98 846 | 51 |
| 10 | 9.35 752 | 54 | 9.36 909 | 57 | 10.63 091 | 9.98 843 | 50 |
| 11 | 9.35 806 | 54 | 9.36 966 | 57 | 10.63 034 | 9.98 840 | 49 |
| 12 | 9.35 860 | 54 | 9.37 023 | 57 | 10.62 977 | 9.98 837 | 48 |
| 13 | 9.35 914 | 54 | 9.37 080 | 57 | 10.62 920 | 9.98 834 | 47 |
| 14 | 9.35 968 | 54 | 9.37 137 | 56 | 10.62 863 | 9.98 831 | 46 |
| 15 | 9.36 022 | 53 | 9.37 193 | 57 | 10.62 807 | 9.98 828 | 45 |
| 16 | 9.36 075 | 54 | 9.37 250 | 56 | 10.62 750 | 9.98 825 | 44 |
| 17 | 9.36 129 | 53 | 9.37 306 | 57 | 10.62 694 | 9.98 822 | 43 |
| 18 | 9.36 182 | 54 | 9.37 363 | 56 | 10.62 637 | 9.98 819 | 42 |
| 19 | 9.36 236 | 53 | 9.37 419 | 57 | 10.62 581 | 9.98 816 | 41 |
| 20 | 9.36 289 | 53 | 9.37 476 | 56 | 10.62 524 | 9.98 813 | 40 |
| 21 | 9.36 342 | 53 | 9.37 532 | 56 | 10.62 468 | 9.98 810 | 39 |
| 22 | 9.36 395 | 54 | 9.37 588 | 56 | 10.62 412 | 9.98 807 | 38 |
| 23 | 9.36 449 | 53 | 9.37 644 | 56 | 10.62 356 | 9.98 804 | 37 |
| 24 | 9.36 502 | 53 | 9.37 700 | 56 | 10.62 300 | 9.98 801 | 36 |
| 25 | 9.36 555 | 53 | 9.37 756 | 56 | 10.62 244 | 9.98 798 | 35 |
| 26 | 9.36 608 | 52 | 9.37 812 | 56 | 10.62 188 | 9.98 795 | 34 |
| 27 | 9.36 660 | 53 | 9.37 868 | 56 | 10.62 132 | 9.98 792 | 33 |
| 28 | 9.36 713 | 53 | 9.37 924 | 56 | 10.62 076 | 9.98 789 | 32 |
| 29 | 9.36 766 | 53 | 9.37 980 | 55 | 10.62 020 | 9.98 786 | 31 |
| 30 | 9.36 819 | 52 | 9.38 035 | 56 | 10.61 965 | 9.98 783 | 30 |
| 31 | 9.36 871 | 53 | 9.38 091 | 56 | 10.61 909 | 9.98 780 | 29 |
| 32 | 9.36 924 | 52 | 9.38 147 | 55 | 10.61 853 | 9.98 777 | 28 |
| 33 | 9.36 976 | 52 | 9.38 202 | 55 | 10.61 798 | 9.98 774 | 27 |
| 34 | 9.37 028 | 53 | 9.38 257 | 56 | 10.61 743 | 9.98 771 | 26 |
| 35 | 9.37 081 | 52 | 9.38 313 | 55 | 10.61 687 | 9.98 768 | 25 |
| 36 | 9.37 133 | 52 | 9.38 368 | 55 | 10.61 632 | 9.98 765 | 24 |
| 37 | 9.37 185 | 52 | 9.38 423 | 56 | 10.61 577 | 9.98 762 | 23 |
| 38 | 9.37 237 | 52 | 9.38 479 | 55 | 10.61 521 | 9.98 759 | 22 |
| 39 | 9.37 289 | 52 | 9.38 534 | 55 | 10.61 466 | 9.98 756 | 21 |
| 40 | 9.37 341 | 52 | 9.38 589 | 55 | 10.61 411 | 9.98 753 | 20 |
| 41 | 9.37 393 | 52 | 9.38 644 | 55 | 10.61 356 | 9.98 750 | 19 |
| 42 | 9.37 445 | 52 | 9.38 699 | 55 | 10.61 301 | 9.98 746 | 18 |
| 43 | 9.37 497 | 52 | 9.38 754 | 54 | 10.61 246 | 9.98 743 | 17 |
| 44 | 9.37 549 | 51 | 9.38 808 | 55 | 10.61 192 | 9.98 740 | 16 |
| 45 | 9.37 600 | 52 | 9.38 863 | 55 | 10.61 137 | 9.98 737 | 15 |
| 46 | 9.37 652 | 51 | 9.38 918 | 54 | 10.61 082 | 9.98 734 | 14 |
| 47 | 9.37 703 | 52 | 9.38 972 | 55 | 10.61 028 | 9.98 731 | 13 |
| 48 | 9.37 755 | 51 | 9.39 027 | 55 | 10.60 973 | 9.98 728 | 12 |
| 49 | 9.37 806 | 52 | 9.39 082 | 54 | 10.60 918 | 9.98 725 | 11 |
| 50 | 9.37 858 | 51 | 9.39 136 | 54 | 10.60 864 | 9.98 722 | 10 |
| 51 | 9.37 909 | 51 | 9.39 190 | 55 | 10.60 810 | 9.98 719 | 9 |
| 52 | 9.37 960 | 51 | 9.39 245 | 54 | 10.60 755 | 9.98 715 | 8 |
| 53 | 9.38 011 | 51 | 9.39 299 | 54 | 10.60 701 | 9.98 712 | 7 |
| 54 | 9.38 062 | 51 | 9.39 353 | 54 | 10.60 647 | 9.98 709 | 6 |
| 55 | 9.38 113 | 51 | 9.39 407 | 54 | 10.60 593 | 9.98 706 | 5 |
| 56 | 9.38 164 | 51 | 9.39 461 | 54 | 10.60 539 | 9.98 703 | 4 |
| 57 | 9.38 215 | 51 | 9.39 515 | 54 | 10.60 485 | 9.98 700 | 3 |
| 58 | 9.38 266 | 51 | 9.39 569 | 54 | 10.60 431 | 9.98 697 | 2 |
| 59 | 9.38 317 | 51 | 9.39 623 | 54 | 10.60 377 | 9.98 694 | 1 |
| 60 | 9.38 368 | | 9.39 677 | | 10.60 323 | 9.98 690 | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

**Prop. Pts.**

| ″ | 58 | 57 | 56 |
|---|---|---|---|
| 6 | 5.8 | 5.7 | 5.6 |
| 7 | 6.8 | 6.6 | 6.5 |
| 8 | 7.7 | 7.6 | 7.5 |
| 9 | 8.7 | 8.6 | 8.4 |
| 10 | 9.7 | 9.5 | 9.3 |
| 20 | 19.3 | 19.0 | 18.7 |
| 30 | 29.0 | 28.5 | 28.0 |
| 40 | 38.7 | 38.0 | 37.3 |
| 50 | 48.3 | 47.5 | 46.7 |

| ″ | 55 | 54 | 53 |
|---|---|---|---|
| 6 | 5.5 | 5.4 | 5.3 |
| 7 | 6.4 | 6.3 | 6.2 |
| 8 | 7.3 | 7.2 | 7.1 |
| 9 | 8.2 | 8.1 | 8.0 |
| 10 | 9.2 | 9.0 | 8.8 |
| 20 | 18.3 | 18.0 | 17.7 |
| 30 | 27.5 | 27.0 | 26.5 |
| 40 | 36.7 | 36.0 | 35.3 |
| 50 | 45.8 | 45.0 | 44.2 |

| ″ | 52 | 51 | 4 |
|---|---|---|---|
| 6 | 5.2 | 5.1 | 0.4 |
| 7 | 6.1 | 6.0 | 0.5 |
| 8 | 6.9 | 6.8 | 0.5 |
| 9 | 7.8 | 7.6 | 0.6 |
| 10 | 8.7 | 8.5 | 0.7 |
| 20 | 17.3 | 17.0 | 1.3 |
| 30 | 26.0 | 25.5 | 2.0 |
| 40 | 34.7 | 34.0 | 2.7 |
| 50 | 43.3 | 42.5 | 3.3 |

| ″ | 3 | 2 |
|---|---|---|
| 6 | 0.3 | 0.2 |
| 7 | 0.4 | 0.2 |
| 8 | 0.4 | 0.3 |
| 9 | 0.4 | 0.3 |
| 10 | 0.5 | 0.3 |
| 20 | 1.0 | 0.7 |
| 30 | 1.5 | 1.0 |
| 40 | 2.0 | 1.3 |
| 50 | 2.5 | 1.7 |

## 76°

## 14°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | |
|---|---|---|---|---|---|---|---|
| **0** | 9.38 368 | | 9.39 677 | | 10.60 323 | 9.98 690 | **60** |
| 1 | 9.38 418 | 50 | 9.39 731 | 54 | 10.60 269 | 9.98 687 | 59 |
| 2 | 9.38 469 | 51 | 9.39 785 | 54 | 10.60 215 | 9.98 684 | 58 |
| 3 | 9.38 519 | 50 | 9.39 838 | 53 | 10.60 162 | 9.98 681 | 57 |
| 4 | 9.38 570 | 51 | 9.39 892 | 54 | 10.60 108 | 9.98 678 | 56 |
| 5 | 9.38 620 | 50 | 9.39 945 | 53 | 10.60 055 | 9.98 675 | 55 |
| 6 | 9.38 670 | 50 | 9.39 999 | 54 | 10.60 001 | 9.98 671 | 54 |
| 7 | 9.38 721 | 51 | 9.40 052 | 53 | 10.59 948 | 9.98 668 | 53 |
| 8 | 9.38 771 | 50 | 9.40 106 | 54 | 10.59 894 | 9.98 665 | 52 |
| 9 | 9.38 821 | 50 | 9.40 159 | 53 | 10.59 841 | 9.98 662 | 51 |
| **10** | 9.38 871 | 50 | 9.40 212 | 53 | 10.59 788 | 9.98 659 | **50** |
| 11 | 9.38 921 | 50 | 9.40 266 | 54 | 10.59 734 | 9.98 656 | 49 |
| 12 | 9.38 971 | 50 | 9.40 319 | 53 | 10.59 681 | 9.98 652 | 48 |
| 13 | 9.39 021 | 50 | 9.40 372 | 53 | 10.59 628 | 9.98 649 | 47 |
| 14 | 9.39 071 | 50 | 9.40 425 | 53 | 10.59 575 | 9.98 646 | 46 |
| 15 | 9.39 121 | 50 | 9.40 478 | 53 | 10.59 522 | 9.98 643 | 45 |
| 16 | 9.39 170 | 49 | 9.40 531 | 53 | 10.59 469 | 9.98 640 | 44 |
| 17 | 9.39 220 | 50 | 9.40 584 | 53 | 10.59 416 | 9.98 636 | 43 |
| 18 | 9.39 270 | 50 | 9.40 636 | 52 | 10.59 364 | 9.98 633 | 42 |
| 19 | 9.39 319 | 49 | 9.40 689 | 53 | 10.59 311 | 9.98 630 | 41 |
| **20** | 9.39 369 | 50 | 9.40 742 | 53 | 10.59 258 | 9.98 627 | **40** |
| 21 | 9.39 418 | 49 | 9.40 795 | 53 | 10.59 205 | 9.98 623 | 39 |
| 22 | 9.39 467 | 49 | 9.40 847 | 52 | 10.59 153 | 9.98 620 | 38 |
| 23 | 9.39 517 | 50 | 9.40 900 | 53 | 10.59 100 | 9.98 617 | 37 |
| 24 | 9.39 566 | 49 | 9.40 952 | 52 | 10.59 048 | 9.98 614 | 36 |
| 25 | 9.39 615 | 49 | 9.41 005 | 53 | 10.58 995 | 9.98 610 | 35 |
| 26 | 9.39 664 | 49 | 9.41 057 | 52 | 10.58 943 | 9.98 607 | 34 |
| 27 | 9.39 713 | 49 | 9.41 109 | 52 | 10.58 891 | 9.98 604 | 33 |
| 28 | 9.39 762 | 49 | 9.41 161 | 52 | 10.58 839 | 9.98 601 | 32 |
| 29 | 9.39 811 | 49 | 9.41 214 | 53 | 10.58 786 | 9.98 597 | 31 |
| **30** | 9.39 860 | 49 | 9.41 266 | 52 | 10.58 734 | 9.98 594 | **30** |
| 31 | 9.39 909 | 49 | 9.41 318 | 52 | 10.58 682 | 9.98 591 | 29 |
| 32 | 9.39 958 | 49 | 9.41 370 | 52 | 10.58 630 | 9.98 588 | 28 |
| 33 | 9.40 006 | 48 | 9.41 422 | 52 | 10.58 578 | 9.98 584 | 27 |
| 34 | 9.40 055 | 49 | 9.41 474 | 52 | 10.58 526 | 9.98 581 | 26 |
| 35 | 9.40 103 | 48 | 9.41 526 | 52 | 10.58 474 | 9.98 578 | 25 |
| 36 | 9.40 152 | 49 | 9.41 578 | 52 | 10.58 422 | 9.98 574 | 24 |
| 37 | 9.40 200 | 48 | 9.41 629 | 51 | 10.58 371 | 9.98 571 | 23 |
| 38 | 9.40 249 | 49 | 9.41 681 | 52 | 10.58 319 | 9.98 568 | 22 |
| 39 | 9.40 297 | 48 | 9.41 733 | 52 | 10.58 267 | 9.98 565 | 21 |
| **40** | 9.40 346 | 49 | 9.41 784 | 51 | 10.58 216 | 9.98 561 | **20** |
| 41 | 9.40 394 | 48 | 9.41 836 | 52 | 10.58 164 | 9.98 558 | 19 |
| 42 | 9.40 442 | 48 | 9.41 887 | 51 | 10.58 113 | 9.98 555 | 18 |
| 43 | 9.40 490 | 48 | 9.41 939 | 52 | 10.58 061 | 9.98 551 | 17 |
| 44 | 9.40 538 | 48 | 9.41 990 | 51 | 10.58 010 | 9.98 548 | 16 |
| 45 | 9.40 586 | 48 | 9.42 041 | 51 | 10.57 959 | 9.98 545 | 15 |
| 46 | 9.40 634 | 48 | 9.42 093 | 52 | 10.57 907 | 9.98 541 | 14 |
| 47 | 9.40 682 | 48 | 9.42 144 | 51 | 10.57 856 | 9.98 538 | 13 |
| 48 | 9.40 730 | 48 | 9.42 195 | 51 | 10.57 805 | 9.98 535 | 12 |
| 49 | 9.40 778 | 48 | 9.42 246 | 51 | 10.57 754 | 9.98 531 | 11 |
| **50** | 9.40 825 | 47 | 9.42 297 | 51 | 10.57 703 | 9.98 528 | **10** |
| 51 | 9.40 873 | 48 | 9.42 348 | 51 | 10.57 652 | 9.98 525 | 9 |
| 52 | 9.40 921 | 48 | 9.42 399 | 51 | 10.57 601 | 9.98 521 | 8 |
| 53 | 9.40 968 | 47 | 9.42 450 | 51 | 10.57 550 | 9.98 518 | 7 |
| 54 | 9.41 016 | 48 | 9.42 501 | 51 | 10.57 499 | 9.98 515 | 6 |
| 55 | 9.41 063 | 47 | 9.42 552 | 51 | 10.57 448 | 9.98 511 | 5 |
| 56 | 9.41 111 | 48 | 9.42 603 | 51 | 10.57 397 | 9.98 508 | 4 |
| 57 | 9.41 158 | 47 | 9.42 653 | 50 | 10.57 347 | 9.98 505 | 3 |
| 58 | 9.41 205 | 47 | 9.42 704 | 51 | 10.57 296 | 9.98 501 | 2 |
| 59 | 9.41 252 | 47 | 9.42 755 | 51 | 10.57 245 | 9.98 498 | 1 |
| **60** | 9.41 300 | 48 | 9.42 805 | 50 | 10.57 195 | 9.98 494 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | ′ |

**Prop. Pts.**

| ″ | 54 | 53 | 52 |
|---|---|---|---|
| 6 | 5.4 | 5.3 | 5.2 |
| 7 | 6.3 | 6.2 | 6.1 |
| 8 | 7.2 | 7.1 | 6.9 |
| 9 | 8.1 | 8.0 | 7.8 |
| 10 | 9.0 | 8.8 | 8.7 |
| 20 | 18.0 | 17.7 | 17.3 |
| 30 | 27.0 | 26.5 | 26.0 |
| 40 | 36.0 | 35.3 | 34.7 |
| 50 | 45.0 | 44.2 | 43.3 |

| ″ | 51 | 50 | 49 |
|---|---|---|---|
| 6 | 5.1 | 5.0 | 4.9 |
| 7 | 6.0 | 5.8 | 5.7 |
| 8 | 6.8 | 6.7 | 6.5 |
| 9 | 7.6 | 7.5 | 7.4 |
| 10 | 8.5 | 8.3 | 8.2 |
| 20 | 17.0 | 16.7 | 16.3 |
| 30 | 25.5 | 25.0 | 24.5 |
| 40 | 34.0 | 33.3 | 32.7 |
| 50 | 42.5 | 41.7 | 40.8 |

| ″ | 48 | 47 |
|---|---|---|
| 6 | 4.8 | 4.7 |
| 7 | 5.6 | 5.5 |
| 8 | 6.4 | 6.3 |
| 9 | 7.2 | 7.0 |
| 10 | 8.0 | 7.8 |
| 20 | 16.0 | 15.7 |
| 30 | 24.0 | 23.5 |
| 40 | 32.0 | 31.3 |
| 50 | 40.0 | 39.2 |

| ″ | 4 | 3 |
|---|---|---|
| 6 | 0.4 | 0.3 |
| 7 | 0.5 | 0.4 |
| 8 | 0.5 | 0.4 |
| 9 | 0.6 | 0.4 |
| 10 | 0.7 | 0.5 |
| 20 | 1.3 | 1.0 |
| 30 | 2.0 | 1.5 |
| 40 | 2.7 | 2.0 |
| 50 | 3.3 | 2.5 |

## 75°

## 15°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.41 300 | 47 | 9.42 805 | 51 | 10.57 195 | 9.98 494 | 3 | **60** |
| 1 | 9.41 347 | 47 | 9.42 856 | 50 | 10.57 144 | 9.98 491 | 3 | 59 |
| 2 | 9.41 394 | 47 | 9.42 906 | 51 | 10.57 094 | 9.98 488 | 4 | 58 |
| 3 | 9.41 441 | 47 | 9.42 957 | 50 | 10.57 043 | 9.98 484 | 3 | 57 |
| 4 | 9.41 488 | 47 | 9.43 007 | 50 | 10.56 993 | 9.98 481 | 4 | 56 |
| 5 | 9.41 535 | 47 | 9.43 057 | 51 | 10.56 943 | 9.98 477 | 3 | 55 |
| 6 | 9.41 582 | 46 | 9.43 108 | 50 | 10.56 892 | 9.98 474 | 3 | 54 |
| 7 | 9.41 628 | 47 | 9.43 158 | 50 | 10.56 842 | 9.98 471 | 4 | 53 |
| 8 | 9.41 675 | 47 | 9.43 208 | 50 | 10.56 792 | 9.98 467 | 3 | 52 |
| 9 | 9.41 722 | 46 | 9.43 258 | 50 | 10.56 742 | 9.98 464 | 4 | 51 |
| **10** | 9.41 768 | 47 | 9.43 308 | 50 | 10.56 692 | 9.98 460 | 3 | **50** |
| 11 | 9.41 815 | 46 | 9.43 358 | 50 | 10.56 642 | 9.98 457 | 4 | 49 |
| 12 | 9.41 861 | 47 | 9.43 408 | 50 | 10.56 592 | 9.98 453 | 3 | 48 |
| 13 | 9.41 908 | 46 | 9.43 458 | 50 | 10.56 542 | 9.98 450 | 3 | 47 |
| 14 | 9.41 954 | 47 | 9.43 508 | 50 | 10.56 492 | 9.98 447 | 4 | 46 |
| 15 | 9.42 001 | 46 | 9.43 558 | 49 | 10.56 442 | 9.98 443 | 3 | 45 |
| 16 | 9.42 047 | 46 | 9.43 607 | 50 | 10.56 393 | 9.98 440 | 4 | 44 |
| 17 | 9.42 093 | 47 | 9.43 657 | 50 | 10.56 343 | 9.98 436 | 3 | 43 |
| 18 | 9.42 140 | 46 | 9.43 707 | 49 | 10.56 293 | 9.98 433 | 4 | 42 |
| 19 | 9.42 186 | 46 | 9.43 756 | 50 | 10.56 244 | 9.98 429 | 3 | 41 |
| **20** | 9.42 232 | 46 | 9.43 806 | 49 | 10.56 194 | 9.98 426 | 4 | **40** |
| 21 | 9.42 278 | 46 | 9.43 855 | 50 | 10.56 145 | 9.98 422 | 3 | 39 |
| 22 | 9.42 324 | 46 | 9.43 905 | 49 | 10.56 095 | 9.98 419 | 4 | 38 |
| 23 | 9.42 370 | 46 | 9.43 954 | 50 | 10.56 046 | 9.98 415 | 3 | 37 |
| 24 | 9.42 416 | 45 | 9.44 004 | 49 | 10.55 996 | 9.98 412 | 3 | 36 |
| 25 | 9.42 461 | 46 | 9.44 053 | 49 | 10.55 947 | 9.98 409 | 4 | 35 |
| 26 | 9.42 507 | 46 | 9.44 102 | 49 | 10.55 898 | 9.98 405 | 3 | 34 |
| 27 | 9.42 553 | 46 | 9.44 151 | 50 | 10.55 849 | 9.98 402 | 4 | 33 |
| 28 | 9.42 599 | 45 | 9.44 201 | 49 | 10.55 799 | 9.98 398 | 3 | 32 |
| 29 | 9.42 644 | 46 | 9.44 250 | 49 | 10.55 750 | 9.98 395 | 4 | 31 |
| **30** | 9.42 690 | 45 | 9.44 299 | 49 | 10.55 701 | 9.98 391 | 3 | **30** |
| 31 | 9.42 735 | 46 | 9.44 348 | 49 | 10.55 652 | 9.98 388 | 4 | 29 |
| 32 | 9.42 781 | 45 | 9.44 397 | 49 | 10.55 603 | 9.98 384 | 3 | 28 |
| 33 | 9.42 826 | 46 | 9.44 446 | 49 | 10.55 554 | 9.98 381 | 4 | 27 |
| 34 | 9.42 872 | 45 | 9.44 495 | 49 | 10.55 505 | 9.98 377 | 4 | 26 |
| 35 | 9.42 917 | 45 | 9.44 544 | 48 | 10.55 456 | 9.98 373 | 3 | 25 |
| 36 | 9.42 962 | 46 | 9.44 592 | 49 | 10.55 408 | 9.98 370 | 4 | 24 |
| 37 | 9.43 008 | 45 | 9.44 641 | 49 | 10.55 359 | 9.98 366 | 3 | 23 |
| 38 | 9.43 053 | 45 | 9.44 690 | 48 | 10.55 310 | 9.98 363 | 4 | 22 |
| 39 | 9.43 098 | 45 | 9.44 738 | 49 | 10.55 262 | 9.98 359 | 3 | 21 |
| **40** | 9.43 143 | 45 | 9.44 787 | 49 | 10.55 213 | 9.98 356 | 4 | **20** |
| 41 | 9.43 188 | 45 | 9.44 836 | 48 | 10.55 164 | 9.98 352 | 3 | 19 |
| 42 | 9.43 233 | 45 | 9.44 884 | 49 | 10.55 116 | 9.98 349 | 4 | 18 |
| 43 | 9.43 278 | 45 | 9.44 933 | 48 | 10.55 067 | 9.98 345 | 3 | 17 |
| 44 | 9.43 323 | 44 | 9.44 981 | 48 | 10.55 019 | 9.98 342 | 4 | 16 |
| 45 | 9.43 367 | 45 | 9.45 029 | 49 | 10.54 971 | 9.98 338 | 4 | 15 |
| 46 | 9.43 412 | 45 | 9.45 078 | 48 | 10.54 922 | 9.98 334 | 3 | 14 |
| 47 | 9.43 457 | 45 | 9.45 126 | 48 | 10.54 874 | 9.98 331 | 4 | 13 |
| 48 | 9.43 502 | 44 | 9.45 174 | 48 | 10.54 826 | 9.98 327 | 3 | 12 |
| 49 | 9.43 546 | 45 | 9.45 222 | 49 | 10.54 778 | 9.98 324 | 4 | 11 |
| **50** | 9.43 591 | 44 | 9.45 271 | 48 | 10.54 729 | 9.98 320 | 3 | **10** |
| 51 | 9.43 635 | 45 | 9.45 319 | 48 | 10.54 681 | 9.98 317 | 4 | 9 |
| 52 | 9.43 680 | 44 | 9.45 367 | 48 | 10.54 633 | 9.98 313 | 4 | 8 |
| 53 | 9.43 724 | 45 | 9.45 415 | 48 | 10.54 585 | 9.98 309 | 3 | 7 |
| 54 | 9.43 769 | 44 | 9.45 463 | 48 | 10.54 537 | 9.98 306 | 4 | 6 |
| 55 | 9.43 813 | 44 | 9.45 511 | 48 | 10.54 489 | 9.98 302 | 3 | 5 |
| 56 | 9.43 857 | 44 | 9.45 559 | 47 | 10.54 441 | 9.98 299 | 4 | 4 |
| 57 | 9.43 901 | 45 | 9.45 606 | 48 | 10.54 394 | 9.98 295 | 4 | 3 |
| 58 | 9.43 946 | 44 | 9.45 654 | 48 | 10.54 346 | 9.98 291 | 3 | 2 |
| 59 | 9.43 990 | 44 | 9.45 702 | 48 | 10.54 298 | 9.98 288 | 4 | 1 |
| **60** | 9.44 034 | | 9.45 750 | | 10.54 250 | 9.98 284 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 51 | 50 | 49 |
|---|---|---|---|
| 6 | 5.1 | 5.0 | 4.9 |
| 7 | 6.0 | 5.8 | 5.7 |
| 8 | 6.8 | 6.7 | 6.5 |
| 9 | 7.7 | 7.5 | 7.4 |
| 10 | 8.5 | 8.3 | 8.2 |
| 20 | 17.0 | 16.7 | 16.3 |
| 30 | 25.5 | 25.0 | 24.5 |
| 40 | 34.0 | 33.3 | 32.7 |
| 50 | 42.5 | 41.7 | 40.8 |

| ″ | 48 | 47 | 46 |
|---|---|---|---|
| 6 | 4.8 | 4.7 | 4.6 |
| 7 | 5.6 | 5.5 | 5.4 |
| 8 | 6.4 | 6.3 | 6.1 |
| 9 | 7.2 | 7.0 | 6.9 |
| 10 | 8.0 | 7.8 | 7.7 |
| 20 | 16.0 | 15.7 | 15.3 |
| 30 | 24.0 | 23.5 | 23.0 |
| 40 | 32.0 | 31.3 | 30.7 |
| 50 | 40.0 | 39.2 | 38.3 |

| ″ | 45 | 44 |
|---|---|---|
| 6 | 4.5 | 4.4 |
| 7 | 5.3 | 5.1 |
| 8 | 6.0 | 5.9 |
| 9 | 6.8 | 6.6 |
| 10 | 7.5 | 7.3 |
| 20 | 15.0 | 14.7 |
| 30 | 22.5 | 22.0 |
| 40 | 30.0 | 29.3 |
| 50 | 37.5 | 36.7 |

| ″ | 4 | 3 |
|---|---|---|
| 6 | 0.4 | 0.3 |
| 7 | 0.5 | 0.4 |
| 8 | 0.5 | 0.4 |
| 9 | 0.6 | 0.5 |
| 10 | 0.7 | 0.5 |
| 20 | 1.3 | 1.0 |
| 30 | 2.0 | 1.5 |
| 40 | 2.7 | 2.0 |
| 50 | 3.3 | 2.5 |

Prop. Pts.

## 74°

## 16°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.44 034 | 44 | 9.45 750 | 47 | 10.54 250 | 9.98 284 | 3 | 60 | |
| 1 | 9.44 078 | 44 | 9.45 797 | 48 | 10.54 203 | 9.98 281 | 4 | 59 | |
| 2 | 9.44 122 | 44 | 9.45 845 | 47 | 10.54 155 | 9.98 277 | 3 | 58 | |
| 3 | 9.44 166 | 44 | 9.45 892 | 48 | 10.54 108 | 9.98 273 | 4 | 57 | |
| 4 | 9.44 210 | 43 | 9.45 940 | 47 | 10.54 060 | 9.98 270 | 4 | 56 | |
| 5 | 9.44 253 | 44 | 9.45 987 | 48 | 10.54 013 | 9.98 266 | 4 | 55 | ″ 48 47 46 |
| 6 | 9.44 297 | 44 | 9.46 035 | 47 | 10.53 965 | 9.98 262 | 3 | 54 | 6 4.8 4.7 4.6 |
| 7 | 9.44 341 | 44 | 9.46 082 | 48 | 10.53 918 | 9.98 259 | 4 | 53 | 7 5.6 5.5 5.4 |
| 8 | 9.44 385 | 43 | 9.46 130 | 47 | 10.53 870 | 9.98 255 | 4 | 52 | 8 6.4 6.3 6.1 |
| 9 | 9.44 428 | 44 | 9.46 177 | 47 | 10.53 823 | 9.98 251 | 3 | 51 | 9 7.2 7.0 6.9 |
| 10 | 9.44 472 | 44 | 9.46 224 | 47 | 10.53 776 | 9.98 248 | 4 | 50 | 10 8.0 7.8 7.7 |
| 11 | 9.44 516 | 43 | 9.46 271 | 48 | 10.53 729 | 9.98 244 | 4 | 49 | 20 16.0 15.7 15.3 |
| 12 | 9.44 559 | 43 | 9.46 319 | 47 | 10.53 681 | 9.98 240 | 3 | 48 | 30 24.0 23.5 23.0 |
| 13 | 9.44 602 | 44 | 9.46 366 | 47 | 10.53 634 | 9.98 237 | 4 | 47 | 40 32.0 31.3 30.7 |
| 14 | 9.44 646 | 43 | 9.46 413 | 47 | 10.53 587 | 9.98 233 | 4 | 46 | 50 40.0 39.2 38.3 |
| 15 | 9.44 689 | 44 | 9.46 460 | 47 | 10.53 540 | 9.98 229 | 3 | 45 | |
| 16 | 9.44 733 | 43 | 9.46 507 | 47 | 10.53 493 | 9.98 226 | 4 | 44 | |
| 17 | 9.44 776 | 43 | 9.46 554 | 47 | 10.53 446 | 9.98 222 | 4 | 43 | |
| 18 | 9.44 819 | 43 | 9.46 601 | 47 | 10.53 399 | 9.98 218 | 3 | 42 | |
| 19 | 9.44 862 | 43 | 9.46 648 | 46 | 10.53 352 | 9.98 215 | 4 | 41 | ″ 45 44 43 |
| 20 | 9.44 905 | 43 | 9.46 694 | 47 | 10.53 306 | 9.98 211 | 4 | 40 | 6 4.5 4.4 4.3 |
| 21 | 9.44 948 | 44 | 9.46 741 | 47 | 10.53 259 | 9.98 207 | 3 | 39 | 7 5.3 5.1 5.0 |
| 22 | 9.44 992 | 43 | 9.46 788 | 47 | 10.53 212 | 9.98 204 | 4 | 38 | 8 6.0 5.9 5.7 |
| 23 | 9.45 035 | 42 | 9.46 835 | 46 | 10.53 165 | 9.98 200 | 4 | 37 | 9 6.8 6.6 6.4 |
| 24 | 9.45 077 | 43 | 9.46 881 | 47 | 10.53 119 | 9.98 196 | 4 | 36 | 10 7.5 7.3 7.2 |
| 25 | 9.45 120 | 43 | 9.46 928 | 47 | 10.53 072 | 9.98 192 | 3 | 35 | 20 15.0 14.7 14.3 |
| 26 | 9.45 163 | 43 | 9.46 975 | 46 | 10.53 025 | 9.98 189 | 4 | 34 | 30 22.5 22.0 21.5 |
| 27 | 9.45 206 | 43 | 9.47 021 | 47 | 10.52 979 | 9.98 185 | 4 | 33 | 40 30.0 29.3 28.7 |
| 28 | 9.45 249 | 43 | 9.47 068 | 46 | 10.52 932 | 9.98 181 | 4 | 32 | 50 37.5 36.7 35.8 |
| 29 | 9.45 292 | 42 | 9.47 114 | 46 | 10.52 886 | 9.98 177 | 3 | 31 | |
| 30 | 9.45 344 | 43 | 9.47 160 | 47 | 10.52 840 | 9.98 174 | 4 | 30 | |
| 31 | 9.45 377 | 42 | 9.47 207 | 46 | 10.52 793 | 9.98 170 | 4 | 29 | |
| 32 | 9.45 419 | 43 | 9.47 253 | 46 | 10.52 747 | 9.98 166 | 4 | 28 | |
| 33 | 9.45 462 | 42 | 9.47 299 | 47 | 10.52 701 | 9.98 162 | 3 | 27 | ″ 42 41 |
| 34 | 9.45 504 | 43 | 9.47 346 | 46 | 10.52 654 | 9.98 159 | 4 | 26 | 6 4.2 4.1 |
| 35 | 9.45 547 | 42 | 9.47 392 | 46 | 10.52 608 | 9.98 155 | 4 | 25 | 7 4.9 4.8 |
| 36 | 9.45 589 | 43 | 9.47 438 | 46 | 10.52 562 | 9.98 151 | 4 | 24 | 8 5.6 5.5 |
| 37 | 9.45 632 | 42 | 9.47 484 | 46 | 10.52 516 | 9.98 147 | 3 | 23 | 9 6.3 6.2 |
| 38 | 9.45 674 | 42 | 9.47 530 | 46 | 10.52 470 | 9.98 144 | 4 | 22 | 10 7.0 6.8 |
| 39 | 9.45 716 | 42 | 9.47 576 | 46 | 10.52 424 | 9.98 140 | 4 | 21 | 20 14.0 13.7 |
| 40 | 9.45 758 | 43 | 9.47 622 | 46 | 10.52 378 | 9.98 136 | 4 | 20 | 30 21.0 20.5 |
| 41 | 9.45 801 | 42 | 9.47 668 | 46 | 10.52 332 | 9.98 132 | 3 | 19 | 40 28.0 27.3 |
| 42 | 9.45 843 | 42 | 9.47 714 | 46 | 10.52 286 | 9.98 129 | 4 | 18 | 50 35.0 34.2 |
| 43 | 9.45 885 | 42 | 9.47 760 | 46 | 10.52 240 | 9.98 125 | 4 | 17 | |
| 44 | 9.45 927 | 42 | 9.47 806 | 46 | 10.52 194 | 9.98 121 | 4 | 16 | |
| 45 | 9.45 969 | 42 | 9.47 852 | 45 | 10.52 148 | 9.98 117 | 4 | 15 | |
| 46 | 9.46 011 | 42 | 9.47 897 | 46 | 10.52 103 | 9.98 113 | 4 | 14 | |
| 47 | 9.46 053 | 42 | 9.47 943 | 46 | 10.52 057 | 9.98 110 | 3 | 13 | ″ 4 3 |
| 48 | 9.46 095 | 41 | 9.47 989 | 46 | 10.52 011 | 9.98 106 | 4 | 12 | 6 0.4 0.3 |
| 49 | 9.46 136 | 42 | 9.48 035 | 45 | 10.51 965 | 9.98 102 | 4 | 11 | 7 0.5 0.4 |
| 50 | 9.46 178 | 42 | 9.48 080 | 46 | 10.51 920 | 9.98 098 | 4 | 10 | 8 0.5 0.4 |
| 51 | 9.46 220 | 42 | 9.48 126 | 45 | 10.51 874 | 9.98 094 | 4 | 9 | 9 0.6 0.5 |
| 52 | 9.46 262 | 41 | 9.48 171 | 46 | 10.51 829 | 9.98 090 | 3 | 8 | 10 0.7 0.5 |
| 53 | 9.46 303 | 42 | 9.48 217 | 45 | 10.51 783 | 9.98 087 | 4 | 7 | 20 1.3 1.0 |
| 54 | 9.46 345 | 41 | 9.48 262 | 45 | 10.51 738 | 9.98 083 | 4 | 6 | 30 2.0 1.5 |
| 55 | 9.46 386 | 42 | 9.48 307 | 46 | 10.51 693 | 9.98 079 | 4 | 5 | 40 2.7 2.0 |
| 56 | 9.46 428 | 41 | 9.48 353 | 45 | 10.51 647 | 9.98 075 | 4 | 4 | 50 3.3 2.5 |
| 57 | 9.46 469 | 42 | 9.48 398 | 45 | 10.51 602 | 9.98 071 | 4 | 3 | |
| 58 | 9.46 511 | 41 | 9.48 443 | 45 | 10.51 557 | 9.98 067 | 4 | 2 | |
| 59 | 9.46 552 | 42 | 9.48 489 | 45 | 10.51 511 | 9.98 063 | 3 | 1 | |
| 60 | 9.46 594 | | 9.48 534 | | 10.51 466 | 9.98 060 | | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 73°

## 17°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.46 594 | | 9.48 534 | | 10.51 466 | 9.98 060 | | 60 |
| 1 | 9.46 635 | 41 | 9.48 579 | 45 | 10.51 421 | 9.98 056 | 4 | 59 |
| 2 | 9.46 676 | 41 | 9.48 624 | 45 | 10.51 376 | 9.98 052 | 4 | 58 |
| 3 | 9.46 717 | 41 | 9.48 669 | 45 | 10.51 331 | 9.98 048 | 4 | 57 |
| 4 | 9.46 758 | 41 | 9.48 714 | 45 | 10.51 286 | 9.98 044 | 4 | 56 |
| 5 | 9.46 800 | 42 | 9.48 759 | 45 | 10.51 241 | 9.98 040 | 4 | 55 |
| 6 | 9.46 841 | 41 | 9.48 804 | 45 | 10.51 196 | 9.98 036 | 4 | 54 |
| 7 | 9.46 882 | 41 | 9.48 849 | 45 | 10.51 151 | 9.98 032 | 4 | 53 |
| 8 | 9.46 923 | 41 | 9.48 894 | 45 | 10.51 106 | 9.98 029 | 3 | 52 |
| 9 | 9.46 964 | 41 | 9.48 939 | 45 | 10.51 061 | 9.98 025 | 4 | 51 |
| 10 | 9.47 005 | 41 | 9.48 984 | 45 | 10.51 016 | 9.98 021 | 4 | 50 |
| 11 | 9.47 045 | 40 | 9.49 029 | 45 | 10.50 971 | 9.98 017 | 4 | 49 |
| 12 | 9.47 086 | 41 | 9.49 073 | 44 | 10.50 927 | 9.98 013 | 4 | 48 |
| 13 | 9.47 127 | 41 | 9.49 118 | 45 | 10.50 882 | 9.98 009 | 4 | 47 |
| 14 | 9.47 168 | 41 | 9.49 163 | 45 | 10.50 837 | 9.98 005 | 4 | 46 |
| 15 | 9.47 209 | 41 | 9.49 207 | 44 | 10.50 793 | 9.98 001 | 4 | 45 |
| 16 | 9.47 249 | 40 | 9.49 252 | 45 | 10.50 748 | 9.97 997 | 4 | 44 |
| 17 | 9.47 290 | 41 | 9.49 296 | 44 | 10.50 704 | 9.97 993 | 4 | 43 |
| 18 | 9.47 330 | 40 | 9.49 341 | 45 | 10.50 659 | 9.97 989 | 4 | 42 |
| 19 | 9.47 371 | 41 | 9.49 385 | 44 | 10.50 615 | 9.97 986 | 3 | 41 |
| 20 | 9.47 411 | 40 | 9.49 430 | 45 | 10.50 570 | 9.97 982 | 4 | 40 |
| 21 | 9.47 452 | 41 | 9.49 474 | 44 | 10.50 526 | 9.97 978 | 4 | 39 |
| 22 | 9.47 492 | 40 | 9.49 519 | 45 | 10.50 481 | 9.97 974 | 4 | 38 |
| 23 | 9.47 533 | 41 | 9.49 563 | 44 | 10.50 437 | 9.97 970 | 4 | 37 |
| 24 | 9.47 573 | 40 | 9.49 607 | 44 | 10.50 393 | 9.97 966 | 4 | 36 |
| 25 | 9.47 613 | 40 | 9.49 652 | 45 | 10.50 348 | 9.97 962 | 4 | 35 |
| 26 | 9.47 654 | 41 | 9.49 696 | 44 | 10.50 304 | 9.97 958 | 4 | 34 |
| 27 | 9.47 694 | 40 | 9.49 740 | 44 | 10.50 260 | 9.97 954 | 4 | 33 |
| 28 | 9.47 734 | 40 | 9.49 784 | 44 | 10.50 216 | 9.97 950 | 4 | 32 |
| 29 | 9.47 774 | 40 | 9.49 828 | 44 | 10.50 172 | 9.97 946 | 4 | 31 |
| 30 | 9.47 814 | 40 | 9.49 872 | 44 | 10.50 128 | 9.97 942 | 4 | 30 |
| 31 | 9.47 854 | 40 | 9.49 916 | 44 | 10.50 084 | 9.97 938 | 4 | 29 |
| 32 | 9.47 894 | 40 | 9.49 960 | 44 | 10.50 040 | 9.97 934 | 4 | 28 |
| 33 | 9.47 934 | 40 | 9.50 004 | 44 | 10.49 996 | 9.97 930 | 4 | 27 |
| 34 | 9.47 974 | 40 | 9.50 048 | 44 | 10.49 952 | 9.97 926 | 4 | 26 |
| 35 | 9.48 014 | 40 | 9.50 092 | 44 | 10.49 908 | 9.97 922 | 4 | 25 |
| 36 | 9.48 054 | 40 | 9.50 136 | 44 | 10.49 864 | 9.97 918 | 4 | 24 |
| 37 | 9.48 094 | 40 | 9.50 180 | 44 | 10.49 820 | 9.97 914 | 4 | 23 |
| 38 | 9.48 133 | 39 | 9.50 223 | 43 | 10.49 777 | 9.97 910 | 4 | 22 |
| 39 | 9.48 173 | 40 | 9.50 267 | 44 | 10.49 733 | 9.97 906 | 4 | 21 |
| 40 | 9.48 213 | 40 | 9.50 311 | 44 | 10.49 689 | 9.97 902 | 4 | 20 |
| 41 | 9.48 252 | 39 | 9.50 355 | 44 | 10.49 645 | 9.97 898 | 4 | 19 |
| 42 | 9.48 292 | 40 | 9.50 398 | 43 | 10.49 602 | 9.97 894 | 4 | 18 |
| 43 | 9.48 332 | 40 | 9.50 442 | 44 | 10.49 558 | 9.97 890 | 4 | 17 |
| 44 | 9.48 371 | 39 | 9.50 485 | 43 | 10.49 515 | 9.97 886 | 4 | 16 |
| 45 | 9.48 411 | 40 | 9.50 529 | 44 | 10.49 471 | 9.97 882 | 4 | 15 |
| 46 | 9.48 450 | 39 | 9.50 572 | 43 | 10.49 428 | 9.97 878 | 4 | 14 |
| 47 | 9.48 490 | 40 | 9.50 616 | 44 | 10.49 384 | 9.97 874 | 4 | 13 |
| 48 | 9.48 529 | 39 | 9.50 659 | 43 | 10.49 341 | 9.97 870 | 4 | 12 |
| 49 | 9.48 568 | 39 | 9.50 703 | 44 | 10.49 297 | 9.97 866 | 4 | 11 |
| 50 | 9.48 607 | 39 | 9.50 746 | 43 | 10.49 254 | 9.97 861 | 5 | 10 |
| 51 | 9.48 647 | 40 | 9.50 789 | 43 | 10.49 211 | 9.97 857 | 4 | 9 |
| 52 | 9.48 686 | 39 | 9.50 833 | 44 | 10.49 167 | 9.97 853 | 4 | 8 |
| 53 | 9.48 725 | 39 | 9.50 876 | 43 | 10.49 124 | 9.97 849 | 4 | 7 |
| 54 | 9.48 764 | 39 | 9.50 919 | 43 | 10.49 081 | 9.97 845 | 4 | 6 |
| 55 | 9.48 803 | 39 | 9.50 962 | 43 | 10.49 038 | 9.97 841 | 4 | 5 |
| 56 | 9.48 842 | 39 | 9.51 005 | 43 | 10.48 995 | 9.97 837 | 4 | 4 |
| 57 | 9.48 881 | 39 | 9.51 048 | 43 | 10.48 952 | 9.97 833 | 4 | 3 |
| 58 | 9.48 920 | 39 | 9.51 092 | 44 | 10.48 908 | 9.97 829 | 4 | 2 |
| 59 | 9.48 959 | 39 | 9.51 135 | 43 | 10.48 865 | 9.97 825 | 4 | 1 |
| 60 | 9.48 998 | 39 | 9.51 178 | 43 | 10.48 822 | 9.97 821 | 4 | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

**Prop. Pts.**

| ″ | 45 | 44 | 43 |
|---|---|---|---|
| 6 | 4.5 | 4.4 | 4.3 |
| 7 | 5.3 | 5.1 | 5.0 |
| 8 | 6.0 | 5.9 | 5.7 |
| 9 | 6.8 | 6.6 | 6.4 |
| 10 | 7.5 | 7.3 | 7.2 |
| 20 | 15.0 | 14.7 | 14.3 |
| 30 | 22.5 | 22.0 | 21.5 |
| 40 | 30.0 | 29.3 | 28.7 |
| 50 | 37.5 | 36.7 | 35.8 |

| ″ | 42 | 41 |
|---|---|---|
| 6 | 4.2 | 4.1 |
| 7 | 4.9 | 4.8 |
| 8 | 5.6 | 5.5 |
| 9 | 6.3 | 6.2 |
| 10 | 7.0 | 6.8 |
| 20 | 14.0 | 13.7 |
| 30 | 21.0 | 20.5 |
| 40 | 28.0 | 27.3 |
| 50 | 35.0 | 34.2 |

| ″ | 40 | 39 |
|---|---|---|
| 6 | 4.0 | 3.9 |
| 7 | 4.7 | 4.6 |
| 8 | 5.3 | 5.2 |
| 9 | 6.0 | 5.9 |
| 10 | 6.7 | 6.5 |
| 20 | 13.3 | 13.0 |
| 30 | 20.0 | 19.5 |
| 40 | 26.7 | 26.0 |
| 50 | 33.3 | 32.5 |

| ″ | 5 | 4 | 3 |
|---|---|---|---|
| 6 | 0.5 | 0.4 | 0.3 |
| 7 | 0.6 | 0.5 | 0.4 |
| 8 | 0.7 | 0.5 | 0.4 |
| 9 | 0.8 | 0.6 | 0.5 |
| 10 | 0.8 | 0.7 | 0.5 |
| 20 | 1.7 | 1.3 | 1.0 |
| 30 | 2.5 | 2.0 | 1.5 |
| 40 | 3.3 | 2.7 | 2.0 |
| 50 | 4.2 | 3.3 | 2.5 |

## 72°

## 18°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.48 998 | | 9.51 178 | | 10.48 822 | 9.97 821 | | **60** |
| 1 | 9.49 037 | 39 | 9.51 221 | 43 | 10.48 779 | 9.97 817 | 4 | 59 |
| 2 | 9.49 076 | 39 | 9.51 264 | 43 | 10.48 736 | 9.97 812 | 5 | 58 |
| 3 | 9.49 115 | 39 | 9.51 306 | 42 | 10.48 694 | 9.97 808 | 4 | 57 |
| 4 | 9.49 153 | 38 | 9.51 349 | 43 | 10.48 651 | 9.97 804 | 4 | 56 |
| 5 | 9.49 192 | 39 | 9.51 392 | 43 | 10.48 608 | 9.97 800 | 4 | 55 |
| 6 | 9.49 231 | 39 | 9.51 435 | 43 | 10.48 565 | 9.97 796 | 4 | 54 |
| 7 | 9.49 269 | 38 | 9.51 478 | 43 | 10.48 522 | 9.97 792 | 4 | 53 |
| 8 | 9.49 308 | 39 | 9.51 520 | 42 | 10.48 480 | 9.97 788 | 4 | 52 |
| 9 | 9.49 347 | 39 | 9.51 563 | 43 | 10.48 437 | 9.97 784 | 4 | 51 |
| **10** | 9.49 385 | 38 | 9.51 606 | 43 | 10.48 394 | 9.97 779 | 5 | **50** |
| 11 | 9.49 424 | 39 | 9.51 648 | 42 | 10.48 352 | 9.97 775 | 4 | 49 |
| 12 | 9.49 462 | 38 | 9.51 691 | 43 | 10.48 309 | 9.97 771 | 4 | 48 |
| 13 | 9.49 500 | 38 | 9.51 734 | 43 | 10.48 266 | 9.97 767 | 4 | 47 |
| 14 | 9.49 539 | 39 | 9.51 776 | 42 | 10.48 224 | 9.97 763 | 4 | 46 |
| 15 | 9.49 577 | 38 | 9.51 819 | 43 | 10.48 181 | 9.97 759 | 4 | 45 |
| 16 | 9.49 615 | 38 | 9.51 861 | 42 | 10.48 139 | 9.97 754 | 5 | 44 |
| 17 | 9.49 654 | 39 | 9.51 903 | 42 | 10.48 097 | 9.97 750 | 4 | 43 |
| 18 | 9.49 692 | 38 | 9.51 946 | 43 | 10.48 054 | 9.97 746 | 4 | 42 |
| 19 | 9.49 730 | 38 | 9.51 988 | 42 | 10.48 012 | 9.97 742 | 4 | 41 |
| **20** | 9.49 768 | 38 | 9.52 031 | 43 | 10.47 969 | 9.97 738 | 4 | **40** |
| 21 | 9.49 806 | 38 | 9.52 073 | 42 | 10.47 927 | 9.97 734 | 4 | 39 |
| 22 | 9.49 844 | 38 | 9.52 115 | 42 | 10.47 885 | 9.97 729 | 5 | 38 |
| 23 | 9.49 882 | 38 | 9.52 157 | 42 | 10.47 843 | 9.97 725 | 4 | 37 |
| 24 | 9.49 920 | 38 | 9.52 200 | 43 | 10.47 800 | 9.97 721 | 4 | 36 |
| 25 | 9.49 958 | 38 | 9.52 242 | 42 | 10.47 758 | 9.97 717 | 4 | 35 |
| 26 | 9.49 996 | 38 | 9.52 284 | 42 | 10.47 716 | 9.97 713 | 4 | 34 |
| 27 | 9.50 034 | 38 | 9.52 326 | 42 | 10.47 674 | 9.97 708 | 5 | 33 |
| 28 | 9.50 072 | 38 | 9.52 368 | 42 | 10.47 632 | 9.97 704 | 4 | 32 |
| 29 | 9.50 110 | 38 | 9.52 410 | 42 | 10.47 590 | 9.97 700 | 4 | 31 |
| **30** | 9.50 148 | 37 | 9.52 452 | 42 | 10.47 548 | 9.97 696 | 4 | **30** |
| 31 | 9.50 185 | 38 | 9.52 494 | 42 | 10.47 506 | 9.97 691 | 5 | 29 |
| 32 | 9.50 223 | 38 | 9.52 536 | 42 | 10.47 464 | 9.97 687 | 4 | 28 |
| 33 | 9.50 261 | 37 | 9.52 578 | 42 | 10.47 422 | 9.97 683 | 4 | 27 |
| 34 | 9.50 298 | 38 | 9.52 620 | 41 | 10.47 380 | 9.97 679 | 4 | 26 |
| 35 | 9.50 336 | 38 | 9.52 661 | 42 | 10.47 339 | 9.97 674 | 5 | 25 |
| 36 | 9.50 374 | 37 | 9.52 703 | 42 | 10.47 297 | 9.97 670 | 4 | 24 |
| 37 | 9.50 411 | 38 | 9.52 745 | 42 | 10.47 255 | 9.97 666 | 4 | 23 |
| 38 | 9.50 449 | 37 | 9.52 787 | 42 | 10.47 213 | 9.97 662 | 4 | 22 |
| 39 | 9.50 486 | 37 | 9.52 829 | 41 | 10.47 171 | 9.97 657 | 5 | 21 |
| **40** | 9.50 523 | 38 | 9.52 870 | 42 | 10.47 130 | 9.97 653 | 4 | **20** |
| 41 | 9.50 561 | 37 | 9.52 912 | 41 | 10.47 088 | 9.97 649 | 4 | 19 |
| 42 | 9.50 598 | 37 | 9.52 953 | 42 | 10.47 047 | 9.97 645 | 4 | 18 |
| 43 | 9.50 635 | 38 | 9.52 995 | 42 | 10.47 005 | 9.97 640 | 5 | 17 |
| 44 | 9.50 673 | 37 | 9.53 037 | 41 | 10.46 963 | 9.97 636 | 4 | 16 |
| 45 | 9.50 710 | 37 | 9.53 078 | 42 | 10.46 922 | 9.97 632 | 4 | 15 |
| 46 | 9.50 747 | 37 | 9.53 120 | 41 | 10.46 880 | 9.97 628 | 4 | 14 |
| 47 | 9.50 784 | 37 | 9.53 161 | 41 | 10.46 839 | 9.97 623 | 5 | 13 |
| 48 | 9.50 821 | 37 | 9.53 202 | 42 | 10.46 798 | 9.97 619 | 4 | 12 |
| 49 | 9.50 858 | 38 | 9.53 244 | 41 | 10.46 756 | 9.97 615 | 4 | 11 |
| **50** | 9.50 896 | 37 | 9.53 285 | 42 | 10.46 715 | 9.97 610 | 5 | **10** |
| 51 | 9.50 933 | 37 | 9.53 327 | 41 | 10.46 673 | 9.97 606 | 4 | 9 |
| 52 | 9.50 970 | 37 | 9.53 368 | 41 | 10.46 632 | 9.97 602 | 4 | 8 |
| 53 | 9.51 007 | 36 | 9.53 409 | 41 | 10.46 591 | 9.97 597 | 5 | 7 |
| 54 | 9.51 043 | 37 | 9.53 450 | 42 | 10.46 550 | 9.97 593 | 4 | 6 |
| 55 | 9.51 080 | 37 | 9.53 492 | 41 | 10.46 508 | 9.97 589 | 4 | 5 |
| 56 | 9.51 117 | 37 | 9.53 533 | 41 | 10.46 467 | 9.97 584 | 5 | 4 |
| 57 | 9.51 154 | 37 | 9.53 574 | 41 | 10.46 426 | 9.97 580 | 4 | 3 |
| 58 | 9.51 191 | 36 | 9.53 615 | 41 | 10.46 385 | 9.97 576 | 4 | 2 |
| 59 | 9.51 227 | 37 | 9.53 656 | 41 | 10.46 344 | 9.97 571 | 5 | 1 |
| **60** | 9.51 264 | | 9.53 697 | | 10.46 303 | 9.97 567 | 4 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 43 | 42 | 41 |
|---|---|---|---|
| 6 | 4.3 | 4.2 | 4.1 |
| 7 | 5.0 | 4.9 | 4.8 |
| 8 | 5.7 | 5.6 | 5.5 |
| 9 | 6.4 | 6.3 | 6.2 |
| 10 | 7.2 | 7.0 | 6.8 |
| 20 | 14.3 | 14.0 | 13.7 |
| 30 | 21.5 | 21.0 | 20.5 |
| 40 | 28.7 | 28.0 | 27.3 |
| 50 | 35.8 | 35.0 | 34.2 |

| ″ | 39 | 38 | 37 |
|---|---|---|---|
| 6 | 3.9 | 3.8 | 3.7 |
| 7 | 4.6 | 4.4 | 4.3 |
| 8 | 5.2 | 5.1 | 4.9 |
| 9 | 5.9 | 5.7 | 5.6 |
| 10 | 6.5 | 6.3 | 6.2 |
| 20 | 13.0 | 12.7 | 12.3 |
| 30 | 19.5 | 19.0 | 18.5 |
| 40 | 26.0 | 25.3 | 24.7 |
| 50 | 32.5 | 31.7 | 30.8 |

| ″ | 36 | 5 | 4 |
|---|---|---|---|
| 6 | 3.6 | 0.5 | 0.4 |
| 7 | 4.2 | 0.6 | 0.5 |
| 8 | 4.8 | 0.7 | 0.5 |
| 9 | 5.4 | 0.8 | 0.6 |
| 10 | 6.0 | 0.8 | 0.7 |
| 20 | 12.0 | 1.7 | 1.3 |
| 30 | 18.0 | 2.5 | 2.0 |
| 40 | 24.0 | 3.3 | 2.7 |
| 50 | 30.0 | 4.2 | 3.3 |

## 71°

## 19°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.51 264 | | 9.53 697 | | 10.46 303 | 9.97 567 | | **60** |
| 1 | 9.51 301 | 37 | 9.53 738 | 41 | 10.46 262 | 9.97 563 | 4 | 59 |
| 2 | 9.51 338 | 37 | 9.53 779 | 41 | 10.46 221 | 9.97 558 | 5 | 58 |
| 3 | 9.51 374 | 36 | 9.53 820 | 41 | 10.46 180 | 9.97 554 | 4 | 57 |
| 4 | 9.51 411 | 37 | 9.53 861 | 41 | 10.46 139 | 9.97 550 | 4 | 56 |
| 5 | 9.51 447 | 36 | 9.53 902 | 41 | 10.46 098 | 9.97 545 | 5 | 55 |
| 6 | 9.51 484 | 37 | 9.53 943 | 41 | 10.46 057 | 9.97 541 | 4 | 54 |
| 7 | 9.51 520 | 36 | 9.53 984 | 41 | 10.46 016 | 9.97 536 | 5 | 53 |
| 8 | 9.51 557 | 37 | 9.54 025 | 41 | 10.45 975 | 9.97 532 | 4 | 52 |
| 9 | 9.51 593 | 36 | 9.54 065 | 40 | 10.45 935 | 9.97 528 | 4 | 51 |
| **10** | 9.51 629 | 36 | 9.54 106 | 41 | 10.45 894 | 9.97 523 | 5 | **50** |
| 11 | 9.51 666 | 37 | 9.54 147 | 41 | 10.45 853 | 9.97 519 | 4 | 49 |
| 12 | 9.51 702 | 36 | 9.54 187 | 40 | 10.45 813 | 9.97 515 | 4 | 48 |
| 13 | 9.51 738 | 36 | 9.54 228 | 41 | 10.45 772 | 9.97 510 | 5 | 47 |
| 14 | 9.51 774 | 36 | 9.54 269 | 41 | 10.45 731 | 9.97 506 | 4 | 46 |
| 15 | 9.51 811 | 37 | 9.54 309 | 40 | 10.45 691 | 9.97 501 | 5 | 45 |
| 16 | 9.51 847 | 36 | 9.54 350 | 41 | 10.45 650 | 9.97 497 | 4 | 44 |
| 17 | 9.51 883 | 36 | 9.54 390 | 40 | 10.45 610 | 9.97 492 | 5 | 43 |
| 18 | 9.51 919 | 36 | 9.54 431 | 41 | 10.45 569 | 9.97 488 | 4 | 42 |
| 19 | 9.51 955 | 36 | 9.54 471 | 40 | 10.45 529 | 9.97 484 | 4 | 41 |
| **20** | 9.51 991 | 36 | 9.54 512 | 41 | 10.45 488 | 9.97 479 | 5 | **40** |
| 21 | 9.52 027 | 36 | 9.54 552 | 40 | 10.45 448 | 9.97 475 | 4 | 39 |
| 22 | 9.52 063 | 36 | 9.54 593 | 41 | 10.45 407 | 9.97 470 | 5 | 38 |
| 23 | 9.52 099 | 36 | 9.54 633 | 40 | 10.45 367 | 9.97 466 | 4 | 37 |
| 24 | 9.52 135 | 36 | 9.54 673 | 40 | 10.45 327 | 9.97 461 | 5 | 36 |
| 25 | 9.52 171 | 36 | 9.54 714 | 41 | 10.45 286 | 9.97 457 | 4 | 35 |
| 26 | 9.52 207 | 36 | 9.54 754 | 40 | 10.45 246 | 9.97 453 | 4 | 34 |
| 27 | 9.52 242 | 35 | 9.54 794 | 40 | 10.45 206 | 9.97 448 | 5 | 33 |
| 28 | 9.52 278 | 36 | 9.54 835 | 41 | 10.45 165 | 9.97 444 | 4 | 32 |
| 29 | 9.52 314 | 36 | 9.54 875 | 40 | 10.45 125 | 9.97 439 | 5 | 31 |
| **30** | 9.52 350 | 36 | 9.54 915 | 40 | 10.45 085 | 9.97 435 | 4 | **30** |
| 31 | 9.52 385 | 35 | 9.54 955 | 40 | 10.45 045 | 9.97 430 | 5 | 29 |
| 32 | 9.52 421 | 36 | 9.54 995 | 40 | 10.45 005 | 9.97 426 | 4 | 28 |
| 33 | 9.52 456 | 35 | 9.55 035 | 40 | 10.44 965 | 9.97 421 | 5 | 27 |
| 34 | 9.52 492 | 36 | 9.55 075 | 40 | 10.44 925 | 9.97 417 | 4 | 26 |
| 35 | 9.52 527 | 35 | 9.55 115 | 40 | 10.44 885 | 9.97 412 | 5 | 25 |
| 36 | 9.52 563 | 36 | 9.55 155 | 40 | 10.44 845 | 9.97 408 | 4 | 24 |
| 37 | 9.52 598 | 35 | 9.55 195 | 40 | 10.44 805 | 9.97 403 | 5 | 23 |
| 38 | 9.52 634 | 36 | 9.55 235 | 40 | 10.44 765 | 9.97 399 | 4 | 22 |
| 39 | 9.52 669 | 35 | 9.55 275 | 40 | 10.44 725 | 9.97 394 | 5 | 21 |
| **40** | 9.52 705 | 36 | 9.55 315 | 40 | 10.44 685 | 9.97 390 | 4 | **20** |
| 41 | 9.52 740 | 35 | 9.55 355 | 40 | 10.44 645 | 9.97 385 | 5 | 19 |
| 42 | 9.52 775 | 35 | 9.55 395 | 40 | 10.44 605 | 9.97 381 | 4 | 18 |
| 43 | 9.52 811 | 36 | 9.55 434 | 39 | 10.44 566 | 9.97 376 | 5 | 17 |
| 44 | 9.52 846 | 35 | 9.55 474 | 40 | 10.44 526 | 9.97 372 | 4 | 16 |
| 45 | 9.52 881 | 35 | 9.55 514 | 40 | 10.44 486 | 9.97 367 | 5 | 15 |
| 46 | 9.52 916 | 35 | 9.55 554 | 40 | 10.44 446 | 9.97 363 | 4 | 14 |
| 47 | 9.52 951 | 35 | 9.55 593 | 39 | 10.44 407 | 9.97 358 | 5 | 13 |
| 48 | 9.52 986 | 35 | 9.55 633 | 40 | 10.44 367 | 9.97 353 | 5 | 12 |
| 49 | 9.53 021 | 35 | 9.55 673 | 40 | 10.44 327 | 9.97 349 | 4 | 11 |
| **50** | 9.53 056 | 35 | 9.55 712 | 39 | 10.44 288 | 9.97 344 | 5 | **10** |
| 51 | 9.53 092 | 36 | 9.55 752 | 40 | 10.44 248 | 9.97 340 | 4 | 9 |
| 52 | 9.53 126 | 34 | 9.55 791 | 39 | 10.44 209 | 9.97 335 | 5 | 8 |
| 53 | 9.53 161 | 35 | 9.55 831 | 40 | 10.44 169 | 9.97 331 | 4 | 7 |
| 54 | 9.53 196 | 35 | 9.55 870 | 39 | 10.44 130 | 9.97 326 | 5 | 6 |
| 55 | 9.53 231 | 35 | 9.55 910 | 40 | 10.44 090 | 9.97 322 | 4 | 5 |
| 56 | 9.53 266 | 35 | 9.55 949 | 39 | 10.44 051 | 9.97 317 | 5 | 4 |
| 57 | 9.53 301 | 35 | 9.55 989 | 40 | 10.44 011 | 9.97 312 | 5 | 3 |
| 58 | 9.53 336 | 35 | 9.56 028 | 39 | 10.43 972 | 9.97 308 | 4 | 2 |
| 59 | 9.53 370 | 34 | 9.56 067 | 39 | 10.43 933 | 9.97 303 | 5 | 1 |
| **60** | 9.53 405 | 35 | 9.56 107 | 40 | 10.43 893 | 9.97 299 | 4 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 41 | 40 | 39 |
|---|---|---|---|
| 6 | 4.1 | 4.0 | 3.9 |
| 7 | 4.8 | 4.7 | 4.6 |
| 8 | 5.5 | 5.3 | 5.2 |
| 9 | 6.2 | 6.0 | 5.9 |
| 10 | 6.8 | 6.7 | 6.5 |
| 20 | 13.7 | 13.3 | 13.0 |
| 30 | 20.5 | 20.0 | 19.5 |
| 40 | 27.3 | 26.7 | 26.0 |
| 50 | 34.2 | 33.3 | 32.5 |

| ″ | 37 | 36 | 35 |
|---|---|---|---|
| 6 | 3.7 | 3.6 | 3.5 |
| 7 | 4.3 | 4.2 | 4.1 |
| 8 | 4.9 | 4.8 | 4.7 |
| 9 | 5.6 | 5.4 | 5.3 |
| 10 | 6.2 | 6.0 | 5.8 |
| 20 | 12.3 | 12.0 | 11.7 |
| 30 | 18.5 | 18.0 | 17.5 |
| 40 | 24.7 | 24.0 | 23.3 |
| 50 | 30.8 | 30.0 | 29.2 |

| ″ | 34 | 5 | 4 |
|---|---|---|---|
| 6 | 3.4 | 0.5 | 0.4 |
| 7 | 4.0 | 0.6 | 0.5 |
| 8 | 4.5 | 0.7 | 0.5 |
| 9 | 5.1 | 0.8 | 0.6 |
| 10 | 5.7 | 0.8 | 0.7 |
| 20 | 11.3 | 1.7 | 1.3 |
| 30 | 17.0 | 2.5 | 2.0 |
| 40 | 22.7 | 3.3 | 2.7 |
| 50 | 28.3 | 4.2 | 3.3 |

## 70°

## 20°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.53 405 | | 9.56 107 | | 10.43 893 | 9.97 299 | | 60 | |
| 1 | 9.53 440 | 35 | 9.56 146 | 39 | 10.43 854 | 9.97 294 | 5 | 59 | |
| 2 | 9.53 475 | 35 | 9.56 185 | 39 | 10.43 815 | 9.97 289 | 5 | 58 | |
| 3 | 9.53 509 | 34 | 9.56 224 | 39 | 10.43 776 | 9.97 285 | 4 | 57 | |
| 4 | 9.53 544 | 35 | 9.56 264 | 40 | 10.43 736 | 9.97 280 | 5 | 56 | |
| 5 | 9.53 578 | 34 | 9.56 303 | 39 | 10.43 697 | 9.97 276 | 4 | 55 | |
| 6 | 9.53 613 | 35 | 9.56 342 | 39 | 10.43 658 | 9.97 271 | 5 | 54 | |
| 7 | 9.53 647 | 34 | 9.56 381 | 39 | 10.43 619 | 9.97 266 | 5 | 53 | |
| 8 | 9.53 682 | 35 | 9.56 420 | 39 | 10.43 580 | 9.97 262 | 4 | 52 | |
| 9 | 9.53 716 | 34 | 9.56 459 | 39 | 10.43 541 | 9.97 257 | 5 | 51 | |
| 10 | 9.53 751 | 35 | 9.56 498 | 39 | 10.43 502 | 9.97 252 | 5 | 50 | |
| 11 | 9.53 785 | 34 | 9.56 537 | 39 | 10.43 463 | 9.97 248 | 4 | 49 | |
| 12 | 9.53 819 | 34 | 9.56 576 | 39 | 10.43 424 | 9.97 243 | 5 | 48 | |
| 13 | 9.53 854 | 35 | 9.56 615 | 39 | 10.43 385 | 9.97 238 | 5 | 47 | |
| 14 | 9.53 888 | 34 | 9.56 654 | 39 | 10.43 346 | 9 97 234 | 4 | 46 | |
| 15 | 9.53 922 | 34 | 9.56 693 | 39 | 10.43 307 | 9.97 229 | 5 | 45 | |
| 16 | 9.53 957 | 35 | 9.56 732 | 39 | 10.43 268 | 9.97 224 | 5 | 44 | |
| 17 | 9.53 991 | 34 | 9.56 771 | 39 | 10.43 229 | 9.97 220 | 4 | 43 | |
| 18 | 9.54 025 | 34 | 9.56 810 | 39 | 10.43 190 | 9.97 215 | 5 | 42 | |
| 19 | 9.54 059 | 34 | 9.56 849 | 39 | 10.43 151 | 9.97 210 | 5 | 41 | |
| 20 | 9.54 093 | 34 | 9.56 887 | 38 | 10.43 113 | 9.97 206 | 4 | 40 | |
| 21 | 9.54 127 | 34 | 9.56 926 | 39 | 10.43 074 | 9.97 201 | 5 | 39 | |
| 22 | 9.54 161 | 34 | 9.56 965 | 39 | 10.43 035 | 9.97 196 | 5 | 38 | |
| 23 | 9.54 195 | 34 | 9.57 004 | 39 | 10.42 996 | 9.97 192 | 4 | 37 | |
| 24 | 9.54 229 | 34 | 9.57 042 | 38 | 10.42 958 | 9.97 187 | 5 | 36 | |
| 25 | 9.54 263 | 34 | 9.57 081 | 39 | 10.42 919 | 9.97 182 | 5 | 35 | |
| 26 | 9.54 297 | 34 | 9.57 120 | 39 | 10.42 880 | 9.97 178 | 4 | 34 | |
| 27 | 9.54 331 | 34 | 9.57 158 | 38 | 10.42 842 | 9.97 173 | 5 | 33 | |
| 28 | 9.54 365 | 34 | 9.57 197 | 39 | 10.42 803 | 9.97 168 | 5 | 32 | |
| 29 | 9.54 399 | 34 | 9.57 235 | 38 | 10.42 765 | 9.97 163 | 5 | 31 | |
| 30 | 9.54 433 | 34 | 9.57 274 | 39 | 10.42 726 | 9.97 159 | 4 | 30 | |
| 31 | 9.54 466 | 33 | 9.57 312 | 38 | 10.42 688 | 9.97 154 | 5 | 29 | |
| 32 | 9.54 500 | 34 | 9.57 351 | 39 | 10.42 649 | 9.97 149 | 5 | 28 | |
| 33 | 9.54 534 | 34 | 9.57 389 | 38 | 10.42 611 | 9.97 145 | 4 | 27 | |
| 34 | 9.54 567 | 33 | 9.57 428 | 39 | 10.42 572 | 9.97 140 | 5 | 26 | |
| 35 | 9.54 601 | 34 | 9.57 466 | 38 | 10.42 534 | 9.97 135 | 5 | 25 | |
| 36 | 9.54 635 | 34 | 9.57 504 | 38 | 10.42 496 | 9.97 130 | 5 | 24 | |
| 37 | 9.54 668 | 33 | 9.57 543 | 39 | 10.42 457 | 9.97 126 | 4 | 23 | |
| 38 | 9.54 702 | 34 | 9.57 581 | 38 | 10.42 419 | 9.97 121 | 5 | 22 | |
| 39 | 9.54 735 | 33 | 9.57 619 | 38 | 10.42 381 | 9.97 116 | 5 | 21 | |
| 40 | 9.54 769 | 34 | 9.57 658 | 39 | 10.42 342 | 9.97 111 | 5 | 20 | |
| 41 | 9.54 802 | 33 | 9.57 696 | 38 | 10.42 304 | 9.97 107 | 4 | 19 | |
| 42 | 9.54 836 | 34 | 9.57 734 | 38 | 10.42 266 | 9.97 102 | 5 | 18 | |
| 43 | 9.54 869 | 33 | 9.57 772 | 38 | 10.42 228 | 9.97 097 | 5 | 17 | |
| 44 | 9.54 903 | 34 | 9.57 810 | 38 | 10.42 190 | 9.97 092 | 5 | 16 | |
| 45 | 9.54 936 | 33 | 9.57 849 | 39 | 10.42 151 | 9.97 087 | 5 | 15 | |
| 46 | 9.54 969 | 33 | 9.57 887 | 38 | 10.42 113 | 9.97 083 | 4 | 14 | |
| 47 | 9.55 003 | 34 | 9.57 925 | 38 | 10.42 075 | 9.97 078 | 5 | 13 | |
| 48 | 9.55 036 | 33 | 9.57 963 | 38 | 10.42 037 | 9.97 073 | 5 | 12 | |
| 49 | 9.55 069 | 33 | 9.58 001 | 38 | 10.41 999 | 9.97 068 | 5 | 11 | |
| 50 | 9.55 102 | 33 | 9.58 039 | 38 | 10.41 961 | 9.97 063 | 5 | 10 | |
| 51 | 9.55 136 | 34 | 9.58 077 | 38 | 10.41 923 | 9.97 059 | 4 | 9 | |
| 52 | 9.55 169 | 33 | 9.58 115 | 38 | 10.41 885 | 9.97 054 | 5 | 8 | |
| 53 | 9.55 202 | 33 | 9.58 153 | 38 | 10.41 847 | 9.97 049 | 5 | 7 | |
| 54 | 9.55 235 | 33 | 9.58 191 | 38 | 10.41 809 | 9.97 044 | 5 | 6 | |
| 55 | 9.55 268 | 33 | 9.58 229 | 38 | 10.41 771 | 9.97 039 | 5 | 5 | |
| 56 | 9.55 301 | 33 | 9.58 267 | 37 | 10.41 733 | 9.97 035 | 4 | 4 | |
| 57 | 9.55 334 | 33 | 9.58 304 | 38 | 10.41 696 | 9.97 030 | 5 | 3 | |
| 58 | 9.55 367 | 33 | 9.58 342 | 38 | 10.41 658 | 9.97 025 | 5 | 2 | |
| 59 | 9.55 400 | 33 | 9.58 380 | 38 | 10.41 620 | 9.97 020 | 5 | 1 | |
| 60 | 9.55 433 | | 9.58 418 | | 10.41 582 | 9.97 015 | | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 40 | 39 |
|---|---|---|
| 6 | 4.0 | 3.9 |
| 7 | 4.7 | 4.6 |
| 8 | 5.3 | 5.2 |
| 9 | 6.0 | 5.9 |
| 10 | 6.7 | 6.5 |
| 20 | 13.3 | 13.0 |
| 30 | 20.0 | 19.5 |
| 40 | 26.7 | 26.0 |
| 50 | 33.3 | 32.5 |

| ″ | 38 | 37 |
|---|---|---|
| 6 | 3.8 | 3.7 |
| 7 | 4.4 | 4.3 |
| 8 | 5.1 | 4.9 |
| 9 | 5.7 | 5.6 |
| 10 | 6.3 | 6.2 |
| 20 | 12.7 | 12.3 |
| 30 | 19.0 | 18.5 |
| 40 | 25.3 | 24.7 |
| 50 | 31.7 | 30.8 |

| ″ | 35 | 34 |
|---|---|---|
| 6 | 3.5 | 3.4 |
| 7 | 4.1 | 4.0 |
| 8 | 4.7 | 4.5 |
| 9 | 5.3 | 5.1 |
| 10 | 5.8 | 5.7 |
| 20 | 11.7 | 11.3 |
| 30 | 17.5 | 17.0 |
| 40 | 23.3 | 22.7 |
| 50 | 29.2 | 28.3 |

| ″ | 33 | 5 | 4 |
|---|---|---|---|
| 6 | 3.3 | 0.5 | 0.4 |
| 7 | 3.8 | 0.6 | 0.5 |
| 8 | 4.4 | 0.7 | 0.5 |
| 9 | 5.0 | 0.8 | 0.6 |
| 10 | 5.5 | 0.8 | 0.7 |
| 20 | 11.0 | 1.7 | 1.3 |
| 30 | 16.5 | 2.5 | 2.0 |
| 40 | 22.0 | 3.3 | 2.7 |
| 50 | 27.5 | 4.2 | 3.3 |

## 69°

## 21°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.55 433 | | 9.58 418 | | 10.41 582 | 9.97 015 | | **60** |
| 1 | 9.55 466 | 33 | 9.58 455 | 37 | 10.41 545 | 9.97 010 | 5 | 59 |
| 2 | 9.55 499 | 33 | 9.58 493 | 38 | 10.41 507 | 9.97 005 | 5 | 58 |
| 3 | 9.55 532 | 33 | 9.58 531 | 38 | 10.41 469 | 9.97 001 | 4 | 57 |
| 4 | 9.55 564 | 32 | 9.58 569 | 38 | 10.41 431 | 9.96 996 | 5 | 56 |
| 5 | 9.55 597 | 33 | 9.58 606 | 37 | 10.41 394 | 9.96 991 | 5 | 55 |
| 6 | 9.55 630 | 33 | 9.58 644 | 38 | 10.41 356 | 9.96 986 | 5 | 54 |
| 7 | 9.55 663 | 33 | 9.58 681 | 37 | 10.41 319 | 9.96 981 | 5 | 53 |
| 8 | 9.55 695 | 32 | 9.58 719 | 38 | 10.41 281 | 9.96 876 | 5 | 52 |
| 9 | 9.55 728 | 33 | 9.58 757 | 38 | 10.41 243 | 9.96 971 | 5 | 51 |
| **10** | 9.55 761 | 33 | 9.58 794 | 37 | 10.41 206 | 9.96 966 | 5 | **50** |
| 11 | 9.55 793 | 32 | 9.58 832 | 38 | 10.41 168 | 9.96 962 | 4 | 49 |
| 12 | 9.55 826 | 33 | 9.58 869 | 37 | 10.41 131 | 9.96 957 | 5 | 48 |
| 13 | 9.55 858 | 32 | 9.58 907 | 38 | 10.41 093 | 9.96 952 | 5 | 47 |
| 14 | 9.55 891 | 33 | 9.58 944 | 37 | 10.41 056 | 9.96 947 | 5 | 46 |
| 15 | 9.55 923 | 32 | 9.58 981 | 37 | 10.41 019 | 9.96 942 | 5 | 45 |
| 16 | 9.55 956 | 33 | 9.59 019 | 38 | 10.40 981 | 9.96 937 | 5 | 44 |
| 17 | 9.55 988 | 32 | 9.59 056 | 37 | 10.40 944 | 9.96 932 | 5 | 43 |
| 18 | 9.56 021 | 33 | 9.59 094 | 38 | 10.40 906 | 9.96 927 | 5 | 42 |
| 19 | 9.56 053 | 32 | 9.59 131 | 37 | 10.40 869 | 9.96 922 | 5 | 41 |
| **20** | 9.56 085 | 32 | 9.59 168 | 37 | 10.40 832 | 9.96 917 | 5 | **40** |
| 21 | 9.56 118 | 33 | 9.59 205 | 37 | 10.40 795 | 9.96 912 | 5 | 39 |
| 22 | 9.56 150 | 32 | 9.59 243 | 38 | 10.40 757 | 9.96 907 | 5 | 38 |
| 23 | 9.56 182 | 32 | 9.59 280 | 37 | 10.40 720 | 9.96 903 | 4 | 37 |
| 24 | 9.56 215 | 33 | 9.59 317 | 37 | 10.40 683 | 9.96 898 | 5 | 36 |
| 25 | 9.56 247 | 32 | 9.59 354 | 37 | 10.40 646 | 9.96 893 | 5 | 35 |
| 26 | 9.56 279 | 32 | 9.59 391 | 37 | 10.40 609 | 9.96 888 | 5 | 34 |
| 27 | 9.56 311 | 32 | 9.59 429 | 38 | 10.40 571 | 9.96 883 | 5 | 33 |
| 28 | 9.56 343 | 32 | 9.59 466 | 37 | 10.40 534 | 9.96 878 | 5 | 32 |
| 29 | 9.56 375 | 32 | 9.59 503 | 37 | 10.40 497 | 9.96 873 | 5 | 31 |
| **30** | 9.56 408 | 33 | 9.59 540 | 37 | 10.40 460 | 9.96 868 | 5 | **30** |
| 31 | 9.56 440 | 32 | 9.59 577 | 37 | 10.40 423 | 9.96 863 | 5 | 29 |
| 32 | 9.56 472 | 32 | 9.59 614 | 37 | 10.40 386 | 9.96 858 | 5 | 28 |
| 33 | 9.56 504 | 32 | 9.59 651 | 37 | 10.40 349 | 9.96 853 | 5 | 27 |
| 34 | 9.56 536 | 32 | 9.59 688 | 37 | 10.40 312 | 9.96 848 | 5 | 26 |
| 35 | 9.56 568 | 32 | 9.59 725 | 37 | 10.40 275 | 9.96 843 | 5 | 25 |
| 36 | 9.56 599 | 31 | 9.59 762 | 37 | 10.40 238 | 9.96 838 | 5 | 24 |
| 37 | 9.56 631 | 32 | 9.59 799 | 37 | 10.40 201 | 9.96 833 | 5 | 23 |
| 38 | 9.56 663 | 32 | 9.59 835 | 36 | 10.40 165 | 9.96 828 | 5 | 22 |
| 39 | 9.56 695 | 32 | 9.59 872 | 37 | 10.40 128 | 9.96 823 | 5 | 21 |
| **40** | 9.56 727 | 32 | 9.59 909 | 37 | 10.40 091 | 9.96 818 | 5 | **20** |
| 41 | 9.56 759 | 32 | 9.59 946 | 37 | 10.40 054 | 9.96 813 | 5 | 19 |
| 42 | 9.56 790 | 31 | 9.59 983 | 37 | 10.40 017 | 9.96 808 | 5 | 18 |
| 43 | 9.56 822 | 32 | 9.60 019 | 36 | 10.39 981 | 9.96 803 | 5 | 17 |
| 44 | 9.56 854 | 32 | 9.60 056 | 37 | 10.39 944 | 9.96 798 | 5 | 16 |
| 45 | 9.56 886 | 32 | 9.60 093 | 37 | 10.39 907 | 9.96 793 | 5 | 15 |
| 46 | 9.56 917 | 31 | 9.60 130 | 37 | 10.39 870 | 9.96 788 | 5 | 14 |
| 47 | 9.56 949 | 32 | 9.60 166 | 36 | 10.39 834 | 9.96 783 | 5 | 13 |
| 48 | 9.56 980 | 31 | 9.60 203 | 37 | 10.39 797 | 9.96 778 | 5 | 12 |
| 49 | 9.57 012 | 32 | 9.60 240 | 37 | 10.39 760 | 9.96 772 | 6 | 11 |
| **50** | 9.57 044 | 32 | 9.60 276 | 36 | 10.39 724 | 9.96 767 | 5 | **10** |
| 51 | 9.57 075 | 31 | 9.60 313 | 37 | 10.39 687 | 9.96 762 | 5 | 9 |
| 52 | 9.57 107 | 32 | 9.60 349 | 36 | 10.39 651 | 9.96 757 | 5 | 8 |
| 53 | 9.57 138 | 31 | 9.60 386 | 37 | 10.39 614 | 9.96 752 | 5 | 7 |
| 54 | 9.57 169 | 31 | 9.60 422 | 36 | 10.39 578 | 9.96 747 | 5 | 6 |
| 55 | 9.57 201 | 32 | 9.60 459 | 37 | 10.39 541 | 9.96 742 | 5 | 5 |
| 56 | 9.57 232 | 31 | 9.60 495 | 36 | 10.39 505 | 9.96 737 | 5 | 4 |
| 57 | 9.57 264 | 32 | 9.60 532 | 37 | 10.39 468 | 9.96 732 | 5 | 3 |
| 58 | 9.57 295 | 31 | 9.60 568 | 36 | 10.39 432 | 9.96 727 | 5 | 2 |
| 59 | 9.57 326 | 31 | 9.60 605 | 37 | 10.39 395 | 9.96 722 | 5 | 1 |
| **60** | 9.57 358 | 32 | 9.60 641 | 36 | 10.39 359 | 9.96 717 | 5 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 38 | 37 | 36 |
|---|---|---|---|
| 6 | 3.8 | 3.7 | 3.6 |
| 7 | 4.4 | 4.3 | 4.2 |
| 8 | 5.1 | 4.9 | 4.8 |
| 9 | 5.7 | 5.6 | 5.4 |
| 10 | 6.3 | 6.2 | 6.0 |
| 20 | 12.7 | 12.3 | 12.0 |
| 30 | 19.0 | 18.5 | 18.0 |
| 40 | 25.3 | 24.7 | 24.0 |
| 50 | 31.7 | 30.8 | 30.0 |

| ″ | 33 | 32 | 31 |
|---|---|---|---|
| 6 | 3.3 | 3.2 | 3.1 |
| 7 | 3.9 | 3.7 | 3.6 |
| 8 | 4.4 | 4.3 | 4.1 |
| 9 | 5.0 | 4.8 | 4.6 |
| 10 | 5.5 | 5.3 | 5.2 |
| 20 | 11.0 | 10.7 | 10.3 |
| 30 | 16.5 | 16.0 | 15.5 |
| 40 | 22.0 | 21.3 | 20.7 |
| 50 | 27.5 | 26.7 | 25.8 |

| ″ | 6 | 5 | 4 |
|---|---|---|---|
| 6 | 0.6 | 0.5 | 0.4 |
| 7 | 0.7 | 0.6 | 0.5 |
| 8 | 0.8 | 0.7 | 0.5 |
| 9 | 0.9 | 0.8 | 0.6 |
| 10 | 1.0 | 0.8 | 0.7 |
| 20 | 2.0 | 1.7 | 1.3 |
| 30 | 3.0 | 2.5 | 2.0 |
| 40 | 4.0 | 3.3 | 2.7 |
| 50 | 5.0 | 4.2 | 3.3 |

## 68°

## 22°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.57 358 | 31 | 9.60 641 | 36 | 10.39 359 | 9.96 717 | 6 | **60** |
| 1 | 9.57 389 | 31 | 9.60 677 | 37 | 10.39 323 | 9.96 711 | 5 | 59 |
| 2 | 9.57 420 | 31 | 9.60 714 | 36 | 10.39 286 | 9.96 706 | 5 | 58 |
| 3 | 9.57 451 | 31 | 9.60 750 | 36 | 10.39 250 | 9.96 701 | 5 | 57 |
| 4 | 9.57 482 | 32 | 9.60 786 | 37 | 10.39 214 | 9.96 696 | 5 | 56 |
| 5 | 9.57 514 | 31 | 9.60 823 | 36 | 10.39 177 | 9.96 691 | 5 | 55 |
| 6 | 9.57 545 | 31 | 9.60 859 | 36 | 10.39 141 | 9.96 686 | 5 | 54 |
| 7 | 9.57 576 | 31 | 9.60 895 | 36 | 10.39 105 | 9.96 681 | 6 | 53 |
| 8 | 9.57 607 | 31 | 9.60 931 | 36 | 10.39 069 | 9.96 676 | 5 | 52 |
| 9 | 9.57 638 | 31 | 9.60 967 | 37 | 10.39 033 | 9.96 670 | 5 | 51 |
| **10** | 9.57 669 | 31 | 9.61 004 | 36 | 10.38 996 | 9.96 665 | 5 | **50** |
| 11 | 9.57 700 | 31 | 9.61 040 | 36 | 10.38 960 | 9.96 660 | 5 | 49 |
| 12 | 9.57 731 | 31 | 9.61 076 | 36 | 10.38 924 | 9.96 655 | 5 | 48 |
| 13 | 9.57 762 | 31 | 9.61 112 | 36 | 10.38 888 | 9.96 650 | 5 | 47 |
| 14 | 9.57 793 | 31 | 9.61 148 | 36 | 10.38 852 | 9.96 645 | 5 | 46 |
| 15 | 9.57 824 | 31 | 9.61 184 | 36 | 10.38 816 | 9.96 640 | 6 | 45 |
| 16 | 9.57 855 | 30 | 9.61 220 | 36 | 10.38 780 | 9.96 634 | 5 | 44 |
| 17 | 9.57 885 | 31 | 9.61 256 | 36 | 10.38 744 | 9.96 629 | 5 | 43 |
| 18 | 9.57 916 | 31 | 9.61 292 | 36 | 10.38 708 | 9.96 624 | 5 | 42 |
| 19 | 9.57 947 | 31 | 9.61 328 | 36 | 10.38 672 | 9.96 619 | 5 | 41 |
| **20** | 9.57 978 | 30 | 9.61 364 | 36 | 10.38 636 | 9.96 614 | 6 | **40** |
| 21 | 9.58 008 | 31 | 9.61 400 | 36 | 10.38 600 | 9.96 608 | 5 | 39 |
| 22 | 9.58 039 | 31 | 9.61 436 | 36 | 10.38 564 | 9.96 603 | 5 | 38 |
| 23 | 9.58 070 | 31 | 9.61 472 | 36 | 10.38 528 | 9.96 598 | 5 | 37 |
| 24 | 9.58 101 | 30 | 9.61 508 | 36 | 10.38 492 | 9.96 593 | 5 | 36 |
| 25 | 9.58 131 | 31 | 9.61 544 | 35 | 10.38 456 | 9.96 588 | 6 | 35 |
| 26 | 9.58 162 | 30 | 9.61 579 | 36 | 10.38 421 | 9.96 582 | 5 | 34 |
| 27 | 9.58 192 | 31 | 9.61 615 | 36 | 10.38 385 | 9.96 577 | 5 | 33 |
| 28 | 9.58 223 | 30 | 9.61 651 | 36 | 10.38 349 | 9.96 572 | 5 | 32 |
| 29 | 9.58 253 | 31 | 9.61 687 | 35 | 10.38 313 | 9.96 567 | 5 | 31 |
| **30** | 9.58 284 | 30 | 9.61 722 | 36 | 10.38 278 | 9.96 562 | 6 | **30** |
| 31 | 9.58 314 | 31 | 9.61 758 | 36 | 10.38 242 | 9.96 556 | 5 | 29 |
| 32 | 9.58 345 | 30 | 9.61 794 | 36 | 10.38 206 | 9.96 551 | 5 | 28 |
| 33 | 9.58 375 | 31 | 9.61 830 | 35 | 10.38 170 | 9.96 546 | 5 | 27 |
| 34 | 9.58 406 | 30 | 9.61 865 | 36 | 10.38 135 | 9.96 541 | 6 | 26 |
| 35 | 9.58 436 | 31 | 9.61 901 | 35 | 10.38 099 | 9.96 535 | 5 | 25 |
| 36 | 9.58 467 | 30 | 9.61 936 | 36 | 10.38 064 | 9.96 530 | 5 | 24 |
| 37 | 9.58 497 | 30 | 9.61 972 | 36 | 10.38 028 | 9.96 525 | 5 | 23 |
| 38 | 9.58 527 | 30 | 9.62 008 | 35 | 10.37 992 | 9.96 520 | 6 | 22 |
| 39 | 9.58 557 | 31 | 9.62 043 | 36 | 10.37 957 | 9.96 514 | 5 | 21 |
| **40** | 9.58 588 | 30 | 9.62 079 | 35 | 10.37 921 | 9.96 509 | 5 | **20** |
| 41 | 9.58 618 | 30 | 9.62 114 | 36 | 10.37 886 | 9.96 504 | 6 | 19 |
| 42 | 9.58 648 | 30 | 9.62 150 | 35 | 10.37 850 | 9.96 498 | 5 | 18 |
| 43 | 9.58 678 | 31 | 9.62 185 | 36 | 10.37 815 | 9.96 493 | 5 | 17 |
| 44 | 9.58 709 | 30 | 9.62 221 | 35 | 10.37 779 | 9.96 488 | 5 | 16 |
| 45 | 9.58 739 | 30 | 9.62 256 | 36 | 10.37 744 | 9.96 483 | 6 | 15 |
| 46 | 9.58 769 | 30 | 9.62 292 | 35 | 10.37 708 | 9.96 477 | 5 | 14 |
| 47 | 9.58 799 | 30 | 9.62 327 | 35 | 10.37 673 | 9.96 472 | 5 | 13 |
| 48 | 9.58 829 | 30 | 9.62 362 | 36 | 10.37 638 | 9.96 467 | 6 | 12 |
| 49 | 9.58 859 | 30 | 9.62 398 | 35 | 10.37 602 | 9.96 461 | 5 | 11 |
| **50** | 9.58 889 | 30 | 9.62 433 | 35 | 10.37 567 | 9.96 456 | 5 | **10** |
| 51 | 9.58 919 | 30 | 9.62 468 | 36 | 10.37 532 | 9.96 451 | 6 | 9 |
| 52 | 9.58 949 | 30 | 9.62 504 | 35 | 10.37 496 | 9.96 445 | 5 | 8 |
| 53 | 9.58 979 | 30 | 9.62 539 | 35 | 10.37 461 | 9.96 440 | 5 | 7 |
| 54 | 9.59 009 | 30 | 9.62 574 | 35 | 10.37 426 | 9.96 435 | 6 | 6 |
| 55 | 9.59 039 | 30 | 9.62 609 | 36 | 10.37 391 | 9.96 429 | 5 | 5 |
| 56 | 9.59 069 | 29 | 9.62 645 | 35 | 10.37 355 | 9.96 424 | 5 | 4 |
| 57 | 9.59 098 | 30 | 9.62 680 | 35 | 10.37 320 | 9.96 419 | 6 | 3 |
| 58 | 9.59 128 | 30 | 9.62 715 | 35 | 10.37 285 | 9.96 413 | 5 | 2 |
| 59 | 9.59 158 | 30 | 9.62 750 | 35 | 10.37 250 | 9.96 408 | 5 | 1 |
| **60** | 9.59 188 | | 9.62 785 | | 10.37 215 | 9.96 403 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 37 | 36 | 35 |
|---|---|---|---|
| 6 | 3.7 | 3.6 | 3.5 |
| 7 | 4.3 | 4.2 | 4.1 |
| 8 | 4.9 | 4.8 | 4.7 |
| 9 | 5.6 | 5.4 | 5.2 |
| 10 | 6.2 | 6.0 | 5.8 |
| 20 | 12.3 | 12.0 | 11.7 |
| 30 | 18.5 | 18.0 | 17.5 |
| 40 | 24.7 | 24.0 | 23.3 |
| 50 | 30.8 | 30.0 | 29.2 |

| ″ | 32 | 31 | 30 |
|---|---|---|---|
| 6 | 3.2 | 3.1 | 3.0 |
| 7 | 3.7 | 3.6 | 3.5 |
| 8 | 4.3 | 4.1 | 4.0 |
| 9 | 4.8 | 4.6 | 4.5 |
| 10 | 5.3 | 5.2 | 5.0 |
| 20 | 10.7 | 10.3 | 10.0 |
| 30 | 16.0 | 15.5 | 15.0 |
| 40 | 21.3 | 20.7 | 20.0 |
| 50 | 26.7 | 25.8 | 25.0 |

| ″ | 29 | 6 | 5 |
|---|---|---|---|
| 6 | 2.9 | 0.6 | 0.5 |
| 7 | 3.4 | 0.7 | 0.6 |
| 8 | 3.9 | 0.8 | 0.7 |
| 9 | 4.4 | 0.9 | 0.8 |
| 10 | 4.8 | 1.0 | 0.8 |
| 20 | 9.7 | 2.0 | 1.7 |
| 30 | 14.5 | 3.0 | 2.5 |
| 40 | 19.3 | 4.0 | 3.3 |
| 50 | 24.2 | 5.0 | 4.2 |

## 67°

## 23°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.59 188 | | 9.62 785 | | 10.37 215 | 9.96 403 | | 60 | |
| 1 | 9.59 218 | 30 | 9.62 820 | 35 | 10.37 180 | 9.96 397 | 6 | 59 | |
| 2 | 9.59 247 | 29 | 9.62 855 | 35 | 10.37 145 | 9.96 392 | 5 | 58 | |
| 3 | 9.59 277 | 30 | 9.62 890 | 35 | 10.37 110 | 9.96 387 | 5 | 57 | |
| 4 | 9.59 307 | 30 | 9.62 926 | 36 | 10.37 074 | 9.96 381 | 6 | 56 | |
| 5 | 9.59 336 | 29 | 9.62 961 | 35 | 10.37 039 | 9.96 376 | 5 | 55 | |
| 6 | 9.59 366 | 30 | 9.62 996 | 35 | 10.37 004 | 9.96 370 | 6 | 54 | |
| 7 | 9.59 396 | 30 | 9.63 031 | 35 | 10.36 969 | 9.96 365 | 5 | 53 | |
| 8 | 9.59 425 | 29 | 9.63 066 | 35 | 10.36 934 | 9.96 360 | 5 | 52 | |
| 9 | 9.59 455 | 30 | 9.63 101 | 35 | 10.36 899 | 9.96 354 | 6 | 51 | |
| 10 | 9.59 484 | 29 | 9.63 135 | 34 | 10.36 865 | 9.96 349 | 5 | 50 | |
| 11 | 9.59 514 | 30 | 9.63 170 | 35 | 10.36 830 | 9.96 343 | 6 | 49 | |
| 12 | 9.59 543 | 29 | 9.63 205 | 35 | 10.36 795 | 9.96 338 | 5 | 48 | |
| 13 | 9.59 573 | 30 | 9.63 240 | 35 | 10.36 760 | 9.96 333 | 5 | 47 | |
| 14 | 9.59 602 | 29 | 9.63 275 | 35 | 10.36 725 | 9.96 327 | 6 | 46 | |
| 15 | 9.59 632 | 30 | 9.63 310 | 35 | 10.36 690 | 9.96 322 | 5 | 45 | |
| 16 | 9.59 661 | 29 | 9.63 345 | 35 | 10.36 655 | 9.96 316 | 6 | 44 | |
| 17 | 9.59 690 | 29 | 9.63 379 | 34 | 10.36 621 | 9.96 311 | 5 | 43 | |
| 18 | 9.59 720 | 30 | 9.63 414 | 35 | 10.36 586 | 9.96 305 | 6 | 42 | |
| 19 | 9.59 749 | 29 | 9.63 449 | 35 | 10.36 551 | 9.96 300 | 5 | 41 | |
| 20 | 9.59 778 | 29 | 9.63 484 | 35 | 10.36 516 | 9.96 294 | 6 | 40 | |
| 21 | 9.59 808 | 30 | 9.63 519 | 35 | 10.36 481 | 9.96 289 | 5 | 39 | |
| 22 | 9.59 837 | 29 | 9.63 553 | 34 | 10.36 447 | 9.96 284 | 5 | 38 | |
| 23 | 9.59 866 | 29 | 9.63 588 | 35 | 10.36 412 | 9.96 278 | 6 | 37 | |
| 24 | 9.59 895 | 29 | 9.63 623 | 35 | 10.36 377 | 9.96 273 | 5 | 36 | |
| 25 | 9.59 924 | 29 | 9.63 657 | 34 | 10.36 343 | 9.96 267 | 6 | 35 | |
| 26 | 9.59 954 | 30 | 9.63 692 | 35 | 10.36 308 | 9.96 262 | 5 | 34 | |
| 27 | 9.59 983 | 29 | 9.63 726 | 34 | 10.36 274 | 9.96 256 | 6 | 33 | |
| 28 | 9.60 012 | 29 | 9.63 761 | 35 | 10.36 239 | 9.96 251 | 5 | 32 | |
| 29 | 9.60 041 | 29 | 9.63 796 | 35 | 10.36 204 | 9.96 245 | 6 | 31 | |
| 30 | 9.60 070 | 29 | 9.63 830 | 34 | 10.36 170 | 9.96 240 | 5 | 30 | |
| 31 | 9.60 099 | 29 | 9.63 865 | 35 | 10.36 135 | 9.96 234 | 6 | 29 | |
| 32 | 9.60 128 | 29 | 9.63 899 | 34 | 10.36 101 | 9.96 229 | 5 | 28 | |
| 33 | 9.60 157 | 29 | 9.63 934 | 35 | 10.36 066 | 9.96 223 | 6 | 27 | |
| 34 | 9.60 186 | 29 | 9.63 968 | 34 | 10.36 032 | 9.96 218 | 5 | 26 | |
| 35 | 9.60 215 | 29 | 9.64 003 | 35 | 10.35 997 | 9.96 212 | 6 | 25 | |
| 36 | 9.60 244 | 29 | 9.64 037 | 34 | 10.35 963 | 9.96 207 | 5 | 24 | |
| 37 | 9.60 273 | 29 | 9.64 072 | 35 | 10.35 928 | 9.96 201 | 6 | 23 | |
| 38 | 9.60 302 | 29 | 9.64 106 | 34 | 10.35 894 | 9.96 196 | 5 | 22 | |
| 39 | 9.60 331 | 29 | 9.64 140 | 34 | 10.35 860 | 9.96 190 | 6 | 21 | |
| 40 | 9.60 359 | 28 | 9.64 175 | 35 | 10.35 825 | 9.96 185 | 5 | 20 | |
| 41 | 9.60 388 | 29 | 9.64 209 | 34 | 10.35 791 | 9.96 179 | 6 | 19 | |
| 42 | 9.60 417 | 29 | 9.64 243 | 34 | 10.35 757 | 9.96 174 | 5 | 18 | |
| 43 | 9.60 446 | 29 | 9.64 278 | 35 | 10.35 722 | 9.96 168 | 6 | 17 | |
| 44 | 9.60 474 | 28 | 9.64 312 | 34 | 10.35 688 | 9.96 162 | 6 | 16 | |
| 45 | 9.60 503 | 29 | 9.64 346 | 34 | 10.35 654 | 9.96 157 | 5 | 15 | |
| 46 | 9.60 532 | 29 | 9.64 381 | 35 | 10.35 619 | 9.96 151 | 6 | 14 | |
| 47 | 9.60 561 | 29 | 9.64 415 | 34 | 10.35 585 | 9.96 146 | 5 | 13 | |
| 48 | 9.60 589 | 28 | 9.64 449 | 34 | 10.35 551 | 9.96 140 | 6 | 12 | |
| 49 | 9.60 618 | 29 | 9.64 483 | 34 | 10.35 517 | 9.96 135 | 5 | 11 | |
| 50 | 9.60 646 | 28 | 9.64 517 | 34 | 10.35 483 | 9.96 129 | 6 | 10 | |
| 51 | 9.60 675 | 29 | 9.64 552 | 35 | 10.35 448 | 9.96 123 | 6 | 9 | |
| 52 | 9.60 704 | 29 | 9.64 586 | 34 | 10.35 414 | 9.96 118 | 5 | 8 | |
| 53 | 9.60 732 | 28 | 9.64 620 | 34 | 10.35 380 | 9.96 112 | 6 | 7 | |
| 54 | 9.60 761 | 29 | 9.64 654 | 34 | 10.35 346 | 9.96 107 | 5 | 6 | |
| 55 | 9.60 789 | 28 | 9.64 688 | 34 | 10.35 312 | 9.96 101 | 6 | 5 | |
| 56 | 9.60 818 | 29 | 9.64 722 | 34 | 10.35 278 | 9.96 095 | 6 | 4 | |
| 57 | 9.60 846 | 28 | 9.64 756 | 34 | 10.35 244 | 9.96 090 | 5 | 3 | |
| 58 | 9.60 875 | 29 | 9.64 790 | 34 | 10.35 210 | 9.96 084 | 6 | 2 | |
| 59 | 9.60 903 | 28 | 9.64 824 | 34 | 10.35 176 | 9.96 079 | 5 | 1 | |
| 60 | 9.60 931 | 28 | 9.64 858 | 34 | 10.35 142 | 9.96 073 | 6 | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 36 | 35 | 34 |
|---|---|---|---|
| 6 | 3.6 | 3.5 | 3.4 |
| 7 | 4.2 | 4.1 | 4.0 |
| 8 | 4.8 | 4.7 | 4.5 |
| 9 | 5.4 | 5.2 | 5.1 |
| 10 | 6.0 | 5.8 | 5.7 |
| 20 | 12.0 | 11.7 | 11.3 |
| 30 | 18.0 | 17.5 | 17.0 |
| 40 | 24.0 | 23.3 | 22.7 |
| 50 | 30.0 | 29.2 | 28.3 |

| ″ | 30 | 29 | 28 |
|---|---|---|---|
| 6 | 3.0 | 2.9 | 2.8 |
| 7 | 3.5 | 3.4 | 3.3 |
| 8 | 4.0 | 3.9 | 3.7 |
| 9 | 4.5 | 4.4 | 4.2 |
| 10 | 5.0 | 4.8 | 4.7 |
| 20 | 10.0 | 9.7 | 9.3 |
| 30 | 15.0 | 14.5 | 14.0 |
| 40 | 20.0 | 19.3 | 18.7 |
| 50 | 25.0 | 24.2 | 23.3 |

| ″ | 6 | 5 |
|---|---|---|
| 6 | 0.6 | 0.5 |
| 7 | 0.7 | 0.6 |
| 8 | 0.8 | 0.7 |
| 9 | 0.9 | 0.8 |
| 10 | 1.0 | 0.8 |
| 20 | 2.0 | 1.7 |
| 30 | 3.0 | 2.5 |
| 40 | 4.0 | 3.3 |
| 50 | 5.0 | 4 2 |

## 66°

## 24°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.60 931 | | 9.64 858 | | 10.35 142 | 9.96 073 | | 60 | |
| 1 | 9.60 960 | 29 | 9.64 892 | 34 | 10.35 108 | 9.96 067 | 6 | 59 | |
| 2 | 9.60 988 | 28 | 9.64 926 | 34 | 10.35 074 | 9.96 062 | 5 | 58 | |
| 3 | 9.61 016 | 28 | 9.64 960 | 34 | 10.35 040 | 9.96 056 | 6 | 57 | |
| 4 | 9.61 045 | 29 | 9.64 994 | 34 | 10.35 006 | 9.96 050 | 6 | 56 | |
| 5 | 9.61 073 | 28 | 9.65 028 | 34 | 10.34 972 | 9.96 045 | 5 | 55 | |
| 6 | 9.61 101 | 28 | 9.65 062 | 34 | 10.34 938 | 9.96 039 | 6 | 54 | |
| 7 | 9.61 129 | 28 | 9.65 096 | 34 | 10.34 904 | 9.96 034 | 5 | 53 | |
| 8 | 9.61 158 | 29 | 9.65 130 | 34 | 10.34 870 | 9.96 028 | 6 | 52 | |
| 9 | 9.61 186 | 28 | 9.65 164 | 34 | 10.34 836 | 9.96 022 | 6 | 51 | |
| 10 | 9.61 214 | 28 | 9.65 197 | 33 | 10.34 803 | 9.96 017 | 5 | 50 | |
| 11 | 9.61 242 | 28 | 9.65 231 | 34 | 10.34 769 | 9.96 011 | 6 | 49 | |
| 12 | 9.61 270 | 28 | 9.65 265 | 34 | 10.34 735 | 9.96 005 | 6 | 48 | |
| 13 | 9.61 298 | 28 | 9.65 299 | 34 | 10.34 701 | 9.96 000 | 5 | 47 | |
| 14 | 9.61 326 | 28 | 9.65 333 | 34 | 10.34 667 | 9.95 994 | 6 | 46 | |
| 15 | 9.61 354 | 28 | 9.65 366 | 33 | 10.34 634 | 9.95 988 | 6 | 45 | |
| 16 | 9.61 382 | 28 | 9.65 400 | 34 | 10.34 600 | 9.95 982 | 6 | 44 | |
| 17 | 9.61 411 | 29 | 9.65 434 | 34 | 10.34 566 | 9.95 977 | 5 | 43 | |
| 18 | 9.61 438 | 27 | 9.65 467 | 33 | 10.34 533 | 9.95 971 | 6 | 42 | |
| 19 | 9.61 466 | 28 | 9.65 501 | 34 | 10.34 499 | 9.95 965 | 6 | 41 | |
| 20 | 9.61 494 | 28 | 9.65 535 | 34 | 10.34 465 | 9.95 960 | 5 | 40 | |
| 21 | 9.61 522 | 28 | 9.65 568 | 33 | 10.34 432 | 9.95 954 | 6 | 39 | |
| 22 | 9.61 550 | 28 | 9.65 602 | 34 | 10.34 398 | 9.95 948 | 6 | 38 | |
| 23 | 9.61 578 | 28 | 9.65 636 | 34 | 10.34 364 | 9.95 942 | 6 | 37 | |
| 24 | 9.61 606 | 28 | 9.65 669 | 33 | 10.34 331 | 9.95 937 | 5 | 36 | |
| 25 | 9.61 634 | 28 | 9.65 703 | 34 | 10.34 297 | 9.95 931 | 6 | 35 | |
| 26 | 9.61 662 | 28 | 9.65 736 | 33 | 10.34 264 | 9.95 925 | 6 | 34 | |
| 27 | 9.61 689 | 27 | 9.65 770 | 34 | 10.34 230 | 9.95 920 | 5 | 33 | |
| 28 | 9.61 717 | 28 | 9.65 803 | 33 | 10.34 197 | 9.95 914 | 6 | 32 | |
| 29 | 9.61 745 | 28 | 9.65 837 | 34 | 10.34 163 | 9.95 908 | 6 | 31 | |
| 30 | 9.61 773 | 28 | 9.65 870 | 33 | 10.34 130 | 9.95 902 | 6 | 30 | |
| 31 | 9.61 800 | 27 | 9.65 904 | 34 | 10.34 096 | 9.95 897 | 5 | 29 | |
| 32 | 9.61 828 | 28 | 9.65 937 | 33 | 10.34 063 | 9.95 891 | 6 | 28 | |
| 33 | 9.61 856 | 28 | 9.65 971 | 34 | 10.34 029 | 9.95 885 | 6 | 27 | |
| 34 | 9.61 883 | 27 | 9.66 004 | 33 | 10.33 996 | 9.95 879 | 6 | 26 | |
| 35 | 9.61 911 | 28 | 9.66 038 | 34 | 10.33 962 | 9.95 873 | 6 | 25 | |
| 36 | 9.61 939 | 28 | 9.66 071 | 33 | 10.33 929 | 9.95 868 | 5 | 24 | |
| 37 | 9.61 966 | 27 | 9.66 104 | 33 | 10.33 896 | 9.95 862 | 6 | 23 | |
| 38 | 9.61 994 | 28 | 9.66 138 | 34 | 10.33 862 | 9.95 856 | 6 | 22 | |
| 39 | 9.62 021 | 27 | 9.66 171 | 33 | 10.33 829 | 9.95 850 | 6 | 21 | |
| 40 | 9.62 049 | 28 | 9.66 204 | 33 | 10.33 796 | 9.95 844 | 6 | 20 | |
| 41 | 9.62 076 | 27 | 9.66 238 | 34 | 10.33 762 | 9.95 839 | 5 | 19 | |
| 42 | 9.62 104 | 28 | 9.66 271 | 33 | 10.33 729 | 9.95 833 | 6 | 18 | |
| 43 | 9.62 131 | 27 | 9.66 304 | 33 | 10.33 696 | 9.95 827 | 6 | 17 | |
| 44 | 9.62 159 | 28 | 9.66 337 | 33 | 10.33 663 | 9.95 821 | 6 | 16 | |
| 45 | 9.62 186 | 27 | 9.66 371 | 34 | 10.33 629 | 9.95 815 | 6 | 15 | |
| 46 | 9.62 214 | 28 | 9.66 404 | 33 | 10.33 596 | 9.95 810 | 5 | 14 | |
| 47 | 9.62 241 | 27 | 9.66 437 | 33 | 10.33 563 | 9.95 804 | 6 | 13 | |
| 48 | 9.62 268 | 27 | 9.66 470 | 33 | 10.33 530 | 9.95 798 | 6 | 12 | |
| 49 | 9.62 296 | 28 | 9.66 503 | 33 | 10.33 497 | 9.95 792 | 6 | 11 | |
| 50 | 9.62 323 | 27 | 9.66 537 | 34 | 10.33 463 | 9.95 786 | 6 | 10 | |
| 51 | 9.62 350 | 27 | 9.66 570 | 33 | 10.33 430 | 9.95 780 | 6 | 9 | |
| 52 | 9.62 377 | 27 | 9.66 603 | 33 | 10.33 397 | 9.95 775 | 5 | 8 | |
| 53 | 9.62 405 | 28 | 9.66 636 | 33 | 10.33 364 | 9.95 769 | 6 | 7 | |
| 54 | 9.62 432 | 27 | 9.66 669 | 33 | 10.33 331 | 9.95 763 | 6 | 6 | |
| 55 | 9.62 459 | 27 | 9.66 702 | 33 | 10.33 298 | 9.95 757 | 6 | 5 | |
| 56 | 9.62 486 | 27 | 9.66 735 | 33 | 10.33 265 | 9.95 751 | 6 | 4 | |
| 57 | 9.62 513 | 27 | 9.66 768 | 33 | 10.33 232 | 9.95 745 | 6 | 3 | |
| 58 | 9.62 541 | 28 | 9.66 801 | 33 | 10.33 199 | 9.95 739 | 6 | 2 | |
| 59 | 9.62 568 | 27 | 9.66 834 | 33 | 10.33 166 | 9.95 733 | 6 | 1 | |
| 60 | 9.62 595 | 27 | 9.66 867 | 33 | 10.33 133 | 9.95 728 | 5 | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 34 | 33 |
|---|---|---|
| 6 | 3.4 | 3.3 |
| 7 | 4.0 | 3.8 |
| 8 | 4.5 | 4.4 |
| 9 | 5.1 | 5.0 |
| 10 | 5.7 | 5.5 |
| 20 | 11.3 | 11.0 |
| 30 | 17.0 | 16.5 |
| 40 | 22.7 | 22.0 |
| 50 | 28.3 | 27.5 |

| ″ | 29 | 28 | 27 |
|---|---|---|---|
| 6 | 2.9 | 2.8 | 2.7 |
| 7 | 3.4 | 3.3 | 3.2 |
| 8 | 3.9 | 3.7 | 3.6 |
| 9 | 4.4 | 4.2 | 4.0 |
| 10 | 4.8 | 4.7 | 4.5 |
| 20 | 9.7 | 9.3 | 9.0 |
| 30 | 14.5 | 14.0 | 13.5 |
| 40 | 19.3 | 18.7 | 18.0 |
| 50 | 24.2 | 23.3 | 22.5 |

| ″ | 6 | 5 |
|---|---|---|
| 6 | 0.6 | 0.5 |
| 7 | 0.7 | 0.6 |
| 8 | 0.8 | 0.7 |
| 9 | 0.9 | 0.8 |
| 10 | 1.0 | 0.8 |
| 20 | 2.0 | 1.7 |
| 30 | 3.0 | 2.5 |
| 40 | 4.0 | 3.3 |
| 50 | 5.0 | 4.2 |

## 65°

## 25°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.62 595 | | 9.66 867 | | 10.33 133 | 9.95 728 | | **60** |
| 1 | 9.62 622 | 27 | 9.66 900 | 33 | 10.33 100 | 9.95 722 | 6 | 59 |
| 2 | 9.62 649 | 27 | 9.66 933 | 33 | 10.33 067 | 9.95 716 | 6 | 58 |
| 3 | 9.62 676 | 27 | 9.66 966 | 33 | 10.33 034 | 9.95 710 | 6 | 57 |
| 4 | 9.62 703 | 27 | 9.66 999 | 33 | 10.33 001 | 9.95 704 | 6 | 56 |
| 5 | 9.62 730 | 27 | 9.67 032 | 33 | 10.32 968 | 9.95 698 | 6 | 55 |
| 6 | 9.62 757 | 27 | 9.67 065 | 33 | 10.32 935 | 9.95 692 | 6 | 54 |
| 7 | 9.62 784 | 27 | 9.67 098 | 33 | 10.32 902 | 9.95 686 | 6 | 53 |
| 8 | 9.62 811 | 27 | 9.67 131 | 32 | 10.32 869 | 9.95 680 | 6 | 52 |
| 9 | 9.62 838 | 27 | 9.67 163 | 33 | 10.32 837 | 9.95 674 | 6 | 51 |
| **10** | 9.62 865 | 27 | 9.67 196 | 33 | 10.32 804 | 9.95 668 | 6 | **50** |
| 11 | 9.62 892 | 27 | 9.67 229 | 33 | 10.32 771 | 9.95 663 | 5 | 49 |
| 12 | 9.62 918 | 26 | 9.67 262 | 33 | 10.32 738 | 9.95 657 | 6 | 48 |
| 13 | 9.62 945 | 27 | 9.67 295 | 32 | 10.32 705 | 9.95 651 | 6 | 47 |
| 14 | 9.62 972 | 27 | 9.67 327 | 33 | 10.32 673 | 9.95 645 | 6 | 46 |
| 15 | 9.62 999 | 27 | 9.67 360 | 33 | 10.32 640 | 9.95 639 | 6 | 45 |
| 16 | 9.63 026 | 26 | 9.67 393 | 33 | 10.32 607 | 9.95 633 | 6 | 44 |
| 17 | 9.63 052 | 27 | 9.67 426 | 32 | 10.32 574 | 9.95 627 | 6 | 43 |
| 18 | 9.63 079 | 27 | 9.67 458 | 33 | 10.32 542 | 9.95 621 | 6 | 42 |
| 19 | 9.63 106 | 27 | 9.67 491 | 33 | 10.32 509 | 9.95 615 | 6 | 41 |
| **20** | 9.63 133 | 26 | 9.67 524 | 32 | 10.32 476 | 9.95 609 | 6 | **40** |
| 21 | 9.63 159 | 27 | 9.67 556 | 33 | 10.32 444 | 9.95 603 | 6 | 39 |
| 22 | 9.63 186 | 27 | 9.67 589 | 33 | 10.32 411 | 9.95 597 | 6 | 38 |
| 23 | 9.63 213 | 26 | 9.67 622 | 32 | 10.32 378 | 9.95 591 | 6 | 37 |
| 24 | 9.63 239 | 27 | 9.67 654 | 33 | 10.32 346 | 9.95 585 | 6 | 36 |
| 25 | 9.63 266 | 26 | 9.67 687 | 32 | 10.32 313 | 9.95 579 | 6 | 35 |
| 26 | 9.63 292 | 27 | 9.67 719 | 33 | 10.32 281 | 9.95 573 | 6 | 34 |
| 27 | 9.63 319 | 26 | 9.67 752 | 33 | 10.32 248 | 9.95 567 | 6 | 33 |
| 28 | 9.63 345 | 27 | 9.67 785 | 32 | 10.32 215 | 9.95 561 | 6 | 32 |
| 29 | 9.63 372 | 26 | 9.67 817 | 33 | 10.32 183 | 9.95 555 | 6 | 31 |
| **30** | 9.63 398 | 27 | 9.67 850 | 32 | 10.32 150 | 9.95 549 | 6 | **30** |
| 31 | 9.63 425 | 26 | 9.67 882 | 33 | 10.32 118 | 9.95 543 | 6 | 29 |
| 32 | 9.63 451 | 27 | 9.67 915 | 32 | 10.32 085 | 9.95 537 | 6 | 28 |
| 33 | 9.63 478 | 26 | 9.67 947 | 33 | 10.32 053 | 9.95 531 | 6 | 27 |
| 34 | 9.63 504 | 27 | 9.67 980 | 32 | 10.32 020 | 9.95 525 | 6 | 26 |
| 35 | 9.63 531 | 26 | 9.68 012 | 32 | 10.31 988 | 9.95 519 | 6 | 25 |
| 36 | 9.63 557 | 26 | 9.68 044 | 33 | 10.31 956 | 9.95 513 | 6 | 24 |
| 37 | 9.63 583 | 27 | 9.68 077 | 32 | 10.31 923 | 9.95 507 | 7 | 23 |
| 38 | 9.63 610 | 26 | 9.68 109 | 33 | 10.31 891 | 9.95 500 | 6 | 22 |
| 39 | 9.63 636 | 26 | 9.68 142 | 32 | 10.31 858 | 9.95 494 | 6 | 21 |
| **40** | 9.63 662 | 27 | 9.68 174 | 32 | 10.31 826 | 9.95 488 | 6 | **20** |
| 41 | 9.63 689 | 26 | 9.68 206 | 33 | 10.31 794 | 9.95 482 | 6 | 19 |
| 42 | 9.63 715 | 26 | 9.68 239 | 32 | 10.31 761 | 9.95 476 | 6 | 18 |
| 43 | 9.63 741 | 26 | 9.68 271 | 32 | 10.31 729 | 9.95 470 | 6 | 17 |
| 44 | 9.63 767 | 27 | 9.68 303 | 33 | 10.31 697 | 9.95 464 | 6 | 16 |
| 45 | 9.63 794 | 26 | 9.68 336 | 32 | 10.31 664 | 9.95 458 | 6 | 15 |
| 46 | 9.63 820 | 26 | 9.68 368 | 32 | 10.31 632 | 9.95 452 | 6 | 14 |
| 47 | 9.63 846 | 26 | 9.68 400 | 33 | 10.31 600 | 9.95 446 | 6 | 13 |
| 48 | 9.63 872 | 26 | 9.68 432 | 32 | 10.31 568 | 9.95 440 | 6 | 12 |
| 49 | 9.63 898 | 26 | 9.68 465 | 32 | 10.31 535 | 9.95 434 | 7 | 11 |
| **50** | 9.63 924 | 26 | 9.68 497 | 32 | 10.31 503 | 9.95 427 | 6 | **10** |
| 51 | 9.63 950 | 26 | 9.68 529 | 32 | 10.31 471 | 9.95 421 | 6 | 9 |
| 52 | 9.63 976 | 26 | 9.68 561 | 32 | 10.31 439 | 9.95 415 | 6 | 8 |
| 53 | 9.64 002 | 26 | 9.68 593 | 33 | 10.31 407 | 9.95 409 | 6 | 7 |
| 54 | 9.64 028 | 26 | 9.68 626 | 32 | 10.31 374 | 9.95 403 | 6 | 6 |
| 55 | 9.64 054 | 26 | 9.68 658 | 32 | 10.31 342 | 9.95 397 | 6 | 5 |
| 56 | 9.64 080 | 26 | 9.68 690 | 32 | 10.31 310 | 9.95 391 | 7 | 4 |
| 57 | 9.64 106 | 26 | 9.68 722 | 32 | 10.31 278 | 9.95 384 | 6 | 3 |
| 58 | 9.64 132 | 26 | 9.68 754 | 32 | 10.31 246 | 9.95 378 | 6 | 2 |
| 59 | 9.64 158 | 26 | 9.68 786 | 32 | 10.31 214 | 9.95 372 | 6 | 1 |
| **60** | 9.64 184 | | 9.68 818 | | 10.31 182 | 9.95 366 | | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 33 | 32 |
|---|---|---|
| 6 | 3.3 | 3.2 |
| 7 | 3.8 | 3.7 |
| 8 | 4.4 | 4.3 |
| 8 | 5.0 | 4.8 |
| 10 | 5.5 | 5.3 |
| 20 | 11.0 | 10.7 |
| 30 | 16.5 | 16.0 |
| 40 | 22.0 | 21.3 |
| 50 | 27.5 | 26.7 |

| ″ | 27 | 26 |
|---|---|---|
| 6 | 2.7 | 2.6 |
| 7 | 3.2 | 3.0 |
| 8 | 3.6 | 3.5 |
| 9 | 4.0 | 3.9 |
| 10 | 4.5 | 4.3 |
| 20 | 9.0 | 8.7 |
| 30 | 13.5 | 13.0 |
| 40 | 18.0 | 17.3 |
| 50 | 22.5 | 21.7 |

| ″ | 7 | 6 | 5 |
|---|---|---|---|
| 6 | 0.7 | 0.6 | 0.5 |
| 7 | 0.8 | 0.7 | 0.6 |
| 8 | 0.9 | 0.8 | 0.7 |
| 9 | 1.0 | 0.9 | 0.8 |
| 10 | 1.2 | 1.0 | 0.8 |
| 20 | 2.3 | 2.0 | 1.7 |
| 30 | 3.5 | 3.0 | 2.5 |
| 40 | 4.7 | 4.0 | 3.3 |
| 50 | 5.8 | 5.0 | 4.2 |

## 64°

## 26°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.64 184 | | 9.68 818 | | 10.31 182 | 9.95 366 | | **60** |
| 1 | 9.64 210 | 26 | 9.68 850 | 32 | 10.31 150 | 9.95 360 | 6 | 59 |
| 2 | 9.64 236 | 26 | 9.68 882 | 32 | 10.31 118 | 9.95 354 | 6 | 58 |
| 3 | 9.64 262 | 26 | 9.68 914 | 32 | 10.31 086 | 9.95 348 | 6 | 57 |
| 4 | 9.64 288 | 26 | 9.68 946 | 32 | 10.31 054 | 9.95 341 | 7 | 56 |
| 5 | 9.64 313 | 25 | 9.68 978 | 32 | 10.31 022 | 9.95 335 | 6 | 55 |
| 6 | 9.64 339 | 26 | 9.69 010 | 32 | 10.30 990 | 9.95 329 | 6 | 54 |
| 7 | 9.64 365 | 26 | 9.69 042 | 32 | 10.30 958 | 9.95 323 | 6 | 53 |
| 8 | 9.64 391 | 26 | 9.69 074 | 32 | 10.30 926 | 9.95 317 | 6 | 52 |
| 9 | 9.64 417 | 26 | 9.69 106 | 32 | 10.30 894 | 9.95 310 | 7 | 51 |
| **10** | 9.64 442 | 25 | 9.69 138 | 32 | 10.30 862 | 9.95 304 | 6 | **50** |
| 11 | 9.64 468 | 26 | 9.69 170 | 32 | 10.30 830 | 9.95 298 | 6 | 49 |
| 12 | 9.64 494 | 26 | 9.69 202 | 32 | 10.30 798 | 9.95 292 | 6 | 48 |
| 13 | 9.64 519 | 25 | 9.69 234 | 32 | 10.30 766 | 9.95 286 | 6 | 47 |
| 14 | 9.64 545 | 26 | 9.68 266 | 32 | 10.30 734 | 9.95 279 | 7 | 46 |
| 15 | 9.64 571 | 26 | 9.69 298 | 31 | 10.30 702 | 9.95 273 | 6 | 45 |
| 16 | 9.64 596 | 25 | 9.69 329 | 32 | 10.30 671 | 9.95 267 | 6 | 44 |
| 17 | 9.64 622 | 26 | 9.69 361 | 32 | 10.30 639 | 9.95 261 | 6 | 43 |
| 18 | 9.64 647 | 25 | 9.69 393 | 32 | 10.30 607 | 9.95 254 | 7 | 42 |
| 19 | 9.64 673 | 26 | 9.69 425 | 32 | 10.30 575 | 9.95 248 | 6 | 41 |
| **20** | 9.64 698 | 25 | 9.69 457 | 31 | 10.30 543 | 9.95 242 | 6 | **40** |
| 21 | 9.64 724 | 26 | 9.69 488 | 32 | 10.30 512 | 9.95 236 | 6 | 39 |
| 22 | 9.64 749 | 25 | 9.69 520 | 32 | 10.30 480 | 9.95 229 | 7 | 38 |
| 23 | 9.64 775 | 26 | 9.69 552 | 32 | 10.30 448 | 9.95 223 | 6 | 37 |
| 24 | 9.64 800 | 25 | 9.69 584 | 31 | 10.30 416 | 9.95 217 | 6 | 36 |
| 25 | 9.64 826 | 26 | 9.69 615 | 32 | 10.30 385 | 9.95 211 | 6 | 35 |
| 26 | 9.64 851 | 25 | 9.69 647 | 32 | 10.30 353 | 9.95 204 | 7 | 34 |
| 27 | 9.64 877 | 26 | 9.69 679 | 31 | 10.30 321 | 9.95 198 | 6 | 33 |
| 28 | 9.64 902 | 25 | 9.69 710 | 32 | 10.30 290 | 9.95 192 | 6 | 32 |
| 29 | 9.64 927 | 25 | 9.69 742 | 32 | 10.30 258 | 9.95 185 | 7 | 31 |
| **30** | 9.64 953 | 26 | 9.69 774 | 31 | 10.30 226 | 9.95 179 | 6 | **30** |
| 31 | 9.64 978 | 25 | 9.69 805 | 32 | 10.30 195 | 9.95 173 | 6 | 29 |
| 32 | 9.65 003 | 25 | 9.69 837 | 31 | 10.30 163 | 9.95 167 | 6 | 28 |
| 33 | 9.65 029 | 26 | 9.69 868 | 32 | 10.30 132 | 9.95 160 | 7 | 27 |
| 34 | 9.65 054 | 25 | 9.69 900 | 32 | 10.30 100 | 9.95 154 | 6 | 26 |
| 35 | 9.65 079 | 25 | 9.69 932 | 31 | 10.30 068 | 9.95 148 | 6 | 25 |
| 36 | 9.65 104 | 25 | 9.69 963 | 32 | 10.30 037 | 9.95 141 | 7 | 24 |
| 37 | 9.65 130 | 26 | 9.69 995 | 31 | 10.30 005 | 9.95 135 | 6 | 23 |
| 38 | 9.65 155 | 25 | 9.70 026 | 32 | 10.29 974 | 9.95 129 | 6 | 22 |
| 39 | 9.65 180 | 25 | 9.70 058 | 31 | 10.29 942 | 9.95 122 | 7 | 21 |
| **40** | 9.65 205 | 25 | 9.70 089 | 32 | 10.29 911 | 9.95 116 | 6 | **20** |
| 41 | 9.65 230 | 25 | 9.70 121 | 31 | 10.29 879 | 9.95 110 | 6 | 19 |
| 42 | 9.65 255 | 25 | 9.70 152 | 32 | 10.29 848 | 9.95 103 | 7 | 18 |
| 43 | 9.65 281 | 26 | 9.70 184 | 31 | 10.29 816 | 9.95 097 | 6 | 17 |
| 44 | 9.65 306 | 25 | 9.70 215 | 32 | 10.29 785 | 9.95 090 | 7 | 16 |
| 45 | 9.65 331 | 25 | 9.70 247 | 31 | 10.29 753 | 9.95 084 | 6 | 15 |
| 46 | 9.65 356 | 25 | 9.70 278 | 31 | 10.29 722 | 9.95 078 | 6 | 14 |
| 47 | 9.65 381 | 25 | 9.70 309 | 32 | 10.29 691 | 9.95 071 | 7 | 13 |
| 48 | 9.65 406 | 25 | 9.70 341 | 31 | 10.29 659 | 9.95 065 | 6 | 12 |
| 49 | 9.65 431 | 25 | 9.70 372 | 32 | 10.29 628 | 9.95 059 | 6 | 11 |
| **50** | 9.65 456 | 25 | 9.70 404 | 31 | 10.29 596 | 9.95 052 | 7 | **10** |
| 51 | 9.65 481 | 25 | 9.70 435 | 31 | 10.29 565 | 9.95 046 | 6 | 9 |
| 52 | 9.65 506 | 25 | 9.70 466 | 32 | 10.29 534 | 9.95 039 | 7 | 8 |
| 53 | 9.65 531 | 25 | 9.70 498 | 31 | 10.29 502 | 9.95 033 | 6 | 7 |
| 54 | 9.65 556 | 25 | 9.70 529 | 31 | 10.29 471 | 9.95 027 | 6 | 6 |
| 55 | 9.65 580 | 24 | 9.70 560 | 32 | 10.29 440 | 9.95 020 | 7 | 5 |
| 56 | 9.65 605 | 25 | 9.70 592 | 31 | 10.29 408 | 9.95 014 | 6 | 4 |
| 57 | 9.65 630 | 25 | 9.70 623 | 31 | 10.29 377 | 9.95 007 | 7 | 3 |
| 58 | 9.65 655 | 25 | 9.70 654 | 32 | 10.29 346 | 9.95 001 | 6 | 2 |
| 59 | 9.65 680 | 25 | 9.70 685 | 31 | 10.29 315 | 9.94 995 | 6 | 1 |
| **60** | 9.65 705 | 25 | 9.70 717 | 32 | 10.29 283 | 9.94 988 | 7 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 32 | 31 |
|---|---|---|
| 6 | 3.2 | 3.1 |
| 7 | 3.7 | 3.6 |
| 8 | 4.3 | 4.1 |
| 9 | 4.8 | 4.6 |
| 10 | 5.3 | 5.2 |
| 20 | 10.7 | 10.3 |
| 30 | 16.0 | 15.5 |
| 40 | 21.3 | 20.7 |
| 50 | 26.7 | 25.8 |

| ″ | 26 | 25 | 24 |
|---|---|---|---|
| 6 | 2.6 | 2.5 | 2.4 |
| 7 | 3.0 | 2.9 | 2.8 |
| 8 | 3.5 | 3.3 | 3.2 |
| 9 | 3.9 | 3.8 | 3.6 |
| 10 | 4.3 | 4.2 | 4.0 |
| 20 | 8.7 | 8.3 | 8.0 |
| 30 | 13.0 | 12.5 | 12.0 |
| 40 | 17.3 | 16.7 | 16.0 |
| 50 | 21.7 | 20.8 | 20.0 |

| ″ | 7 | 6 |
|---|---|---|
| 6 | 0.7 | 0.6 |
| 7 | 0.8 | 0.7 |
| 8 | 0.9 | 0.8 |
| 9 | 1.0 | 0.9 |
| 10 | 1.2 | 1.0 |
| 20 | 2.3 | 2.0 |
| 30 | 3.5 | 3.0 |
| 40 | 4.7 | 4.0 |
| 50 | 5.8 | 5.0 |

## 63°

## 27°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.65 705 | | 9.70 717 | | 10.29 283 | 9.94 988 | | 60 | |
| 1 | 9.65 729 | 24 | 9.70 748 | 31 | 10.29 252 | 9.94 982 | 6 | 59 | |
| 2 | 9.65 754 | 25 | 9.70 779 | 31 | 10.29 221 | 9.94 975 | 7 | 58 | |
| 3 | 9.65 779 | 25 | 9.70 810 | 31 | 10.29 190 | 9.94 969 | 6 | 57 | |
| 4 | 9.65 804 | 25 | 9.70 841 | 31 | 10.29 159 | 9.94 962 | 7 | 56 | |
| 5 | 9.65 828 | 24 | 9.70 873 | 32 | 10.29 127 | 9.94 956 | 6 | 55 | |
| 6 | 9.65 853 | 25 | 9.70 904 | 31 | 10.29 096 | 9.94 949 | 7 | 45 | |
| 7 | 9.65 878 | 25 | 9.70 935 | 31 | 10.29 065 | 9.94 943 | 6 | 53 | |
| 8 | 9.65 902 | 24 | 9.70 966 | 31 | 10.29 034 | 9.94 936 | 7 | 52 | |
| 9 | 9.65 927 | 25 | 9.70 997 | 31 | 10.29 003 | 9.94 930 | 6 | 51 | |
| 10 | 9.65 952 | 25 | 9.71 028 | 31 | 10.28 972 | 9.94 923 | 7 | 50 | |
| 11 | 9.65 976 | 24 | 9.71 059 | 31 | 10.28 941 | 9.94 917 | 6 | 49 | |
| 12 | 9.66 001 | 25 | 9.71 090 | 31 | 10.28 910 | 9.94 911 | 6 | 48 | |
| 13 | 9.66 025 | 24 | 9.71 121 | 31 | 10.28 879 | 9.94 904 | 7 | 47 | |
| 14 | 9.66 050 | 25 | 9.71 153 | 32 | 10.28 847 | 9.94 898 | 6 | 46 | |
| 15 | 9.66 075 | 25 | 9.71 184 | 31 | 10.28 816 | 9.94 891 | 7 | 45 | |
| 16 | 9.66 099 | 24 | 9.71 215 | 31 | 10.28 785 | 9.94 885 | 6 | 44 | |
| 17 | 9.66 124 | 25 | 9.71 246 | 31 | 10.28 754 | 9.94 878 | 7 | 43 | |
| 18 | 9.66 148 | 24 | 9.71 277 | 31 | 10.28 723 | 9.94 871 | 7 | 42 | |
| 19 | 9.66 173 | 25 | 9.71 308 | 31 | 10.28 692 | 9.94 865 | 6 | 41 | |
| 20 | 9.66 197 | 24 | 9.71 339 | 31 | 10.28 661 | 9.94 858 | 7 | 40 | |
| 21 | 9.66 221 | 24 | 9.71 370 | 31 | 10.28 630 | 9.94 852 | 6 | 39 | |
| 22 | 9.66 246 | 25 | 9.71 401 | 31 | 10.28 599 | 9.94 845 | 7 | 38 | |
| 23 | 9.66 270 | 24 | 9.71 431 | 30 | 10.28 569 | 9.94 839 | 6 | 37 | |
| 24 | 9.66 295 | 25 | 9.71 462 | 31 | 10.28 538 | 9.94 832 | 7 | 36 | |
| 25 | 9.66 319 | 24 | 9.71 493 | 31 | 10.28 507 | 9.94 826 | 6 | 35 | |
| 26 | 9.66 343 | 24 | 9.71 524 | 31 | 10.28 476 | 9.94 819 | 7 | 34 | |
| 27 | 9.66 368 | 25 | 9.71 555 | 31 | 10.28 445 | 9.94 813 | 6 | 33 | |
| 28 | 9.66 392 | 24 | 9.71 586 | 31 | 10.28 414 | 9.94 806 | 7 | 32 | |
| 29 | 9.66 416 | 24 | 9.71 617 | 31 | 10.28 383 | 9.94 799 | 7 | 31 | |
| 30 | 9.66 441 | 25 | 9.71 648 | 31 | 10.28 352 | 9.94 793 | 6 | 30 | |
| 31 | 9.66 465 | 24 | 9.71 679 | 31 | 10.28 321 | 9.94 786 | 7 | 29 | |
| 32 | 9.66 489 | 24 | 9.71 709 | 30 | 10.28 291 | 9.94 780 | 6 | 28 | |
| 33 | 9.66 513 | 24 | 9.71 740 | 31 | 10.28 260 | 9.94 773 | 7 | 27 | |
| 34 | 9.66 537 | 24 | 9.71 771 | 31 | 10.28 229 | 9.94 767 | 6 | 26 | |
| 35 | 9.66 562 | 25 | 9.71 802 | 31 | 10.28 198 | 9.94 760 | 7 | 25 | |
| 36 | 9.66 586 | 24 | 9.71 833 | 31 | 10.28 167 | 9.94 753 | 7 | 24 | |
| 37 | 9.66 610 | 24 | 9.71 863 | 30 | 10.28 137 | 9.94 747 | 6 | 23 | |
| 38 | 9.66 634 | 24 | 9.71 894 | 31 | 10.28 106 | 9.94 740 | 7 | 22 | |
| 39 | 9.66 658 | 24 | 9.71 925 | 31 | 10.28 075 | 9.94 734 | 6 | 21 | |
| 40 | 9.66 682 | 24 | 9.71 955 | 30 | 10.28 045 | 9.94 727 | 7 | 20 | |
| 41 | 9.66 706 | 24 | 9.71 986 | 31 | 10.28 014 | 9.94 720 | 7 | 19 | |
| 42 | 9.66 731 | 25 | 9.72 017 | 31 | 10.27 983 | 9.94 714 | 6 | 18 | |
| 43 | 9.66 755 | 24 | 9.72 048 | 31 | 10.27 952 | 9.94 707 | 7 | 17 | |
| 44 | 9.66 779 | 24 | 9.72 078 | 30 | 10.27 922 | 9.94 700 | 7 | 16 | |
| 45 | 9.66 803 | 24 | 9.72 109 | 31 | 10.27 891 | 9.94 694 | 6 | 15 | |
| 46 | 9.66 827 | 24 | 9.72 140 | 31 | 10.27 860 | 9.94 687 | 7 | 14 | |
| 47 | 9.66 851 | 24 | 9.72 170 | 30 | 10.27 830 | 9.94 680 | 7 | 13 | |
| 48 | 9.66 875 | 24 | 9.72 201 | 31 | 10.27 799 | 9.94 674 | 6 | 12 | |
| 49 | 9.66 899 | 24 | 9.72 231 | 30 | 10.27 769 | 9.94 667 | 7 | 11 | |
| 50 | 9.66 922 | 23 | 9.72 262 | 31 | 10.27 738 | 9.94 660 | 7 | 10 | |
| 51 | 9.66 946 | 24 | 9.72 293 | 31 | 10.27 707 | 9.94 654 | 6 | 9 | |
| 52 | 9.66 970 | 24 | 9.72 323 | 30 | 10.27 677 | 9.94 647 | 7 | 8 | |
| 53 | 9.66 994 | 24 | 9.72 354 | 31 | 10.27 646 | 9.94 640 | 7 | 7 | |
| 54 | 9.67 018 | 24 | 9.72 384 | 30 | 10.27 616 | 9.94 634 | 6 | 6 | |
| 55 | 9.67 042 | 24 | 9.72 415 | 31 | 10.27 585 | 9.94 627 | 7 | 5 | |
| 56 | 9.67 066 | 24 | 9.72 445 | 30 | 10.27 555 | 9.94 620 | 7 | 4 | |
| 57 | 9.67 090 | 24 | 9.72 476 | 31 | 10.27 524 | 9.94 614 | 6 | 3 | |
| 58 | 9.67 113 | 23 | 9.72 506 | 30 | 10.27 494 | 9.94 607 | 7 | 2 | |
| 59 | 9.67 137 | 24 | 9.72 537 | 31 | 10.27 463 | 9.94 600 | 7 | 1 | |
| 60 | 9.67 161 | 24 | 9.72 567 | 30 | 10.27 433 | 9.94 593 | 7 | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 32 | 31 | 30 |
|---|---|---|---|
| 6 | 3.2 | 3.1 | 3.0 |
| 7 | 3.7 | 3.6 | 3.5 |
| 8 | 4.3 | 4.1 | 4.0 |
| 9 | 4.8 | 4.6 | 4.5 |
| 10 | 5.3 | 5.2 | 5.0 |
| 20 | 10.7 | 10.3 | 10.0 |
| 30 | 16.0 | 15.5 | 15.0 |
| 40 | 21.3 | 20.7 | 20.0 |
| 50 | 26.7 | 25.8 | 25.0 |

| ″ | 25 | 24 | 23 |
|---|---|---|---|
| 6 | 2.5 | 2.4 | 2.3 |
| 7 | 2.9 | 2.8 | 2.7 |
| 8 | 3.3 | 3.2 | 3.1 |
| 9 | 3.8 | 3.6 | 3.4 |
| 10 | 4.2 | 4.0 | 3.8 |
| 20 | 8.3 | 8.0 | 7.7 |
| 30 | 12.5 | 12.0 | 11.5 |
| 40 | 16.7 | 16.0 | 15.3 |
| 50 | 20.8 | 20.0 | 19.2 |

| ″ | 7 | 6 |
|---|---|---|
| 6 | 0.7 | 0.6 |
| 7 | 0.8 | 0.7 |
| 8 | 0.9 | 0.8 |
| 9 | 1.0 | 0.9 |
| 10 | 1.2 | 1.0 |
| 20 | 2.3 | 2.0 |
| 30 | 3.5 | 3.0 |
| 40 | 4.7 | 4.0 |
| 50 | 5.8 | 5.0 |

## 62°

## 28°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.67 161 | | 9.72 567 | | 10.27 433 | 9.94 593 | | **60** | |
| 1 | 9.67 185 | 24 | 9.72 598 | 31 | 10.27 402 | 9.94 587 | 6 | 59 | |
| 2 | 9.67 208 | 23 | 9.72 628 | 30 | 10.27 372 | 9.94 580 | 7 | 58 | |
| 3 | 9.67 232 | 24 | 9.72 659 | 31 | 10.27 341 | 9.94 573 | 7 | 57 | |
| 4 | 9.67 256 | 24 | 9.72 689 | 30 | 10.27 311 | 9.94 567 | 6 | 56 | |
| 5 | 9.67 280 | 24 | 9.72 720 | 31 | 10.27 280 | 9.94 560 | 7 | 55 | |
| 6 | 9.67 303 | 23 | 9.72 750 | 30 | 10.27 250 | 9.94 553 | 7 | 54 | |
| 7 | 9.67 327 | 24 | 9.72 780 | 30 | 10.27 220 | 9.94 546 | 7 | 53 | |
| 8 | 9.67 350 | 23 | 9.72 811 | 31 | 10.27 189 | 9.94 540 | 6 | 52 | |
| 9 | 9.67 374 | 24 | 9.72 841 | 30 | 10.27 159 | 9.94 533 | 7 | 51 | |
| **10** | 9.67 398 | 24 | 9.72 872 | 31 | 10.27 128 | 9.94 526 | 7 | **50** | |
| 11 | 9.67 421 | 23 | 9.72 902 | 30 | 10.27 098 | 9.94 519 | 7 | 49 | |
| 12 | 9.67 445 | 24 | 9.72 932 | 30 | 10.27 068 | 9.94 513 | 6 | 48 | |
| 13 | 9.67 468 | 23 | 9.72 963 | 31 | 10.27 037 | 9.94 506 | 7 | 47 | |
| 14 | 9.67 492 | 24 | 9.72 993 | 30 | 10.27 007 | 9.94 499 | 7 | 46 | |
| 15 | 9.67 515 | 23 | 9.73 023 | 30 | 10.26 977 | 9.94 492 | 7 | 45 | |
| 16 | 9.67 539 | 24 | 9.73 054 | 31 | 10.26 946 | 9.94 485 | 7 | 44 | |
| 17 | 9.67 562 | 23 | 9.73 084 | 30 | 10.26 916 | 9.94 479 | 6 | 43 | |
| 18 | 9.67 586 | 24 | 9.73 114 | 30 | 10.26 886 | 9.94 472 | 7 | 42 | |
| 19 | 9.67 609 | 23 | 0.73 144 | 30 | 10.26 856 | 9.94 465 | 7 | 41 | |
| **20** | 9.67 633 | 24 | 9.73 175 | 31 | 10.26 825 | 9.94 458 | 7 | **40** | |
| 21 | 9.67 656 | 23 | 9.73 205 | 30 | 10.26 795 | 9.94 451 | 7 | 39 | |
| 22 | 9.67 680 | 24 | 9.73 235 | 30 | 10.26 765 | 9.94 445 | 6 | 38 | |
| 23 | 9.67 703 | 23 | 9.73 265 | 30 | 10.26 735 | 9.94 438 | 7 | 37 | |
| 24 | 9.67 726 | 23 | 9.73 295 | 30 | 10.26 705 | 9.94 431 | 7 | 36 | |
| 25 | 9.67 750 | 24 | 9.73 326 | 31 | 10.26 674 | 9.94 424 | 7 | 35 | |
| 26 | 9.67 773 | 23 | 9.73 356 | 30 | 10.26 644 | 9.94 417 | 7 | 34 | |
| 27 | 9.67 796 | 23 | 9.73 386 | 30 | 10.26 614 | 9.94 410 | 7 | 33 | |
| 28 | 9.67 820 | 24 | 9.73 416 | 30 | 10.26 584 | 9.94 404 | 6 | 32 | |
| 29 | 9.67 843 | 23 | 9.73 446 | 30 | 10.26 554 | 9.94 397 | 7 | 31 | |
| **30** | 9.67 866 | 23 | 9.73 476 | 30 | 10.26 524 | 9.94 390 | 7 | **30** | |
| 31 | 9.67 890 | 24 | 9.73 507 | 31 | 10.26 493 | 9.94 383 | 7 | 29 | |
| 32 | 9.67 913 | 23 | 9.73 537 | 30 | 10.26 463 | 9.94 376 | 7 | 28 | |
| 33 | 9.67 936 | 23 | 9.73 567 | 30 | 10.26 433 | 9.94 369 | 7 | 27 | |
| 34 | 9.67 959 | 23 | 9.73 597 | 30 | 10.26 403 | 9.94 362 | 7 | 26 | |
| 35 | 9.67 982 | 23 | 9.73 627 | 30 | 10.26 373 | 9.94 355 | 7 | 25 | |
| 36 | 9.68 006 | 24 | 9.73 657 | 30 | 10.26 343 | 9.94 349 | 6 | 24 | |
| 37 | 9.68 029 | 23 | 9.73 687 | 30 | 10.26 313 | 9.94 342 | 7 | 23 | |
| 38 | 9.68 052 | 23 | 9.73 717 | 30 | 10.26 283 | 9.94 335 | 7 | 22 | |
| 39 | 9.68 075 | 23 | 9.73 747 | 30 | 10.26 253 | 9.94 328 | 7 | 21 | |
| **40** | 9.68 098 | 23 | 9.73 777 | 30 | 10.26 223 | 9.94 321 | 7 | **20** | |
| 41 | 9.68 121 | 23 | 9.73 807 | 30 | 10.26 193 | 9.94 314 | 7 | 19 | |
| 42 | 9.68 144 | 23 | 9.73 837 | 30 | 10.26 163 | 9.94 307 | 7 | 18 | |
| 43 | 9.68 167 | 23 | 9.73 867 | 30 | 10.26 133 | 9.94 300 | 7 | 17 | |
| 44 | 9.68 190 | 23 | 9.73 897 | 30 | 10.26 103 | 9.94 293 | 7 | 16 | |
| 45 | 9.68 213 | 23 | 9.73 927 | 30 | 10.26 073 | 9.94 286 | 7 | 15 | |
| 46 | 9.68 237 | 24 | 9.73 957 | 30 | 10.26 043 | 9.94 279 | 7 | 14 | |
| 47 | 9.68 260 | 23 | 9.73 987 | 30 | 10.26 013 | 9.94 273 | 6 | 13 | |
| 48 | 9.68 283 | 23 | 9.74 017 | 30 | 10.25 983 | 9.94 266 | 7 | 12 | |
| 49 | 9.68 305 | 22 | 9.74 047 | 30 | 10.25 953 | 9.94 259 | 7 | 11 | |
| **50** | 9.68 328 | 23 | 9.74 077 | 30 | 10.25 923 | 9.94 252 | 7 | **10** | |
| 51 | 9.68 351 | 23 | 9.74 107 | 30 | 10.25 893 | 9.94 245 | 7 | 9 | |
| 52 | 9.68 374 | 23 | 9.74 137 | 30 | 10.25 863 | 9.94 238 | 7 | 8 | |
| 53 | 9.68 397 | 23 | 9.74 166 | 29 | 10.25 834 | 9.94 231 | 7 | 7 | |
| 54 | 9.68 420 | 23 | 9.74 196 | 30 | 10.25 804 | 9.94 224 | 7 | 6 | |
| 55 | 9.68 443 | 23 | 9.74 226 | 30 | 10.25 774 | 9.94 217 | 7 | 5 | |
| 56 | 9.68 466 | 23 | 9.74 256 | 30 | 10.25 744 | 9.94 210 | 7 | 4 | |
| 57 | 9.68 489 | 23 | 9.74 286 | 30 | 10.25 714 | 9.94 203 | 7 | 3 | |
| 58 | 9.68 512 | 23 | 9.74 316 | 30 | 10.25 684 | 9.94 196 | 7 | 2 | |
| 59 | 9.68 534 | 22 | 9.74 345 | 29 | 10.25 655 | 9.94 189 | 7 | 1 | |
| **60** | 9.68 557 | 23 | 9.74 375 | 30 | 10.25 625 | 9.94 182 | 7 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 31 | 30 | 29 |
|---|---|---|---|
| 6 | 3.1 | 3.0 | 2.9 |
| 7 | 3.6 | 3.5 | 3.4 |
| 8 | 4.1 | 4.0 | 3.9 |
| 9 | 4.6 | 4.5 | 4.4 |
| 10 | 5.2 | 5.0 | 4.8 |
| 20 | 10.3 | 10.0 | 9.7 |
| 30 | 15.5 | 15.0 | 14.5 |
| 40 | 20.7 | 20.0 | 19.3 |
| 50 | 25.8 | 25.0 | 24.2 |

| ″ | 24 | 23 | 22 |
|---|---|---|---|
| 6 | 2.4 | 2.3 | 2.2 |
| 7 | 2.8 | 2.7 | 2.6 |
| 8 | 3.2 | 3.1 | 2.9 |
| 9 | 3.6 | 3.4 | 3.3 |
| 10 | 4.0 | 3.8 | 3.7 |
| 20 | 8.0 | 7.7 | 7.3 |
| 30 | 12.0 | 11.5 | 11.0 |
| 40 | 16.0 | 15.3 | 14.7 |
| 50 | 20.0 | 19.2 | 18.3 |

| ″ | 7 | 6 |
|---|---|---|
| 6 | 0.7 | 0.6 |
| 7 | 0.8 | 0.7 |
| 8 | 0.9 | 0.8 |
| 9 | 1.0 | 0.9 |
| 10 | 1.2 | 1.0 |
| 20 | 2.3 | 2.0 |
| 30 | 3.5 | 3.0 |
| 40 | 4.7 | 4.0 |
| 50 | 5.8 | 5.0 |

## 61°

## 29°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.68 557 | 23 | 9.74 375 | 30 | 10.25 625 | 9.94 182 | 7 | **60** |
| 1 | 9.68 580 | 23 | 9.74 405 | 30 | 10.25 595 | 9.94 175 | 7 | 59 |
| 2 | 9.68 603 | 22 | 9.74 435 | 30 | 10.25 565 | 9.94 168 | 7 | 58 |
| 3 | 9.68 625 | 23 | 9.74 465 | 29 | 10.25 535 | 9.94 161 | 7 | 57 |
| 4 | 9.68 648 | 23 | 9.74 494 | 30 | 10.25 506 | 9.94 154 | 7 | 56 |
| 5 | 9.68 671 | 23 | 9.74 524 | 30 | 10.25 476 | 9.94 147 | 7 | 55 |
| 6 | 9.68 694 | 22 | 9.74 554 | 29 | 10.25 446 | 9.94 140 | 7 | 54 |
| 7 | 9.68 716 | 23 | 9.74 583 | 30 | 10.25 417 | 9.94 133 | 7 | 53 |
| 8 | 9.68 739 | 23 | 9.74 613 | 30 | 10.25 387 | 9.94 126 | 7 | 52 |
| 9 | 9.68 762 | 22 | 9.74 643 | 30 | 10.25 357 | 9.94 119 | 7 | 51 |
| **10** | 9.68 784 | 23 | 9.74 673 | 29 | 10.25 327 | 9.94 112 | 7 | **50** |
| 11 | 9.68 807 | 22 | 9.74 702 | 30 | 10.25 298 | 9.94 105 | 7 | 49 |
| 12 | 9.68 829 | 23 | 9.74 732 | 30 | 10.25 268 | 9.94 098 | 8 | 48 |
| 13 | 9.68 852 | 23 | 9.74 762 | 29 | 10.25 238 | 9.94 090 | 7 | 47 |
| 14 | 9.68 875 | 22 | 9.74 791 | 30 | 10.25 209 | 9.94 083 | 7 | 46 |
| 15 | 9.68 897 | 23 | 9.74 821 | 30 | 10.25 179 | 9.94 076 | 7 | 45 |
| 16 | 9.68 920 | 22 | 9.74 851 | 29 | 10.25 149 | 9.94 069 | 7 | 44 |
| 17 | 9.68 942 | 23 | 9.74 880 | 30 | 10.25 120 | 9.94 062 | 7 | 43 |
| 18 | 9.68 965 | 22 | 9.74 910 | 29 | 10.25 090 | 9.94 055 | 7 | 42 |
| 19 | 9.68 987 | 23 | 9.74 939 | 30 | 10.25 061 | 9.94 048 | 7 | 41 |
| **20** | 9.69 010 | 22 | 9.74 969 | 29 | 10.25 031 | 9.94 041 | 7 | **40** |
| 21 | 9.69 032 | 23 | 9.74 998 | 30 | 10.25 002 | 9.94 034 | 7 | 39 |
| 22 | 9.69 055 | 22 | 9.75 028 | 30 | 10.24 972 | 9.94 027 | 7 | 38 |
| 23 | 9.69 077 | 23 | 9.75 058 | 29 | 10.24 942 | 9.94 020 | 8 | 37 |
| 24 | 9.69 100 | 22 | 9.75 087 | 30 | 10.24 913 | 9.94 012 | 7 | 36 |
| 25 | 9.69 122 | 22 | 9.75 117 | 29 | 10.24 883 | 9.94 005 | 7 | 35 |
| 26 | 9.69 144 | 23 | 9.75 146 | 30 | 10.24 854 | 9.93 998 | 7 | 34 |
| 27 | 9.69 167 | 22 | 9.75 176 | 29 | 10.24 824 | 9.93 991 | 7 | 33 |
| 28 | 9.69 189 | 23 | 9.75 205 | 30 | 10.24 795 | 9.93 984 | 7 | 32 |
| 29 | 9.69 212 | 22 | 9.75 235 | 29 | 10.24 765 | 9.93 977 | 7 | 31 |
| **30** | 9.69 234 | 22 | 9.75 264 | 30 | 10.24 736 | 9.93 970 | 7 | **30** |
| 31 | 9.69 256 | 23 | 9.75 294 | 29 | 10.24 706 | 9.93 963 | 8 | 29 |
| 32 | 9.69 279 | 22 | 9.75 323 | 30 | 10.24 677 | 9.93 955 | 7 | 28 |
| 33 | 9.69 301 | 22 | 9.75 353 | 29 | 10.24 647 | 9.93 948 | 7 | 27 |
| 34 | 9.69 323 | 22 | 9.75 382 | 29 | 10.24 618 | 9.93 941 | 7 | 26 |
| 35 | 9.69 345 | 23 | 9.75 411 | 30 | 10.24 589 | 9.93 934 | 7 | 25 |
| 36 | 9.69 368 | 22 | 9.75 441 | 29 | 10.24 559 | 9.93 927 | 7 | 24 |
| 37 | 9.69 390 | 22 | 9.75 470 | 30 | 10.24 530 | 9.93 920 | 8 | 23 |
| 38 | 9.69 412 | 22 | 9.75 500 | 29 | 10.24 500 | 9.93 912 | 7 | 22 |
| 39 | 9.69 434 | 22 | 9.75 529 | 29 | 10.24 471 | 9.93 905 | 7 | 21 |
| **40** | 9.69 456 | 23 | 9.75 558 | 30 | 10.24 442 | 9.93 898 | 7 | **20** |
| 41 | 9.69 479 | 22 | 9.75 588 | 29 | 10.24 412 | 9.93 891 | 7 | 19 |
| 42 | 9.69 501 | 22 | 9.75 617 | 30 | 10.24 383 | 9.93 884 | 8 | 18 |
| 43 | 9.69 523 | 22 | 9.75 647 | 29 | 10.24 353 | 9.93 876 | 7 | 17 |
| 44 | 9.69 545 | 22 | 9.75 676 | 29 | 10.24 324 | 9.93 869 | 7 | 16 |
| 45 | 9.69 567 | 22 | 9.75 705 | 30 | 10.24 295 | 9.93 862 | 7 | 15 |
| 46 | 9.69 589 | 22 | 9.75 735 | 29 | 10.24 265 | 9.93 855 | 8 | 14 |
| 47 | 9.69 611 | 22 | 9.75 764 | 29 | 10.24 236 | 9.93 847 | 7 | 13 |
| 48 | 9.69 633 | 22 | 9.75 793 | 29 | 10.24 207 | 9.93 840 | 7 | 12 |
| 49 | 9.69 655 | 22 | 9.75 822 | 30 | 10.24 178 | 9.93 833 | 7 | 11 |
| **50** | 9.69 677 | 22 | 9.75 852 | 29 | 10.24 148 | 9.93 826 | 7 | **10** |
| 51 | 9.69 699 | 22 | 9.75 881 | 29 | 10.24 119 | 9.93 819 | 8 | 9 |
| 52 | 9.69 721 | 22 | 9.75 910 | 29 | 10.24 090 | 9.93 811 | 7 | 8 |
| 53 | 9.69 743 | 22 | 9.75 939 | 30 | 10.24 061 | 9.93 804 | 7 | 7 |
| 54 | 9.69 765 | 22 | 9.75 969 | 29 | 10.24 031 | 9.93 797 | 8 | 6 |
| 55 | 9.69 787 | 22 | 9.75 998 | 29 | 10.24 002 | 9.93 789 | 7 | 5 |
| 56 | 9.69 809 | 22 | 9.76 027 | 29 | 10.23 973 | 9.93 782 | 7 | 4 |
| 57 | 9.69 831 | 22 | 9.76 056 | 30 | 10.23 944 | 9.93 775 | 7 | 3 |
| 58 | 9.69 853 | 22 | 9.76 086 | 29 | 10.23 914 | 9.93 768 | 8 | 2 |
| 59 | 9.69 875 | 22 | 9.76 115 | 29 | 10.23 885 | 9.93 760 | 7 | 1 |
| **60** | 9.69 897 | | 9.76 144 | | 10.23 856 | 9.93 753 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

**Prop. Pts.**

| ″ | 30 | 29 |
|---|---|---|
| 6 | 3.0 | 2.9 |
| 7 | 3.5 | 3.5 |
| 8 | 4.0 | 3.9 |
| 9 | 4.5 | 4.4 |
| 10 | 5.0 | 4.8 |
| 20 | 10.0 | 9.7 |
| 30 | 15.0 | 14.5 |
| 40 | 20.0 | 19.3 |
| 50 | 25.0 | 24.2 |

| ″ | 23 | 22 |
|---|---|---|
| 6 | 2.3 | 2.2 |
| 7 | 2.7 | 2.6 |
| 8 | 3.1 | 2.9 |
| 9 | 3.4 | 3.3 |
| 10 | 3.8 | 3.7 |
| 20 | 7.7 | 7.3 |
| 30 | 11.5 | 11.0 |
| 40 | 15.3 | 14.7 |
| 50 | 19.2 | 18.3 |

| ″ | 8 | 7 |
|---|---|---|
| 6 | 0.8 | 0.7 |
| 7 | 0.9 | 0.8 |
| 8 | 1.1 | 0.9 |
| 9 | 1.2 | 1.0 |
| 10 | 1.3 | 1.2 |
| 20 | 2.7 | 2.3 |
| 30 | 4.0 | 3.5 |
| 40 | 5.3 | 4.7 |
| 50 | 6.7 | 5.8 |

## 60°

## 30°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.69 897 | | 9.76 144 | | 10.23 856 | 9.93 753 | | 60 |
| 1 | 9.69 919 | 22 | 9.76 173 | 29 | 10.23 827 | 9.93 746 | 7 | 59 |
| 2 | 9.69 941 | 22 | 9.76 202 | 29 | 10.23 798 | 9.93 738 | 8 | 58 |
| 3 | 9.69 963 | 22 | 9.76 231 | 29 | 10.23 769 | 9.93 731 | 7 | 57 |
| 4 | 9.69 984 | 21 | 9.76 261 | 30 | 10.23 739 | 9.93 724 | 7 | 56 |
| 5 | 9.70 006 | 22 | 9.76 290 | 29 | 10.23 710 | 9.93 717 | 7 | 55 |
| 6 | 9.70 028 | 22 | 9.76 319 | 29 | 10.23 681 | 9.93 709 | 8 | 54 |
| 7 | 9.70 050 | 22 | 9.76 348 | 29 | 10.23 652 | 9.93 702 | 7 | 53 |
| 8 | 9.70 072 | 22 | 9.76 377 | 29 | 10.23 623 | 9.93 695 | 7 | 52 |
| 9 | 9.70 093 | 21 | 9.76 406 | 29 | 10.23 594 | 9.93 687 | 8 | 51 |
| 10 | 9.70 115 | 22 | 9.76 435 | 29 | 10.23 565 | 9.93 680 | 7 | 50 |
| 11 | 9.70 137 | 22 | 9.76 464 | 29 | 10.23 536 | 9.93 673 | 7 | 49 |
| 12 | 9.70 159 | 22 | 9.76 493 | 29 | 10.23 507 | 9.93 665 | 8 | 48 |
| 13 | 9.70 180 | 21 | 9.76 522 | 29 | 10.23 478 | 9.93 658 | 7 | 47 |
| 14 | 9.70 202 | 22 | 9.76 551 | 29 | 10.23 449 | 9.93 650 | 8 | 46 |
| 15 | 9.70 224 | 22 | 9.76 580 | 29 | 10.23 420 | 9.93 643 | 7 | 45 |
| 16 | 9.70 245 | 21 | 9.76 609 | 29 | 10.23 391 | 9.93 636 | 7 | 44 |
| 17 | 9.70 267 | 22 | 9.76 639 | 30 | 10.23 361 | 9.93 628 | 8 | 43 |
| 18 | 9.70 288 | 21 | 9.76 668 | 29 | 10.23 332 | 9.93 621 | 7 | 42 |
| 19 | 9.70 310 | 22 | 9.76 697 | 29 | 10.23 303 | 9.93 614 | 7 | 41 |
| 20 | 9.70 332 | 22 | 9.76 725 | 28 | 10.23 275 | 9.93 606 | 8 | 40 |
| 21 | 9.70 353 | 21 | 9.76 754 | 29 | 10.23 246 | 9.93 599 | 7 | 39 |
| 22 | 9.70 375 | 22 | 9.76 783 | 29 | 10.23 217 | 9.93 591 | 8 | 38 |
| 23 | 9.70 396 | 21 | 9.76 812 | 29 | 10.23 188 | 9.93 584 | 7 | 37 |
| 24 | 9.70 418 | 22 | 9.76 841 | 29 | 10.23 159 | 9.93 577 | 7 | 36 |
| 25 | 9.70 439 | 21 | 9.76 870 | 29 | 10.23 130 | 9.93 569 | 8 | 35 |
| 26 | 9.70 461 | 22 | 9.76 899 | 29 | 10.23 101 | 9.93 562 | 7 | 34 |
| 27 | 9.70 482 | 21 | 9.76 928 | 29 | 10.23 072 | 9.93 554 | 8 | 33 |
| 28 | 9.70 504 | 22 | 9.76 957 | 29 | 10.23 043 | 9.93 547 | 7 | 32 |
| 29 | 9.70 525 | 21 | 9.76 986 | 29 | 10.23 014 | 9.93 539 | 8 | 31 |
| 30 | 9.70 547 | 22 | 9.77 015 | 29 | 10.22 985 | 9.93 532 | 7 | 30 |
| 31 | 9.70 568 | 21 | 9.77 044 | 29 | 10.22 956 | 9.93 525 | 7 | 29 |
| 32 | 9.70 590 | 22 | 9.77 073 | 29 | 10.22 927 | 9.93 517 | 8 | 28 |
| 33 | 9.70 611 | 21 | 9.77 101 | 28 | 10.22 899 | 9.93 510 | 7 | 27 |
| 34 | 9.70 633 | 22 | 9.77 130 | 29 | 10.22 870 | 9.93 502 | 8 | 26 |
| 35 | 9.70 654 | 21 | 9.77 159 | 29 | 10.22 841 | 9.93 495 | 7 | 25 |
| 36 | 9.70 675 | 21 | 9.77 188 | 29 | 10.22 812 | 9.93 487 | 8 | 24 |
| 37 | 9.70 697 | 22 | 9.77 217 | 29 | 10.22 783 | 9.93 480 | 7 | 23 |
| 38 | 9.70 718 | 21 | 9.77 246 | 29 | 10.22 754 | 9.93 472 | 8 | 22 |
| 39 | 9.70 739 | 21 | 9.77 274 | 28 | 10.22 726 | 9.93 465 | 7 | 21 |
| 40 | 9.70 761 | 22 | 9.77 303 | 29 | 10.22 697 | 9.93 457 | 8 | 20 |
| 41 | 9.70 782 | 21 | 9.77 332 | 29 | 10.22 668 | 9.93 450 | 7 | 19 |
| 42 | 9.70 803 | 21 | 9.77 361 | 29 | 10.22 639 | 9.93 442 | 8 | 18 |
| 43 | 9.70 824 | 21 | 9.77 390 | 29 | 10.22 610 | 9.93 435 | 7 | 17 |
| 44 | 9.70 846 | 22 | 9.77 418 | 28 | 10.22 582 | 9.93 427 | 8 | 16 |
| 45 | 9.70 867 | 21 | 9.77 447 | 29 | 10.22 553 | 9.93 420 | 7 | 15 |
| 46 | 9.70 888 | 21 | 9.77 476 | 29 | 10.22 524 | 9.93 412 | 8 | 14 |
| 47 | 9.70 909 | 21 | 9.77 505 | 29 | 10.22 495 | 9.93 405 | 7 | 13 |
| 48 | 9.70 931 | 22 | 9.77 533 | 28 | 10.22 467 | 9.93 397 | 8 | 12 |
| 49 | 9.70 952 | 21 | 9.77 562 | 29 | 10.22 438 | 9.93 390 | 7 | 11 |
| 50 | 9.70 973 | 21 | 9.77 591 | 28 | 10.22 409 | 9.93 382 | 8 | 10 |
| 51 | 9.70 994 | 21 | 9.77 619 | 29 | 10.22 381 | 9.93 375 | 7 | 9 |
| 52 | 9.71 015 | 21 | 9.77 648 | 29 | 10.22 352 | 9.93 367 | 8 | 8 |
| 53 | 9.71 036 | 22 | 9.77 677 | 29 | 10.22 323 | 9.93 360 | 7 | 7 |
| 54 | 9.71 058 | 21 | 9.77 706 | 28 | 10.22 294 | 9.93 352 | 8 | 6 |
| 55 | 9.71 079 | 21 | 9.77 734 | 29 | 10.22 266 | 9.93 344 | 7 | 5 |
| 56 | 9.71 100 | 21 | 9.77 763 | 28 | 10.22 237 | 9.93 337 | 8 | 4 |
| 57 | 9.71 121 | 21 | 9.77 791 | 29 | 10.22 209 | 9.93 329 | 7 | 3 |
| 58 | 9.71 142 | 21 | 9.77 820 | 29 | 10.22 180 | 9.93 322 | 8 | 2 |
| 59 | 9.71 163 | 21 | 9.77 849 | 28 | 10.22 151 | 9.93 314 | 7 | 1 |
| 60 | 9.71 184 | | 9.77 877 | | 10.22 123 | 9.93 307 | | 0 |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 30 | 29 | 28 |
|---|---|---|---|
| 6 | 3.0 | 2.9 | 2.8 |
| 7 | 3.5 | 3.4 | 3.3 |
| 8 | 4.0 | 3.9 | 3.7 |
| 9 | 4.5 | 4.4 | 4.2 |
| 10 | 5.0 | 4.8 | 4.7 |
| 20 | 10.0 | 9.7 | 9.3 |
| 30 | 15.0 | 14.5 | 14.0 |
| 40 | 20.0 | 19.3 | 18.7 |
| 50 | 25.0 | 24.2 | 23.3 |

| ″ | 22 | 21 |
|---|---|---|
| 6 | 2.2 | 2.1 |
| 7 | 2.6 | 2.4 |
| 8 | 2.9 | 2.8 |
| 9 | 3.3 | 3.2 |
| 10 | 3.7 | 3.5 |
| 20 | 7.3 | 7.0 |
| 30 | 11.0 | 10.5 |
| 40 | 14.7 | 14.0 |
| 50 | 18.3 | 17.5 |

| ″ | 8 | 7 |
|---|---|---|
| 6 | 0.8 | 0.7 |
| 7 | 0.9 | 0.8 |
| 8 | 1.1 | 0.9 |
| 9 | 1.2 | 1.0 |
| 10 | 1.3 | 1.2 |
| 20 | 2.7 | 2.3 |
| 30 | 4.0 | 3.5 |
| 40 | 5.3 | 4.7 |
| 50 | 6.7 | 5.8 |

## 59°

## 31°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.71 184 | | 9.77 877 | | 10.22 123 | 9.93 307 | | **60** | |
| 1 | 9.71 205 | 21 | 9.77 906 | 29 | 10.22 094 | 9.93 299 | 8 | 59 | |
| 2 | 9.71 226 | 21 | 9.77 935 | 29 | 10.22 065 | 9.93 291 | 8 | 58 | |
| 3 | 9.71 247 | 21 | 9.77 963 | 28 | 10.22 037 | 9.93 284 | 7 | 57 | |
| 4 | 9.71 268 | 21 | 9.77 992 | 29 | 10.22 008 | 9.93 276 | 8 | 56 | |
| 5 | 9.71 289 | 21 | 9.78 020 | 28 | 10.21 980 | 9.93 269 | 7 | 55 | |
| 6 | 9.71 310 | 21 | 9.78 049 | 29 | 10.21 951 | 9.93 261 | 8 | 54 | |
| 7 | 9.71 331 | 21 | 9.78 077 | 28 | 10.21 923 | 9.93 253 | 8 | 53 | |
| 8 | 9.71 352 | 21 | 9.78 106 | 29 | 10.21 894 | 9.93 246 | 7 | 52 | |
| 9 | 9.71 373 | 21 | 9.78 135 | 29 | 10.21 865 | 9.93 238 | 8 | 51 | |
| **10** | 9.71 393 | 20 | 9.78 163 | 28 | 10.21 837 | 9.93 230 | 8 | **50** | |
| 11 | 9.71 414 | 21 | 9.78 192 | 29 | 10.21 808 | 9.93 223 | 7 | 49 | |
| 12 | 9.71 435 | 21 | 9.78 220 | 28 | 10.21 780 | 9.93 215 | 8 | 48 | |
| 13 | 9.71 456 | 21 | 9.78 249 | 29 | 10.21 751 | 9.93 207 | 8 | 47 | |
| 14 | 9.71 477 | 21 | 9.78 277 | 28 | 10.21 723 | 9.93 200 | 7 | 46 | |
| 15 | 9.71 498 | 21 | 9.78 306 | 29 | 10.21 694 | 9.93 192 | 8 | 45 | |
| 16 | 9.71 519 | 21 | 9.78 334 | 28 | 10.21 666 | 9.93 184 | 8 | 44 | |
| 17 | 9.71 539 | 20 | 9.78 363 | 29 | 10.21 637 | 9.93 177 | 7 | 43 | |
| 18 | 9.71 560 | 21 | 9.78 391 | 28 | 10.21 609 | 9.93 169 | 8 | 42 | |
| 19 | 9.71 581 | 21 | 9.78 419 | 28 | 10.21 581 | 9.93 161 | 8 | 41 | |
| **20** | 9.71 602 | 21 | 9.78 448 | 29 | 10.21 552 | 9.93 154 | 7 | **40** | |
| 21 | 9.71 622 | 20 | 9.78 476 | 28 | 10.21 524 | 9.93 146 | 8 | 39 | |
| 22 | 9.71 643 | 21 | 9.78 505 | 29 | 10.21 495 | 9.93 138 | 8 | 38 | |
| 23 | 9.71 664 | 21 | 9.78 533 | 28 | 10.21 467 | 9.93 131 | 7 | 37 | |
| 24 | 9.71 685 | 21 | 9.78 562 | 29 | 10.21 438 | 9.93 123 | 8 | 36 | |
| 25 | 9.71 705 | 20 | 9.78 590 | 28 | 10.21 410 | 9.93 115 | 8 | 35 | |
| 26 | 9.71 726 | 21 | 9.78 618 | 28 | 10.21 382 | 9.93 108 | 7 | 34 | |
| 27 | 9.71 747 | 21 | 9.78 647 | 29 | 10.21 353 | 9.93 100 | 8 | 33 | |
| 28 | 9.71 767 | 20 | 9.78 675 | 28 | 10.21 325 | 9.93 092 | 8 | 32 | |
| 29 | 9.71 788 | 21 | 9.78 704 | 29 | 10.21 296 | 9.93 084 | 8 | 31 | |
| **30** | 9.71 809 | 21 | 9.78 732 | 28 | 10.21 268 | 9.93 077 | 7 | **30** | |
| 31 | 9.71 829 | 20 | 9.78 760 | 28 | 10.21 240 | 9.93 069 | 8 | 29 | |
| 32 | 9.71 850 | 21 | 9.78 789 | 29 | 10.21 211 | 9.93 061 | 8 | 28 | |
| 33 | 9.71 870 | 20 | 9.78 817 | 28 | 10.21 183 | 9.93 053 | 8 | 27 | |
| 34 | 9.71 891 | 21 | 9.78 845 | 28 | 10.21 155 | 9.93 046 | 7 | 26 | |
| 35 | 9.71 911 | 20 | 9.78 874 | 29 | 10.21 126 | 9.93 038 | 8 | 25 | |
| 36 | 9.71 932 | 21 | 9.78 902 | 28 | 10.21 098 | 9.93 030 | 8 | 24 | |
| 37 | 9.71 952 | 20 | 9.78 930 | 28 | 10.21 070 | 9.93 022 | 8 | 23 | |
| 38 | 9.71 973 | 21 | 9.78 959 | 29 | 10.21 041 | 9.93 014 | 8 | 22 | |
| 39 | 9.71 994 | 21 | 9.78 987 | 28 | 10.21 013 | 9.93 007 | 7 | 21 | |
| **40** | 9.72 014 | 20 | 9.79 015 | 28 | 10.20 985 | 9.92 999 | 8 | **20** | |
| 41 | 9.72 034 | 20 | 9.79 043 | 29 | 10.20 957 | 9.92 991 | 8 | 19 | |
| 42 | 9.72 055 | 21 | 9.79 072 | 28 | 10.20 928 | 9.92 983 | 8 | 18 | |
| 43 | 9.72 075 | 20 | 9.79 100 | 28 | 10.20 900 | 9.92 976 | 7 | 17 | |
| 44 | 9.72 096 | 21 | 9.79 128 | 28 | 10.20 872 | 9.92 968 | 8 | 16 | |
| 45 | 9.72 116 | 20 | 9.79 156 | 29 | 10.20 844 | 9.92 960 | 8 | 15 | |
| 46 | 9.72 137 | 21 | 9.79 185 | 28 | 10.20 815 | 9.92 952 | 8 | 14 | |
| 47 | 9.72 157 | 20 | 9.79 213 | 28 | 10.20 787 | 9.92 944 | 8 | 13 | |
| 48 | 9.72 177 | 20 | 9.79 241 | 28 | 10.20 759 | 9.92 936 | 8 | 12 | |
| 49 | 9.72 198 | 21 | 9.79 269 | 28 | 10.20 731 | 9.92 929 | 7 | 11 | |
| **50** | 9.72 218 | 20 | 9.79 297 | 28 | 10.20 703 | 9.92 921 | 8 | **10** | |
| 51 | 9.72 238 | 20 | 9.79 326 | 29 | 10.20 674 | 9.92 913 | 8 | 9 | |
| 52 | 9.72 259 | 21 | 9.79 354 | 28 | 10.20 646 | 9.92 905 | 8 | 8 | |
| 53 | 9.72 279 | 20 | 9.79 382 | 28 | 10.20 618 | 9.92 897 | 8 | 7 | |
| 54 | 9.72 299 | 20 | 9.79 410 | 28 | 10.20 590 | 9.92 889 | 8 | 6 | |
| 55 | 9.72 320 | 21 | 9.79 438 | 28 | 10.20 562 | 9.92 881 | 8 | 5 | |
| 56 | 9.72 340 | 20 | 9.79 466 | 28 | 10.20 534 | 9.92 874 | 7 | 4 | |
| 57 | 9.72 360 | 20 | 9.79 495 | 29 | 10.20 505 | 9.92 866 | 8 | 3 | |
| 58 | 9.72 381 | 21 | 9.79 523 | 28 | 10.20 477 | 9.92 858 | 8 | 2 | |
| 59 | 9.72 401 | 20 | 9.79 551 | 28 | 10.20 449 | 9.92 850 | 8 | 1 | |
| **60** | 9.72 421 | 20 | 9.79 579 | 28 | 10.20 421 | 9.92 842 | 8 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 29 | 28 |
|---|---|---|
| 6 | 2.9 | 2.8 |
| 7 | 3.4 | 3.3 |
| 8 | 3.9 | 3.7 |
| 9 | 4.4 | 4.2 |
| 10 | 4.8 | 4.7 |
| 20 | 9.7 | 9.3 |
| 30 | 14.5 | 14.0 |
| 40 | 19.3 | 18.7 |
| 50 | 24.2 | 23.3 |

| ″ | 21 | 20 |
|---|---|---|
| 6 | 2.1 | 2.0 |
| 7 | 2.4 | 2.3 |
| 8 | 2.8 | 2.7 |
| 9 | 3.2 | 3.0 |
| 10 | 3.5 | 3.3 |
| 20 | 7.0 | 6.7 |
| 30 | 10.5 | 10.0 |
| 40 | 14.0 | 13.3 |
| 50 | 17.5 | 16.7 |

| ″ | 8 | 7 |
|---|---|---|
| 6 | 0.8 | 0.7 |
| 7 | 0.9 | 0.8 |
| 8 | 1.1 | 0.9 |
| 9 | 1.2 | 1.0 |
| 10 | 1.3 | 1.2 |
| 20 | 2.7 | 2.3 |
| 30 | 4.0 | 3.5 |
| 40 | 5.3 | 4.7 |
| 50 | 6.7 | 5.8 |

## 58°

## 32°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.72 421 | | 9.79 579 | | 10.20 421 | 9.92 842 | | **60** | |
| 1 | 9.72 441 | 20 | 9.79 607 | 28 | 10.20 393 | 9.92 834 | 8 | 59 | |
| 2 | 9.72 461 | 20 | 9.79 635 | 28 | 10.20 365 | 9.92 826 | 8 | 58 | |
| 3 | 9.72 482 | 21 | 9.79 663 | 28 | 10.20 337 | 9.92 818 | 8 | 57 | |
| 4 | 9.72 502 | 20 | 9.79 691 | 28 | 10.20 309 | 9.92 810 | 8 | 56 | |
| 5 | 9.72 522 | 20 | 9.79 719 | 28 | 10.20 281 | 9.92 803 | 7 | 55 | |
| 6 | 9.72 542 | 20 | 9.79 747 | 29 | 10.20 253 | 9.92 795 | 8 | 54 | |
| 7 | 9.72 562 | 20 | 9.79 776 | 28 | 10.20 224 | 9.92 787 | 8 | 53 | |
| 8 | 9.72 582 | 20 | 9.79 804 | 28 | 10.20 196 | 9.92 779 | 8 | 52 | |
| 9 | 9.72 602 | 20 | 9.79 832 | 28 | 10.20 168 | 9.92 771 | 8 | 51 | |
| **10** | 9.72 622 | 21 | 9.79 860 | 28 | 10.20 140 | 9.92 763 | 8 | **50** | |
| 11 | 9.72 643 | 20 | 9.79 888 | 28 | 10.20 112 | 9.92 755 | 8 | 49 | |
| 12 | 9.72 663 | 20 | 9.79 916 | 28 | 10.20 084 | 9.92 747 | 8 | 48 | |
| 13 | 9.72 683 | 20 | 9.79 944 | 28 | 10.20 056 | 9.92 739 | 8 | 47 | |
| 14 | 9.72 703 | 20 | 9.79 972 | 28 | 10.20 028 | 9.92 731 | 8 | 46 | |
| 15 | 9.72 723 | 20 | 9.80 000 | 28 | 10.20 000 | 9.92 723 | 8 | 45 | |
| 16 | 9.72 743 | 20 | 9.80 028 | 28 | 10.19 972 | 9.92 715 | 8 | 44 | |
| 17 | 9.72 763 | 20 | 9.80 056 | 28 | 10.19 944 | 9.92 707 | 8 | 43 | |
| 18 | 9.72 783 | 20 | 9.80 084 | 28 | 10.19 916 | 9.92 699 | 8 | 42 | |
| 19 | 9.72 803 | 20 | 9.80 112 | 28 | 10.19 888 | 9.92 691 | 8 | 41 | |
| **20** | 9.72 823 | 20 | 9.80 140 | 28 | 10.19 860 | 9.92 683 | 8 | **40** | |
| 21 | 9.72 843 | 20 | 9.80 168 | 27 | 10.19 832 | 9.92 675 | 8 | 39 | |
| 22 | 9.72 863 | 20 | 9.80 195 | 28 | 10.19 805 | 9.92 667 | 8 | 38 | |
| 23 | 9.72 883 | 19 | 9.80 223 | 28 | 10.19 777 | 9.92 659 | 8 | 37 | |
| 24 | 9.72 902 | 20 | 9.80 251 | 28 | 10.19 749 | 9.92 651 | 8 | 36 | |
| 25 | 9.72 922 | 20 | 9.80 279 | 28 | 10.19 721 | 9.92 643 | 8 | 35 | |
| 26 | 9.72 942 | 20 | 9.80 307 | 28 | 10.19 693 | 9.92 635 | 8 | 34 | |
| 27 | 9.72 962 | 20 | 9.80 335 | 28 | 10.19 665 | 9.92 627 | 8 | 33 | |
| 28 | 9.72 982 | 20 | 9.80 363 | 28 | 10.19 637 | 9.92 619 | 8 | 32 | |
| 29 | 9.73 002 | 20 | 9.80 391 | 28 | 10.19 609 | 9.92 611 | 8 | 31 | |
| **30** | 9.73 022 | 19 | 9.80 419 | 28 | 10.19 581 | 9.92 603 | 8 | **30** | |
| 31 | 9.73 041 | 20 | 9.80 447 | 27 | 10.19 553 | 9.92 595 | 8 | 29 | |
| 32 | 9.73 061 | 20 | 9.80 474 | 28 | 10.19 526 | 9.92 587 | 8 | 28 | |
| 33 | 9.73 081 | 20 | 9.80 502 | 28 | 10.19 498 | 9.92 579 | 8 | 27 | |
| 34 | 9.73 101 | 20 | 9.80 530 | 28 | 10.19 470 | 9.92 571 | 8 | 26 | |
| 35 | 9.73 121 | 19 | 9.80 558 | 28 | 10.19 442 | 9.92 563 | 8 | 25 | |
| 36 | 9.73 140 | 20 | 9.80 586 | 28 | 10.19 414 | 9.92 555 | 9 | 24 | |
| 37 | 9.73 160 | 20 | 9.80 614 | 28 | 10.19 386 | 9.92 546 | 9 | 23 | |
| 38 | 9.73 180 | 20 | 9.80 642 | 27 | 10.19 358 | 9.92 538 | 8 | 22 | |
| 39 | 9.73 200 | 19 | 9.80 669 | 28 | 10.19 331 | 9.92 530 | 8 | 21 | |
| **40** | 9.73 219 | 20 | 9.80 697 | 28 | 10.19 303 | 9.92 522 | 8 | **20** | |
| 41 | 9.73 239 | 20 | 9.80 725 | 28 | 10.19 275 | 9.92 514 | 8 | 19 | |
| 42 | 9.73 259 | 19 | 9.80 753 | 28 | 10.19 247 | 9.92 506 | 8 | 18 | |
| 43 | 9.73 278 | 20 | 9.80 781 | 27 | 10.19 219 | 9.92 498 | 8 | 17 | |
| 44 | 9.73 298 | 20 | 9.80 808 | 28 | 10.19 192 | 9.92 490 | 8 | 16 | |
| 45 | 9.73 318 | 19 | 9.80 836 | 28 | 10.19 164 | 9.92 482 | 9 | 15 | |
| 46 | 9.73 337 | 20 | 9.80 864 | 28 | 10.19 136 | 9.92 473 | 8 | 14 | |
| 47 | 9.73 357 | 20 | 9.80 892 | 27 | 10.19 108 | 9.92 465 | 8 | 13 | |
| 48 | 9.73 377 | 19 | 9.80 919 | 28 | 10.19 081 | 9.92 457 | 8 | 12 | |
| 49 | 9.73 396 | 20 | 9.80 947 | 28 | 10.19 053 | 9.92 449 | 8 | 11 | |
| **50** | 9.73 416 | 19 | 9.80 975 | 28 | 10.19 025 | 9.92 441 | 8 | **10** | |
| 51 | 9.73 435 | 20 | 9.81 003 | 27 | 10.18 997 | 9.92 433 | 8 | 9 | |
| 52 | 9.73 455 | 19 | 9.81 030 | 28 | 10.18 970 | 9.92 425 | 9 | 8 | |
| 53 | 9.73 474 | 20 | 9.81 058 | 28 | 10.18 942 | 9.92 416 | 8 | 7 | |
| 54 | 9.73 494 | 19 | 9.81 086 | 27 | 10.18 914 | 9.92 408 | 8 | 6 | |
| 55 | 9.73 513 | 20 | 9.81 113 | 28 | 10.18 887 | 9.92 400 | 8 | 5 | |
| 56 | 9.73 533 | 19 | 9.81 141 | 28 | 10.18 859 | 9.92 392 | 8 | 4 | |
| 57 | 9.73 552 | 20 | 9.81 169 | 27 | 10.18 831 | 9.92 384 | 8 | 3 | |
| 58 | 9.73 572 | 19 | 9.81 196 | 28 | 10.18 804 | 9.92 376 | 9 | 2 | |
| 59 | 9.73 591 | 20 | 9.81 224 | 28 | 10.18 776 | 9.92 367 | 8 | 1 | |
| **60** | 9.73 611 | | 9.81 252 | | 10.18 748 | 9.92 359 | | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 29 | 28 | 27 |
|---|---|---|---|
| 6 | 2.9 | 2.8 | 2.7 |
| 7 | 3.5 | 3.3 | 3.2 |
| 8 | 3.9 | 3.7 | 3.6 |
| 9 | 4.4 | 4.2 | 4.0 |
| 10 | 4.8 | 4.7 | 4.5 |
| 20 | 9.7 | 9.3 | 9.0 |
| 30 | 14.5 | 14.0 | 13.5 |
| 40 | 19.3 | 18.7 | 18.0 |
| 50 | 24.2 | 23.3 | 22.5 |

| ″ | 21 | 20 | 19 |
|---|---|---|---|
| 6 | 2.1 | 2.0 | 1.9 |
| 7 | 2.4 | 2.3 | 2.2 |
| 8 | 2.8 | 2.7 | 2.5 |
| 9 | 3.2 | 3.0 | 2.8 |
| 10 | 3.5 | 3.3 | 3.2 |
| 20 | 7.0 | 6.7 | 6.3 |
| 30 | 10.5 | 10.0 | 9.5 |
| 40 | 14.0 | 13.3 | 12.7 |
| 50 | 17.5 | 16.7 | 15.8 |

| ″ | 9 | 8 | 7 |
|---|---|---|---|
| 6 | 0.9 | 0.8 | 0.7 |
| 7 | 1.0 | 0.9 | 0.8 |
| 8 | 1.2 | 1.1 | 0.9 |
| 9 | 1.4 | 1.2 | 1.0 |
| 10 | 1.5 | 1.3 | 1.2 |
| 20 | 3.0 | 2.7 | 2.3 |
| 30 | 4.5 | 4.0 | 3.5 |
| 40 | 6.0 | 5.3 | 4.7 |
| 50 | 7.5 | 6.7 | 5.8 |

## 57°

## 33°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.73 611 | | 9.81 252 | | 10.18 748 | 9.92 359 | | **60** |
| 1 | 9.73 630 | 19 | 9.81 279 | 27 | 10.18 721 | 9.92 351 | 8 | 59 |
| 2 | 9.73 650 | 20 | 9.81 307 | 28 | 10.18 693 | 9.92 343 | 8 | 58 |
| 3 | 9.73 669 | 19 | 9.81 335 | 28 | 10.18 665 | 9.92 335 | 8 | 57 |
| 4 | 9.73 689 | 20 | 9.81 362 | 27 | 10.18 638 | 9.92 326 | 9 | 56 |
| 5 | 9.73 708 | 19 | 9.81 390 | 28 | 10.18 610 | 9.92 318 | 8 | 55 |
| 6 | 9.73 727 | 19 | 9.81 418 | 28 | 10.18 582 | 9.92 310 | 8 | 54 |
| 7 | 9.73 747 | 20 | 9.81 445 | 27 | 10.18 555 | 9.92 302 | 8 | 53 |
| 8 | 9.73 766 | 19 | 9.81 473 | 28 | 10.18 527 | 9.92 293 | 9 | 52 |
| 9 | 9.73 785 | 19 | 9.81 500 | 27 | 10.18 500 | 9.92 285 | 8 | 51 |
| **10** | 9.73 805 | 20 | 9.81 528 | 28 | 10.18 472 | 9.92 277 | 8 | **50** |
| 11 | 9.73 824 | 19 | 9.81 556 | 28 | 10.18 444 | 9.92 269 | 8 | 49 |
| 12 | 9.73 843 | 19 | 9.81 583 | 27 | 10.18 417 | 9.92 260 | 9 | 48 |
| 13 | 9.73 863 | 20 | 9.81 611 | 28 | 10.18 389 | 9.92 252 | 8 | 47 |
| 14 | 9.73 882 | 19 | 9.81 638 | 27 | 10.18 362 | 9.92 244 | 8 | 46 |
| 15 | 9.73 901 | 19 | 9.81 666 | 28 | 10.18 334 | 9.92 235 | 9 | 45 |
| 16 | 9.73 921 | 20 | 9.81 693 | 27 | 10.18 307 | 9.92 227 | 8 | 44 |
| 17 | 9.73 940 | 19 | 9.81 721 | 28 | 10.18 279 | 9.92 219 | 8 | 43 |
| 18 | 9.73 959 | 19 | 9.81 748 | 27 | 10.18 252 | 9.92 211 | 8 | 42 |
| 19 | 9.73 978 | 19 | 9.81 776 | 28 | 10.18 224 | 9.92 202 | 9 | 41 |
| **20** | 9.73 997 | 19 | 9.81 803 | 27 | 10.18 197 | 9.92 194 | 8 | **40** |
| 21 | 9.74 017 | 20 | 9.81 831 | 28 | 10.18 169 | 9.92 186 | 8 | 39 |
| 22 | 9.74 036 | 19 | 9.81 858 | 27 | 10.18 142 | 9.92 177 | 9 | 38 |
| 23 | 9.74 055 | 19 | 9.81 886 | 28 | 10.18 114 | 9.92 169 | 8 | 37 |
| 24 | 9.74 074 | 19 | 9.81 913 | 27 | 10.18 087 | 9.92 161 | 8 | 36 |
| 25 | 9.74 093 | 19 | 9.81 941 | 28 | 10.18 059 | 9.92 152 | 9 | 35 |
| 26 | 9.74 113 | 20 | 9.81 968 | 27 | 10.18 032 | 9.92 144 | 8 | 34 |
| 27 | 9.74 132 | 19 | 9.81 996 | 28 | 10.18 004 | 9.92 136 | 8 | 33 |
| 28 | 9.74 151 | 19 | 9.82 023 | 27 | 10.17 977 | 9.92 127 | 9 | 32 |
| 29 | 9.74 170 | 19 | 9.82 051 | 28 | 10.17 949 | 9.92 119 | 8 | 31 |
| **30** | 9.74 189 | 19 | 9.82 078 | 27 | 10.17 922 | 9.92 111 | 8 | **30** |
| 31 | 9.74 208 | 19 | 9.82 106 | 28 | 10.17 894 | 9.92 102 | 9 | 29 |
| 32 | 9.74 227 | 19 | 9.82 133 | 27 | 10.17 867 | 9.92 094 | 8 | 28 |
| 33 | 9.74 246 | 19 | 9.82 161 | 28 | 10.17 839 | 9.92 086 | 8 | 27 |
| 34 | 9.74 265 | 19 | 9.82 188 | 27 | 10.17 812 | 9.92 077 | 9 | 26 |
| 35 | 9.74 284 | 19 | 9.82 215 | 27 | 10.17 785 | 9.92 069 | 8 | 25 |
| 36 | 9.74 303 | 19 | 9.82 243 | 28 | 10.17 757 | 9.92 060 | 9 | 24 |
| 37 | 9.74 322 | 19 | 9.82 270 | 27 | 10.17 730 | 9.92 052 | 8 | 23 |
| 38 | 9.74 341 | 19 | 9.82 298 | 28 | 10.17 702 | 9.92 044 | 8 | 22 |
| 39 | 9.74 360 | 19 | 9.82 325 | 27 | 10.17 675 | 9.92 035 | 9 | 21 |
| **40** | 9.74 379 | 19 | 9.82 352 | 27 | 10.17 648 | 9.92 027 | 8 | **20** |
| 41 | 9.74 398 | 19 | 9.82 380 | 28 | 10.17 620 | 9.92 018 | 9 | 19 |
| 42 | 9.74 417 | 19 | 9.82 407 | 27 | 10.17 593 | 9.92 010 | 8 | 18 |
| 43 | 9.74 436 | 19 | 9.82 435 | 28 | 10.17 565 | 9.92 002 | 8 | 17 |
| 44 | 9.74 455 | 19 | 9.82 462 | 27 | 10.17 538 | 9.91 993 | 9 | 16 |
| 45 | 9.74 474 | 19 | 9.82 489 | 27 | 10.17 511 | 9.91 985 | 8 | 15 |
| 46 | 9.74 493 | 19 | 9.82 517 | 28 | 10.17 483 | 9.91 976 | 9 | 14 |
| 47 | 9.74 512 | 19 | 9.82 544 | 27 | 10.17 456 | 9.91 968 | 8 | 13 |
| 48 | 9.74 531 | 19 | 9.82 571 | 27 | 10.17 429 | 9.91 959 | 9 | 12 |
| 49 | 9.74 549 | 18 | 9.82 599 | 28 | 10.17 401 | 9.91 951 | 8 | 11 |
| **50** | 9.74 568 | 19 | 9.82 626 | 27 | 10.17 374 | 9.91 942 | 9 | **10** |
| 51 | 9.74 587 | 19 | 9.82 653 | 27 | 10.17 347 | 9.91 934 | 8 | 9 |
| 52 | 9.74 606 | 19 | 9.82 681 | 28 | 10.17 319 | 9.91 925 | 9 | 8 |
| 53 | 9.74 625 | 19 | 9.82 708 | 27 | 10.17 292 | 9.91 917 | 8 | 7 |
| 54 | 9.74 644 | 19 | 9.82 735 | 27 | 10.17 265 | 9.91 908 | 9 | 6 |
| 55 | 9.74 662 | 18 | 9.82 762 | 27 | 10.17 238 | 9.91 900 | 8 | 5 |
| 56 | 9.74 681 | 19 | 9.82 790 | 28 | 10.17 210 | 9.91 891 | 9 | 4 |
| 57 | 9.74 700 | 19 | 9.82 817 | 27 | 10.17 183 | 9.91 883 | 8 | 3 |
| 58 | 9.74 719 | 19 | 9.82 844 | 27 | 10.17 156 | 9.91 874 | 9 | 2 |
| 59 | 9.74 737 | 18 | 9.82 871 | 27 | 10.17 129 | 9.91 866 | 8 | 1 |
| **60** | 9.74 756 | 19 | 9.82 899 | 28 | 10.17 101 | 9.91 857 | 9 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 28 | 27 | 20 |
|---|---|---|---|
| 6 | 2.8 | 2.7 | 2.0 |
| 7 | 3.3 | 3.2 | 2.3 |
| 8 | 3.7 | 3.6 | 2.7 |
| 9 | 4.2 | 4.0 | 3.0 |
| 10 | 4.7 | 4.5 | 3.3 |
| 20 | 9.3 | 9.0 | 6.7 |
| 30 | 14.0 | 13.5 | 10.0 |
| 40 | 18.7 | 18.0 | 13.3 |
| 50 | 23.3 | 22.5 | 16.7 |

| ″ | 19 | 18 |
|---|---|---|
| 6 | 1.9 | 1.8 |
| 7 | 2.2 | 2.1 |
| 8 | 2.5 | 2.4 |
| 9 | 2.9 | 2.7 |
| 10 | 3.2 | 3.0 |
| 20 | 6.3 | 6.0 |
| 30 | 9.5 | 9.0 |
| 40 | 12.7 | 12.0 |
| 50 | 15.8 | 15.0 |

| ″ | 9 | 8 |
|---|---|---|
| 6 | 0.9 | 0.8 |
| 7 | 1.0 | 0.9 |
| 8 | 1.2 | 1.1 |
| 9 | 1.4 | 1.2 |
| 10 | 1.5 | 1.3 |
| 20 | 3.0 | 2.7 |
| 30 | 4.5 | 4.0 |
| 40 | 6.0 | 5.3 |
| 50 | 7.5 | 6.7 |

Prop. Pts.

## 56°

## 34°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.74 756 | | 9.82 899 | | 10.17 101 | 9.91 857 | | 60 | |
| 1 | 9.74 775 | 19 | 9.82 926 | 27 | 10.17 074 | 9.91 849 | 8 | 59 | |
| 2 | 9.74 794 | 19 | 9.82 953 | 27 | 10.17 047 | 9.91 840 | 9 | 58 | |
| 3 | 9.74 812 | 18 | 9.82 980 | 27 | 10.17 020 | 9.91 832 | 8 | 57 | |
| 4 | 9.74 831 | 19 | 9.83 008 | 28 | 10.16 992 | 9.91 823 | 9 | 56 | |
| 5 | 9.74 850 | 19 | 9.83 035 | 27 | 10.16 965 | 9.91 815 | 8 | 55 | |
| 6 | 9.74 868 | 18 | 9.83 062 | 27 | 10.16 938 | 9.91 806 | 9 | 54 | |
| 7 | 9.74 887 | 19 | 9.83 089 | 27 | 10.16 911 | 9.91 798 | 8 | 53 | |
| 8 | 9.74 906 | 19 | 9.83 117 | 28 | 10.16 883 | 9.91 789 | 9 | 52 | |
| 9 | 9.74 924 | 18 | 9.83 144 | 27 | 10.16 856 | 9.91 781 | 8 | 51 | |
| 10 | 9.74 943 | 19 | 9.83 171 | 27 | 10.16 829 | 9.91 772 | 9 | 50 | |
| 11 | 9.74 961 | 18 | 9.83 198 | 27 | 10.16 802 | 9.91 763 | 9 | 49 | |
| 12 | 9.74 980 | 19 | 9.83 225 | 27 | 10.16 775 | 9.91 755 | 8 | 48 | |
| 13 | 9.74 999 | 19 | 9.83 252 | 27 | 10.16 748 | 9.91 746 | 9 | 47 | |
| 14 | 9.75 017 | 18 | 9.83 280 | 28 | 10.16 720 | 9.91 738 | 8 | 46 | |
| 15 | 9.75 036 | 19 | 9.83 307 | 27 | 10.16 693 | 9.91 729 | 9 | 45 | |
| 16 | 9.75 054 | 18 | 9.83 334 | 27 | 10.16 666 | 9.91 720 | 9 | 44 | |
| 17 | 9.75 073 | 19 | 9.83 361 | 27 | 10.16 639 | 9.91 712 | 8 | 43 | |
| 18 | 9.75 091 | 18 | 9.83 388 | 27 | 10.16 612 | 9.91 703 | 9 | 42 | |
| 19 | 9.75 110 | 19 | 9.83 415 | 27 | 10.16 585 | 9.91 695 | 8 | 41 | |
| 20 | 9.75 128 | 18 | 9.83 442 | 27 | 10.16 558 | 9.91 686 | 9 | 40 | |
| 21 | 9.75 147 | 19 | 9.83 470 | 28 | 10.16 530 | 9.91 677 | 9 | 39 | |
| 22 | 9.75 165 | 18 | 9.83 497 | 27 | 10.16 503 | 9.91 669 | 8 | 38 | |
| 23 | 9.75 184 | 19 | 9.83 524 | 27 | 10.16 476 | 9.91 660 | 9 | 37 | |
| 24 | 9.75 202 | 18 | 9.83 551 | 27 | 10.16 449 | 9.91 651 | 9 | 36 | |
| 25 | 9.75 221 | 19 | 9.83 578 | 27 | 10.16 422 | 9.91 643 | 8 | 35 | |
| 26 | 9.75 239 | 18 | 9.83 605 | 27 | 10.16 395 | 9.91 634 | 9 | 34 | |
| 27 | 9.75 258 | 19 | 9.83 632 | 27 | 10.16 368 | 9.91 625 | 9 | 33 | |
| 28 | 9.75 276 | 18 | 9.83 659 | 27 | 10.16 341 | 9.91 617 | 8 | 32 | |
| 29 | 9.75 294 | 18 | 9.83 686 | 27 | 10.16 314 | 9.91 608 | 9 | 31 | |
| 30 | 9.75 313 | 19 | 9.83 713 | 27 | 10.16 287 | 9.91 599 | 9 | 30 | |
| 31 | 9.75 331 | 18 | 9.83 740 | 27 | 10.16 260 | 9.91 591 | 8 | 29 | |
| 32 | 9.75 350 | 19 | 9.83 768 | 28 | 10.16 232 | 9.91 582 | 9 | 28 | |
| 33 | 9.75 368 | 18 | 9.83 795 | 27 | 10.16 205 | 9.91 573 | 9 | 27 | |
| 34 | 9.75 386 | 18 | 9.83 822 | 27 | 10.16 178 | 9.91 565 | 8 | 26 | |
| 35 | 9.75 405 | 19 | 9.83 849 | 27 | 10.16 151 | 9.91 556 | 9 | 25 | |
| 36 | 9.75 423 | 18 | 9.83 876 | 27 | 10.16 124 | 9.91 547 | 9 | 24 | |
| 37 | 9.75 441 | 18 | 9.83 903 | 27 | 10.16 097 | 9.91 538 | 9 | 23 | |
| 38 | 9.75 459 | 18 | 9.83 930 | 27 | 10.16 070 | 9.91 530 | 8 | 22 | |
| 39 | 9.75 478 | 19 | 9.83 957 | 27 | 10.16 043 | 9.91 521 | 9 | 21 | |
| 40 | 9.75 496 | 18 | 9.83 984 | 27 | 10.16 016 | 9.91 512 | 9 | 20 | |
| 41 | 9.75 514 | 18 | 9.84 011 | 27 | 10.15 989 | 9.91 504 | 8 | 19 | |
| 42 | 9.75 533 | 19 | 9.84 038 | 27 | 10.15 962 | 9.91 495 | 9 | 18 | |
| 43 | 9.75 551 | 18 | 9.84 065 | 27 | 10.15 935 | 9.91 486 | 9 | 17 | |
| 44 | 9.75 569 | 18 | 9.84 092 | 27 | 10.15 908 | 9.91 477 | 9 | 16 | |
| 45 | 9.75 587 | 18 | 9.84 119 | 27 | 10.15 881 | 9.91 469 | 8 | 15 | |
| 46 | 9.75 605 | 18 | 9.84 146 | 27 | 10.15 854 | 9.91 460 | 9 | 14 | |
| 47 | 9.75 624 | 19 | 9.84 173 | 27 | 10.15 827 | 9.91 451 | 9 | 13 | |
| 48 | 9.75 642 | 18 | 9.84 200 | 27 | 10.15 800 | 9.91 442 | 9 | 12 | |
| 49 | 9.75 660 | 18 | 9.84 227 | 27 | 10.15 773 | 9.91 433 | 9 | 11 | |
| 50 | 9.75 678 | 18 | 9.84 254 | 26 | 10.15 746 | 9.91 425 | 8 | 10 | |
| 51 | 9.75 696 | 18 | 9.84 280 | 27 | 10.15 720 | 9.91 416 | 9 | 9 | |
| 52 | 9.75 714 | 18 | 9.84 307 | 27 | 10.15 693 | 9.91 407 | 9 | 8 | |
| 53 | 9.75 733 | 19 | 9.84 334 | 27 | 10.15 666 | 9.91 398 | 9 | 7 | |
| 54 | 9.75 751 | 18 | 9.84 361 | 27 | 10.15 639 | 9.91 389 | 9 | 6 | |
| 55 | 9.75 769 | 18 | 9.84 388 | 27 | 10.15 612 | 9.91 381 | 8 | 5 | |
| 56 | 9.75 787 | 18 | 9.84 415 | 27 | 10.15 585 | 9.91 372 | 9 | 4 | |
| 57 | 9.75 805 | 18 | 9.84 442 | 27 | 10.15 558 | 9.91 363 | 9 | 3 | |
| 58 | 9.75 823 | 18 | 9.84 469 | 27 | 10.15 531 | 9.91 354 | 9 | 2 | |
| 59 | 9.75 841 | 18 | 9.84 496 | 27 | 10.15 504 | 9.91 345 | 9 | 1 | |
| 60 | 9.75 859 | | 9.84 523 | | 10.15 477 | 9.91 336 | | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 28 | 27 | 25 |
|---|---|---|---|
| 6 | 2.8 | 2.7 | 2.6 |
| 7 | 3.3 | 3.2 | 3.0 |
| 8 | 3.7 | 3.6 | 3.5 |
| 9 | 4.2 | 4.0 | 3.9 |
| 10 | 4.7 | 4.5 | 4.3 |
| 20 | 9.3 | 9.0 | 8.7 |
| 30 | 14.0 | 13.5 | 13.0 |
| 40 | 18.7 | 18.0 | 17.3 |
| 50 | 23.3 | 22.5 | 21.7 |

| ″ | 19 | 18 |
|---|---|---|
| 6 | 1.9 | 1.8 |
| 7 | 2.2 | 2.1 |
| 8 | 2.5 | 2.4 |
| 9 | 2.8 | 2.7 |
| 10 | 3.2 | 3.0 |
| 20 | 6.3 | 6.0 |
| 30 | 9.5 | 9.0 |
| 40 | 12.7 | 12.0 |
| 50 | 15.8 | 15.0 |

| ″ | 9 | 8 |
|---|---|---|
| 6 | 0.9 | 0.8 |
| 7 | 1.0 | 0.9 |
| 8 | 1.2 | 1.1 |
| 9 | 1.4 | 1.2 |
| 10 | 1.5 | 1.3 |
| 20 | 3.0 | 2.7 |
| 30 | 4.5 | 4.0 |
| 40 | 6.0 | 5.3 |
| 50 | 7.5 | 6.7 |

## 55°

## 35°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.75 859 | | 9.84 523 | | 10.15 477 | 9.91 336 | | **60** |
| 1 | 9.75 877 | 18 | 9.84 550 | 27 | 10.15 450 | 9.91 328 | 8 | 59 |
| 2 | 9.75 895 | 18 | 9.84 576 | 26 | 10.15 424 | 9.91 319 | 9 | 58 |
| 3 | 9.75 913 | 18 | 9.84 603 | 27 | 10.15 397 | 9.91 310 | 9 | 57 |
| 4 | 9.75 931 | 18 | 9.84 630 | 27 | 10.15 370 | 9.91 301 | 9 | 56 |
| 5 | 9.75 949 | 18 | 9.84 657 | 27 | 10.15 343 | 9.91 292 | 9 | 55 |
| 6 | 9.75 967 | 18 | 9.84 684 | 27 | 10.15 316 | 9.91 283 | 9 | 54 |
| 7 | 9.75 985 | 18 | 9.84 711 | 27 | 10.15 289 | 9.91 274 | 9 | 53 |
| 8 | 9.76 003 | 18 | 9.84 738 | 26 | 10.15 262 | 9.91 266 | 8 | 52 |
| 9 | 9.76 021 | 18 | 9.84 764 | 27 | 10.15 236 | 9.91 257 | 9 | 51 |
| **10** | 9.76 039 | 18 | 9.84 791 | 27 | 10.15 209 | 9.91 248 | 9 | **50** |
| 11 | 9.76 057 | 18 | 9.84 818 | 27 | 10.15 182 | 9.91 239 | 9 | 49 |
| 12 | 9.76 075 | 18 | 9.84 845 | 27 | 10.15 155 | 9.91 230 | 9 | 48 |
| 13 | 9.76 093 | 18 | 9.84 872 | 27 | 10.15 128 | 9.91 221 | 9 | 47 |
| 14 | 9.76 111 | 18 | 9.84 899 | 26 | 10.15 101 | 9.91 212 | 9 | 46 |
| 15 | 9.76 129 | 17 | 9.84 925 | 27 | 10.15 075 | 9.91 203 | 9 | 45 |
| 16 | 9.76 146 | 18 | 9.84 952 | 27 | 10.15 048 | 9.91 194 | 9 | 44 |
| 17 | 9.76 164 | 18 | 9.84 979 | 27 | 10.15 021 | 9.91 185 | 9 | 43 |
| 18 | 9.76 182 | 18 | 9.85 006 | 27 | 10.14 994 | 9.91 176 | 9 | 42 |
| 19 | 9.76 200 | 18 | 9.85 033 | 26 | 10.14 967 | 9.91 167 | 9 | 41 |
| **20** | 9.76 218 | 18 | 9.85 059 | 27 | 10.14 941 | 9.91 158 | 9 | **40** |
| 21 | 9.76 236 | 17 | 9.85 086 | 27 | 10.14 914 | 9.91 149 | 8 | 39 |
| 22 | 9.76 253 | 18 | 9.85 113 | 27 | 10.14 887 | 9.91 141 | 9 | 38 |
| 23 | 9.76 271 | 18 | 9.85 140 | 26 | 10.14 860 | 9.91 132 | 9 | 37 |
| 24 | 9.76 289 | 18 | 9.85 166 | 27 | 10.14 834 | 9.91 123 | 9 | 36 |
| 25 | 9.76 307 | 17 | 9.85 193 | 27 | 10.14 807 | 9.91 114 | 9 | 35 |
| 26 | 9.76 324 | 18 | 9.85 220 | 27 | 10.14 780 | 9.91 105 | 9 | 34 |
| 27 | 9.76 342 | 18 | 9.85 247 | 26 | 10.14 753 | 9.91 096 | 9 | 33 |
| 28 | 9.76 360 | 18 | 9.85 273 | 27 | 10.14 727 | 9.91 087 | 9 | 32 |
| 29 | 9.76 378 | 17 | 9.85 300 | 27 | 10.14 700 | 9.91 078 | 9 | 31 |
| **30** | 9.76 395 | 18 | 9.85 327 | 27 | 10.14 673 | 9.91 069 | 9 | **30** |
| 31 | 9.76 413 | 18 | 9.85 354 | 26 | 10.14 646 | 9.91 060 | 9 | 29 |
| 32 | 9.76 431 | 17 | 9.85 380 | 27 | 10.14 620 | 9.91 051 | 9 | 28 |
| 33 | 9.76 448 | 18 | 9.85 407 | 27 | 10.14 593 | 9.91 042 | 9 | 27 |
| 34 | 9.76 466 | 18 | 9.85 434 | 26 | 10.14 566 | 9.91 033 | 10 | 26 |
| 35 | 9.76 484 | 17 | 9.85 460 | 27 | 10.14 540 | 9.91 023 | 9 | 25 |
| 36 | 9.76 501 | 18 | 9.85 487 | 27 | 10.14 513 | 9.91 014 | 9 | 24 |
| 37 | 9.76 519 | 18 | 9.85 514 | 26 | 10.14 486 | 9.91 005 | 9 | 23 |
| 38 | 9.76 537 | 17 | 9.85 540 | 27 | 10.14 460 | 9.90 996 | 9 | 22 |
| 39 | 9.76 554 | 18 | 9.85 567 | 27 | 10.14 433 | 9.90 987 | 9 | 21 |
| **40** | 9.76 572 | 18 | 9.85 594 | 26 | 10.14 406 | 9.90 978 | 9 | **20** |
| 41 | 9.76 590 | 17 | 9.85 620 | 27 | 10.14 380 | 9.90 969 | 9 | 19 |
| 42 | 9.76 607 | 18 | 9.85 647 | 27 | 10.14 353 | 9.90 960 | 9 | 18 |
| 43 | 9.76 625 | 17 | 9.85 674 | 26 | 10.14 326 | 9.90 951 | 9 | 17 |
| 44 | 9.76 642 | 18 | 9.85 700 | 27 | 10.14 300 | 9.90 942 | 9 | 16 |
| 45 | 9.76 660 | 17 | 9.85 727 | 27 | 10.14 273 | 9.90 933 | 9 | 15 |
| 46 | 9.76 677 | 18 | 9.85 754 | 26 | 10.14 246 | 9.90 924 | 9 | 14 |
| 47 | 9.76 695 | 17 | 9.85 780 | 27 | 10.14 220 | 9.90 915 | 9 | 13 |
| 48 | 9.76 712 | 18 | 9.85 807 | 27 | 10.14 193 | 9.90 906 | 10 | 12 |
| 49 | 9.76 730 | 17 | 9.85 834 | 26 | 10.14 166 | 9.90 896 | 9 | 11 |
| **50** | 9.76 747 | 18 | 9.85 860 | 27 | 10.14 140 | 9.90 887 | 9 | **10** |
| 51 | 9.76 765 | 17 | 9.85 887 | 26 | 10.14 113 | 9.90 878 | 9 | 9 |
| 52 | 9.76 782 | 18 | 9.85 913 | 27 | 10.14 087 | 9.90 869 | 9 | 8 |
| 53 | 9.76 800 | 17 | 9.85 940 | 27 | 10.14 060 | 9.90 860 | 9 | 7 |
| 54 | 9.76 817 | 18 | 9.85 967 | 26 | 10.14 033 | 9.90 851 | 9 | 6 |
| 55 | 9.76 835 | 17 | 9.85 993 | 27 | 10.14 007 | 9.90 842 | 9 | 5 |
| 56 | 9.76 852 | 18 | 9.86 020 | 26 | 10.13 980 | 9.90 832 | 10 | 4 |
| 57 | 9.76 870 | 17 | 9.86 046 | 27 | 10.13 954 | 9.90 823 | 9 | 3 |
| 58 | 9.76 887 | 17 | 9.86 073 | 27 | 10.13 927 | 9.90 814 | 9 | 2 |
| 59 | 9.76 904 | 18 | 9.86 100 | 26 | 10.13 900 | 9.90 805 | 9 | 1 |
| **60** | 9.76 922 | | 9.86 126 | | 10.13 874 | 9.90 796 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

′

| ″ | 27 | 26 | 18 |
|---|---|---|---|
| 6 | 2.7 | 2.6 | 1.8 |
| 7 | 3.2 | 3.0 | 2.1 |
| 8 | 3.6 | 3.5 | 2.4 |
| 9 | 4.0 | 3.9 | 2.7 |
| 10 | 4.5 | 4.3 | 3.0 |
| 20 | 9.0 | 8.7 | 6.0 |
| 30 | 13.5 | 13.0 | 9.0 |
| 40 | 18.0 | 17.3 | 12.0 |
| 50 | 22.5 | 21.7 | 15.0 |

| ″ | 17 | 16 |
|---|---|---|
| 6 | 1.7 | 1.0 |
| 7 | 2.0 | 1.2 |
| 8 | 2.3 | 1.3 |
| 9 | 2.6 | 1.5 |
| 10 | 2.8 | 1.7 |
| 20 | 5.7 | 3.3 |
| 30 | 8.5 | 5.0 |
| 40 | 11.3 | 6.7 |
| 50 | 14.2 | 8.3 |

| ″ | 9 | 8 |
|---|---|---|
| 6 | 0.9 | 0.8 |
| 7 | 1.0 | 0.9 |
| 8 | 1.2 | 1.1 |
| 9 | 1.4 | 1.2 |
| 10 | 1.5 | 1.3 |
| 20 | 3.0 | 2.7 |
| 30 | 4.5 | 4.0 |
| 40 | 6.0 | 5.3 |
| 50 | 7.5 | 6.7 |

## 54°

## 36°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.76 922 | 17 | 9.86 126 | 27 | 10.13 874 | 9.90 796 | 9 | **60** |
| 1 | 9.76 939 | 18 | 9.86 153 | 26 | 10.13 847 | 9.90 787 | 10 | 59 |
| 2 | 9.76 957 | 17 | 9.86 179 | 27 | 10.13 821 | 9.90 777 | 9 | 58 |
| 3 | 9.76 974 | 17 | 9.86 206 | 26 | 10.13 794 | 9.90 768 | 9 | 57 |
| 4 | 9.76 991 | 18 | 9.86 232 | 27 | 10.13 768 | 9.90 759 | 9 | 56 |
| 5 | 9.77 009 | 17 | 9.86 259 | 26 | 10.13 741 | 9.90 750 | 9 | 55 |
| 6 | 9.77 026 | 18 | 9.86 285 | 27 | 10.13 715 | 9.90 741 | 10 | 54 |
| 7 | 9.77 043 | 17 | 9.86 312 | 26 | 10.13 688 | 9.90 731 | 9 | 53 |
| 8 | 9.77 061 | 17 | 9.86 338 | 27 | 10.13 662 | 9.90 722 | 9 | 52 |
| 9 | 9.77 078 | 17 | 9.86 365 | 27 | 10.13 635 | 9.90 713 | 9 | 51 |
| **10** | 9.77 095 | 17 | 9.86 392 | 26 | 10.13 608 | 9.90 704 | 10 | **50** |
| 11 | 9.77 112 | 18 | 9.86 418 | 27 | 10.13 582 | 9.90 694 | 9 | 49 |
| 12 | 9.77 130 | 17 | 9.86 445 | 26 | 10.13 555 | 9.90 685 | 9 | 48 |
| 13 | 9.77 147 | 17 | 9.86 471 | 27 | 10.13 529 | 9.90 676 | 9 | 47 |
| 14 | 9.77 164 | 17 | 9.86 498 | 26 | 10.13 502 | 9.90 667 | 10 | 46 |
| 15 | 9.77 181 | 18 | 9.86 524 | 27 | 10.13 476 | 9.90 657 | 9 | 45 |
| 16 | 9.77 199 | 17 | 9.86 551 | 26 | 10.13 449 | 9.90 648 | 9 | 44 |
| 17 | 9.77 216 | 17 | 9.86 577 | 26 | 10.13 423 | 9.90 639 | 9 | 43 |
| 18 | 9.77 233 | 17 | 9.86 603 | 27 | 10.13 397 | 9.90 630 | 10 | 42 |
| 19 | 9.77 250 | 18 | 9.86 630 | 26 | 10.13 370 | 9.90 620 | 9 | 41 |
| **20** | 9.77 268 | 17 | 9.86 656 | 27 | 10.13 344 | 9.90 611 | 9 | **40** |
| 21 | 9.77 285 | 17 | 9.86 683 | 26 | 10.13 317 | 9.90 602 | 10 | 39 |
| 22 | 9.77 302 | 17 | 9.86 709 | 27 | 10.13 291 | 9.90 592 | 9 | 38 |
| 23 | 9.77 319 | 17 | 9.86 736 | 26 | 10.13 264 | 9.90 583 | 9 | 37 |
| 24 | 9.77 336 | 17 | 9.86 762 | 27 | 10.13 238 | 9.90 574 | 9 | 36 |
| 25 | 9.77 353 | 17 | 9.86 789 | 26 | 10.13 211 | 9.90 565 | 10 | 35 |
| 26 | 9.77 370 | 17 | 9.86 815 | 27 | 10.13 185 | 9.90 555 | 9 | 34 |
| 27 | 9.77 387 | 18 | 9.86 842 | 26 | 10.13 158 | 9.90 546 | 9 | 33 |
| 28 | 9.77 405 | 17 | 9.86 868 | 26 | 10.13 132 | 9.90 537 | 10 | 32 |
| 29 | 9.77 422 | 17 | 9.86 894 | 27 | 10.13 106 | 9.90 527 | 9 | 31 |
| **30** | 9.77 439 | 17 | 9.86 921 | 26 | 10.13 079 | 9.90 518 | 9 | **30** |
| 31 | 9.77 456 | 17 | 9.86 947 | 27 | 10.13 053 | 9.90 509 | 10 | 29 |
| 32 | 9.77 473 | 17 | 9.86 974 | 26 | 10.13 026 | 9.90 499 | 9 | 28 |
| 33 | 9.77 490 | 17 | 9.87 000 | 27 | 10.13 000 | 9.90 490 | 10 | 27 |
| 34 | 9.77 507 | 17 | 9.87 027 | 26 | 10.12 973 | 9.90 480 | 9 | 26 |
| 35 | 9.77 524 | 17 | 9.87 053 | 26 | 10.12 947 | 9.90 471 | 9 | 25 |
| 36 | 9.77 541 | 17 | 9.87 079 | 27 | 10.12 921 | 9.90 462 | 10 | 24 |
| 37 | 9.77 558 | 17 | 9.87 106 | 26 | 10.12 894 | 9.90 452 | 9 | 23 |
| 38 | 9.77 575 | 17 | 9.87 132 | 26 | 10.12 868 | 9.90 443 | 9 | 22 |
| 39 | 9.77 592 | 17 | 9.87 158 | 27 | 10.12 842 | 9.90 434 | 10 | 21 |
| **40** | 9.77 609 | 17 | 9.87 185 | 26 | 10.12 815 | 9.90 424 | 9 | **20** |
| 41 | 9.77 626 | 17 | 9.87 211 | 27 | 10.12 789 | 9.90 415 | 10 | 19 |
| 42 | 9.77 643 | 17 | 9.87 238 | 26 | 10.12 762 | 9.90 405 | 9 | 18 |
| 43 | 9.77 660 | 17 | 9.87 264 | 26 | 10.12 736 | 9.90 396 | 10 | 17 |
| 44 | 9.77 677 | 17 | 9.87 290 | 27 | 10.12 710 | 9.90 386 | 9 | 16 |
| 45 | 9.77 694 | 17 | 9.87 317 | 26 | 10.12 683 | 9.90 377 | 9 | 15 |
| 46 | 9.77 711 | 17 | 9.87 343 | 26 | 10.12 657 | 9.90 368 | 10 | 14 |
| 47 | 9.77 728 | 16 | 9.87 369 | 27 | 10.12 631 | 9.90 358 | 9 | 13 |
| 48 | 9.77 744 | 17 | 9.87 396 | 26 | 10.12 604 | 9.90 349 | 10 | 12 |
| 49 | 9.77 761 | 17 | 9.87 422 | 26 | 10.12 578 | 9.90 339 | 9 | 11 |
| **50** | 9.77 778 | 17 | 9.87 448 | 27 | 10.12 552 | 9.90 330 | 10 | **10** |
| 51 | 9.77 795 | 17 | 9.87 475 | 26 | 10.12 525 | 9.90 320 | 9 | 9 |
| 52 | 9.77 812 | 17 | 9.87 501 | 26 | 10.12 499 | 9.90 311 | 10 | 8 |
| 53 | 9.77 829 | 17 | 9.87 527 | 27 | 10.12 473 | 9.90 301 | 9 | 7 |
| 54 | 9.77 846 | 16 | 9.87 554 | 26 | 10.12 446 | 9.90 292 | 10 | 6 |
| 55 | 9.77 862 | 17 | 9.87 580 | 26 | 10.12 420 | 9.90 282 | 9 | 5 |
| 56 | 9.77 879 | 17 | 9.87 606 | 27 | 10.12 394 | 9.90 273 | 10 | 4 |
| 57 | 9.77 896 | 17 | 9.87 633 | 26 | 10.12 367 | 9.90 263 | 9 | 3 |
| 58 | 9.77 913 | 17 | 9.87 659 | 26 | 10.12 341 | 9.90 254 | 10 | 2 |
| 59 | 9.77 930 | 16 | 9.87 685 | 26 | 10.12 315 | 9.90 244 | 9 | 1 |
| **60** | 9.77 946 | | 9.87 711 | | 10.12 289 | 9.90 235 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

**Prop. Pts.**

| ″ | 27 | 26 | 18 |
|---|---|---|---|
| 6 | 2.7 | 2.6 | 1.8 |
| 7 | 3.2 | 3.0 | 2.1 |
| 8 | 3.6 | 3.5 | 2.4 |
| 9 | 4.0 | 3.9 | 2.7 |
| 10 | 4.5 | 4.3 | 3.0 |
| 20 | 9.0 | 8.7 | 6.0 |
| 30 | 13.5 | 13.0 | 9.0 |
| 40 | 18.0 | 17.3 | 12.0 |
| 50 | 22.5 | 21.7 | 15.0 |

| ″ | 17 | 16 |
|---|---|---|
| 6 | 1.7 | 1.6 |
| 7 | 2.0 | 1.9 |
| 8 | 2.3 | 2.1 |
| 9 | 2.6 | 2.4 |
| 10 | 2.8 | 2.7 |
| 20 | 5.7 | 5.3 |
| 30 | 8.5 | 8.0 |
| 40 | 11.3 | 10.7 |
| 50 | 14.2 | 13.3 |

| ″ | 10 | 9 |
|---|---|---|
| 6 | 1.0 | 0.9 |
| 7 | 1.2 | 1.0 |
| 8 | 1.3 | 1.2 |
| 9 | 1.5 | 1.4 |
| 10 | 1.7 | 1.5 |
| 20 | 3.3 | 3.0 |
| 30 | 5.0 | 4.5 |
| 40 | 6.7 | 6.0 |
| 50 | 8.3 | 7.5 |

Prop. Pts.

## 53°

## 37°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.77 946 | | 9.87 711 | | 10.12 289 | 9.90 235 | | 60 | |
| 1 | 9.77 963 | 17 | 9.87 738 | 27 | 10.12 262 | 9.90 225 | 10 | 59 | |
| 2 | 9.77 980 | 17 | 9.87 764 | 26 | 10.12 236 | 9.90 216 | 9 | 58 | |
| 3 | 9.77 997 | 17 | 9.87 790 | 26 | 10.12 210 | 9.90 206 | 10 | 57 | |
| 4 | 9.78 013 | 16 | 9.87 817 | 27 | 10.12 183 | 9.90 197 | 9 | 56 | |
| 5 | 9.78 030 | 17 | 9.87 843 | 26 | 10.12 157 | 9.90 187 | 10 | 55 | |
| 6 | 9.78 047 | 17 | 9.87 869 | 26 | 10.12 131 | 9.90 178 | 9 | 54 | |
| 7 | 9.78 063 | 16 | 9.87 895 | 26 | 10.12 105 | 9.90 168 | 10 | 53 | |
| 8 | 9.78 080 | 17 | 9.87 922 | 27 | 10.12 078 | 9.90 159 | 9 | 52 | |
| 9 | 9.78 097 | 17 | 9.87 948 | 26 | 10.12 052 | 9.90 149 | 10 | 51 | |
| 10 | 9.78 113 | 16 | 9.87 974 | 26 | 10.12 026 | 9.90 139 | 10 | 50 | |
| 11 | 9.78 130 | 17 | 9.88 000 | 26 | 10.12 000 | 9.90 130 | 9 | 49 | |
| 12 | 9.78 147 | 17 | 9.88 027 | 27 | 10.11 973 | 9.90 120 | 10 | 48 | |
| 13 | 9.78 163 | 16 | 9.88 053 | 26 | 10.11 947 | 9.90 111 | 9 | 47 | |
| 14 | 9.78 180 | 17 | 9.88 079 | 26 | 10.11 921 | 9.90 101 | 10 | 46 | |
| 15 | 9.78 197 | 17 | 9.88 105 | 26 | 10.11 895 | 9.90 091 | 10 | 45 | |
| 16 | 9.78 213 | 16 | 9.88 131 | 26 | 10.11 869 | 9.90 082 | 9 | 44 | |
| 17 | 9.78 230 | 17 | 9.88 158 | 27 | 10.11 842 | 9.90 072 | 10 | 43 | |
| 18 | 9.78 246 | 16 | 9.88 184 | 26 | 10.11 816 | 9.90 063 | 9 | 42 | |
| 19 | 9.78 263 | 17 | 9.88 210 | 26 | 10.11 790 | 9.90 053 | 10 | 41 | |
| 20 | 9.78 280 | 17 | 9.88 236 | 26 | 10.11 764 | 9.90 043 | 10 | 40 | |
| 21 | 9.78 296 | 16 | 9.88 262 | 26 | 10.11 738 | 9.90 034 | 9 | 39 | |
| 22 | 9.78 313 | 17 | 9.88 289 | 27 | 10.11 711 | 9.90 024 | 10 | 38 | |
| 23 | 9.78 329 | 16 | 9.88 315 | 26 | 10.11 685 | 9.90 014 | 10 | 37 | |
| 24 | 9.78 346 | 17 | 9.88 341 | 26 | 10.11 659 | 9.90 005 | 9 | 36 | |
| 25 | 9.78 362 | 16 | 9.88 367 | 26 | 10.11 633 | 9.89 995 | 10 | 35 | |
| 26 | 9.78 379 | 17 | 9.88 393 | 26 | 10.11 607 | 9.89 985 | 10 | 34 | |
| 27 | 9.78 395 | 16 | 9.88 420 | 27 | 10.11 580 | 9.89 976 | 9 | 33 | |
| 28 | 9.78 412 | 17 | 9.88 446 | 26 | 10.11 554 | 9.89 966 | 10 | 32 | |
| 29 | 9.78 428 | 16 | 9.88 472 | 26 | 10.11 528 | 9.89 956 | 10 | 31 | |
| 30 | 9.78 445 | 17 | 9.88 498 | 26 | 10.11 502 | 9.89 947 | 9 | 30 | |
| 31 | 9.78 461 | 16 | 9.88 524 | 26 | 10.11 476 | 9.89 937 | 10 | 29 | |
| 32 | 9.78 478 | 17 | 9.88 550 | 26 | 10.11 450 | 9.89 927 | 10 | 28 | |
| 33 | 9.78 494 | 16 | 9.88 577 | 27 | 10.11 423 | 9.89 918 | 9 | 27 | |
| 34 | 9.78 510 | 16 | 9.88 603 | 26 | 10.11 397 | 9.89 908 | 10 | 26 | |
| 35 | 9.78 527 | 17 | 9.88 629 | 26 | 10.11 371 | 9.89 898 | 10 | 25 | |
| 36 | 9.78 543 | 16 | 9.88 655 | 26 | 10.11 345 | 9.89 888 | 10 | 24 | |
| 37 | 9.78 560 | 17 | 9.88 681 | 26 | 10.11 319 | 9.89 879 | 9 | 23 | |
| 38 | 9.78 576 | 16 | 9.88 707 | 26 | 10.11 293 | 9.89 869 | 10 | 22 | |
| 39 | 9.78 592 | 16 | 9.88 733 | 26 | 10.11 267 | 9.89 859 | 10 | 21 | |
| 40 | 9.78 609 | 17 | 9.88 759 | 26 | 10.11 241 | 9.89 849 | 10 | 20 | |
| 41 | 9.78 625 | 16 | 9.88 786 | 27 | 10.11 214 | 9.89 840 | 9 | 19 | |
| 42 | 9.78 642 | 17 | 9.88 812 | 26 | 10.11 188 | 9.89 830 | 10 | 18 | |
| 43 | 9.78 658 | 16 | 9.88 838 | 26 | 10.11 162 | 9.89 820 | 10 | 17 | |
| 44 | 9.78 674 | 16 | 9.88 864 | 26 | 10.11 136 | 9.89 810 | 10 | 16 | |
| 45 | 9.78 691 | 17 | 9.88 890 | 26 | 10.11 110 | 9.89 801 | 9 | 15 | |
| 46 | 9.78 707 | 16 | 9.88 916 | 26 | 10.11 084 | 9.89 791 | 10 | 14 | |
| 47 | 9.78 723 | 16 | 9.88 942 | 26 | 10.11 058 | 9.89 781 | 10 | 13 | |
| 48 | 9.78 739 | 16 | 9.88 968 | 26 | 10.11 032 | 9.89 771 | 10 | 12 | |
| 49 | 9.78 756 | 17 | 9.88 994 | 26 | 10.11 006 | 9.89 761 | 10 | 11 | |
| 50 | 9.78 772 | 16 | 9.89 020 | 26 | 10.10 980 | 9.89 752 | 9 | 10 | |
| 51 | 9.78 788 | 16 | 9.89 046 | 27 | 10.10 954 | 9.89 742 | 10 | 9 | |
| 52 | 9.78 805 | 17 | 9.89 073 | 26 | 10.10 927 | 9.89 732 | 10 | 8 | |
| 53 | 9.78 821 | 16 | 9.89 099 | 26 | 10.10 901 | 9.89 722 | 10 | 7 | |
| 54 | 9.78 837 | 16 | 9.89 125 | 26 | 10.10 875 | 9.89 712 | 10 | 6 | |
| 55 | 9.78 853 | 16 | 9.89 151 | 26 | 10.10 849 | 9.89 702 | 10 | 5 | |
| 56 | 9.78 869 | 16 | 9.89 177 | 26 | 10.10 823 | 9.89 693 | 9 | 4 | |
| 57 | 9.78 886 | 17 | 9.89 203 | 26 | 10.10 797 | 9.89 683 | 10 | 3 | |
| 58 | 9.78 902 | 16 | 9.89 229 | 26 | 10.10 771 | 9.89 673 | 10 | 2 | |
| 59 | 9.78 918 | 16 | 9.89 255 | 26 | 10.10 745 | 9.89 663 | 10 | 1 | |
| 60 | 9.78 934 | 16 | 9.89 281 | 26 | 10.10 719 | 9.89 653 | 10 | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

| ″ | 27 | 26 | 17 |
|---|---|---|---|
| 6 | 2.7 | 2.6 | 1.7 |
| 7 | 3.2 | 3.0 | 2.0 |
| 8 | 3.6 | 3.5 | 2.3 |
| 9 | 4.0 | 3.9 | 2.6 |
| 10 | 4.5 | 4.3 | 2.8 |
| 20 | 9.0 | 8.7 | 5.7 |
| 30 | 13.5 | 13.0 | 8.5 |
| 40 | 18.0 | 17.3 | 11.3 |
| 50 | 22.5 | 21.7 | 14.2 |

| ″ | 16 | 10 | 9 |
|---|---|---|---|
| 6 | 1.6 | 1.0 | 0.9 |
| 7 | 1.9 | 1.2 | 1.0 |
| 8 | 2.1 | 1.3 | 1.2 |
| 9 | 2.4 | 1.5 | 1.4 |
| 10 | 2.7 | 1.7 | 1.5 |
| 20 | 5.3 | 3.3 | 3.0 |
| 30 | 8.0 | 5.0 | 4.5 |
| 40 | 10.7 | 6.7 | 6.0 |
| 50 | 13.3 | 8.3 | 7.5 |

## 52°

## 38°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.78 934 | | 9.89 281 | | 10.10 719 | 9.89 653 | | **60** | |
| 1 | 9.78 950 | 16 | 9.89 307 | 26 | 10.10 693 | 9.89 643 | 10 | 59 | |
| 2 | 9.78 967 | 17 | 9.89 333 | 26 | 10.10 667 | 9.89 633 | 10 | 58 | |
| 3 | 9.78 983 | 16 | 9.89 359 | 26 | 10.10 641 | 9.89 624 | 9 | 57 | |
| 4 | 9.78 999 | 16 | 9.89 385 | 26 | 10.10 615 | 9.89 614 | 10 | 56 | |
| 5 | 9.79 015 | 16 | 9.89 411 | 26 | 10.10 589 | 9.89 604 | 10 | 55 | ″ \| **26** \| **25** |
| 6 | 9.79 031 | 16 | 9.89 437 | 26 | 10.10 563 | 9.89 594 | 10 | 54 | |
| 7 | 9.79 047 | 16 | 9.89 463 | 26 | 10.10 537 | 9.89 584 | 10 | 53 | 6 \| 2.6 \| 2.5 |
| 8 | 9.79 063 | 16 | 9.89 489 | 26 | 10.10 511 | 9.89 574 | 10 | 52 | 7 \| 3.0 \| 2.9 |
| 9 | 9.79 079 | 16 | 9.89 515 | 26 | 10.10 485 | 9.89 564 | 10 | 51 | 8 \| 3.5 \| 3.3 |
| **10** | 9.79 095 | 16 | 9.89 541 | 26 | 10.10 459 | 9.89 554 | 10 | **50** | 9 \| 3.9 \| 3.8 |
| 11 | 9.79 111 | 16 | 9.89 567 | 26 | 10.10 433 | 9.89 544 | 10 | 49 | 10 \| 4.3 \| 4.2 |
| 12 | 9.79 128 | 17 | 9.89 593 | 26 | 10.10 407 | 9.89 534 | 10 | 48 | 20 \| 8.7 \| 8.3 |
| 13 | 9.79 144 | 16 | 9.89 619 | 26 | 10.10 381 | 9.89 524 | 10 | 47 | 30 \| 13.0 \| 12.5 |
| 14 | 9.79 160 | 16 | 9.89 645 | 26 | 10.10 355 | 9.89 514 | 10 | 46 | 40 \| 17.3 \| 16.7 |
| 15 | 9.79 176 | 16 | 9.89 671 | 26 | 10.10 329 | 9.89 504 | 10 | 45 | 50 \| 21.7 \| 20.8 |
| 16 | 9.79 192 | 16 | 9.89 697 | 26 | 10.10 303 | 9.89 495 | 9 | 44 | |
| 17 | 9.79 208 | 16 | 9.89 723 | 26 | 10.10 277 | 9.89 485 | 10 | 43 | |
| 18 | 9.79 224 | 16 | 9.89 749 | 26 | 10.10 251 | 9.89 475 | 10 | 42 | |
| 19 | 9.79 240 | 16 | 9.89 775 | 26 | 10.10 225 | 9.89 465 | 10 | 41 | ″ \| **17** \| **16** |
| **20** | 9.79 256 | 16 | 9.89 801 | 26 | 10.10 199 | 9.89 455 | 10 | **40** | |
| 21 | 9.79 272 | 16 | 9.89 827 | 26 | 10.10 173 | 9.89 445 | 10 | 39 | 6 \| 1.7 \| 1.6 |
| 22 | 9.79 288 | 16 | 9.89 853 | 26 | 10.10 147 | 9.89 435 | 10 | 38 | 7 \| 2.0 \| 1.9 |
| 23 | 9.79 304 | 16 | 9.89 879 | 26 | 10.10 121 | 9.89 425 | 10 | 37 | 8 \| 2.3 \| 2.1 |
| 24 | 9.79 319 | 15 | 9.89 905 | 26 | 10.10 095 | 9.89 415 | 10 | 36 | 9 \| 2.6 \| 2.4 |
| 25 | 9.79 335 | 16 | 9.89 931 | 26 | 10.10 069 | 9.89 405 | 10 | 35 | 10 \| 2.8 \| 2.7 |
| 26 | 9.79 351 | 16 | 9.89 957 | 26 | 10.10 043 | 9.89 395 | 10 | 34 | 20 \| 5.7 \| 5.3 |
| 27 | 9.79 367 | 16 | 9.89 983 | 26 | 10.10 017 | 9.89 385 | 10 | 33 | 30 \| 8.5 \| 8.0 |
| 28 | 9.79 383 | 16 | 9.90 009 | 26 | 10.09 991 | 9.89 375 | 10 | 32 | 40 \| 11.3 \| 10.7 |
| 29 | 9.79 399 | 16 | 9.90 035 | 26 | 10.09 965 | 9.89 364 | 11 | 31 | 50 \| 14.2 \| 13.3 |
| **30** | 9.79 415 | 16 | 9.90 061 | 25 | 10.09 939 | 9.89 354 | 10 | **30** | |
| 31 | 9.79 431 | 16 | 9.90 086 | 26 | 10.09 914 | 9.89 344 | 10 | 29 | |
| 32 | 9.79 447 | 16 | 9.90 112 | 26 | 10.09 888 | 9.89 334 | 10 | 28 | |
| 33 | 9.79 463 | 16 | 9.90 138 | 26 | 10.09 862 | 9.89 324 | 10 | 27 | ″ \| **15** \| **11** |
| 34 | 9.79 478 | 15 | 9.90 164 | 26 | 10.09 836 | 9.89 314 | 10 | 26 | |
| 35 | 9.79 494 | 16 | 9.90 190 | 26 | 10.09 810 | 9.89 304 | 10 | 25 | 6 \| 1.5 \| 1.1 |
| 36 | 9.79 510 | 16 | 9.90 216 | 26 | 10.09 784 | 9.89 294 | 10 | 24 | 7 \| 1.8 \| 1.3 |
| 37 | 9.79 526 | 16 | 9.90 242 | 26 | 10.09 758 | 9.89 284 | 10 | 23 | 8 \| 2.0 \| 1.5 |
| 38 | 9.79 542 | 16 | 9.90 268 | 26 | 10.09 732 | 9.89 274 | 10 | 22 | 9 \| 2.2 \| 1.6 |
| 39 | 9.79 558 | 16 | 9.90 294 | 26 | 10.09 706 | 9.89 264 | 10 | 21 | 10 \| 2.5 \| 1.8 |
| **40** | 9.79 573 | 15 | 9.90 320 | 26 | 10.09 680 | 9.89 254 | 10 | **20** | 20 \| 5.0 \| 3.7 |
| 41 | 9.79 589 | 16 | 9.90 346 | 26 | 10.09 654 | 9.89 244 | 10 | 19 | 30 \| 7.5 \| 5.5 |
| 42 | 9.79 605 | 16 | 9.90 371 | 25 | 10.09 629 | 9.89 233 | 11 | 18 | 40 \| 10.0 \| 7.3 |
| 43 | 9.79 621 | 16 | 9.90 397 | 26 | 10.09 603 | 9.89 223 | 10 | 17 | 50 \| 12.5 \| 9.2 |
| 44 | 9.79 636 | 15 | 9.90 423 | 26 | 10.09 577 | 9.89 213 | 10 | 16 | |
| 45 | 9.79 652 | 16 | 9.90 449 | 26 | 10.09 551 | 9.89 203 | 10 | 15 | |
| 46 | 9.79 668 | 16 | 9.90 475 | 26 | 10.09 525 | 9.89 193 | 10 | 14 | |
| 47 | 9.79 684 | 16 | 9.90 501 | 26 | 10.09 499 | 9.89 183 | 10 | 13 | ″ \| **10** \| **9** |
| 48 | 9.79 699 | 15 | 9.90 527 | 26 | 10.09 473 | 9.89 173 | 10 | 12 | |
| 49 | 9.79 715 | 16 | 9.90 553 | 25 | 10.09 447 | 9.89 162 | 11 | 11 | 6 \| 1.0 \| 0.9 |
| **50** | 9.79 731 | 16 | 9.90 578 | 26 | 10.09 422 | 9.89 152 | 10 | **10** | 7 \| 1.2 \| 1.0 |
| 51 | 9.79 746 | 15 | 9.90 604 | 26 | 10.09 396 | 9.89 142 | 10 | 9 | 8 \| 1.3 \| 1.2 |
| 52 | 9.79 762 | 16 | 9.90 630 | 26 | 10.09 370 | 9.89 132 | 10 | 8 | 9 \| 1.5 \| 1.4 |
| 53 | 9.79 778 | 16 | 9.90 656 | 26 | 10.09 344 | 9.89 122 | 10 | 7 | 10 \| 1.7 \| 1.5 |
| 54 | 9.79 793 | 15 | 9.90 682 | 26 | 10.09 318 | 9.89 112 | 10 | 6 | 20 \| 3.3 \| 3.0 |
| 55 | 9.79 809 | 16 | 9.90 708 | 26 | 10.09 292 | 9.89 101 | 11 | 5 | 30 \| 5.0 \| 4.5 |
| 56 | 9.79 825 | 16 | 9.90 734 | 26 | 10.09 266 | 9.89 091 | 10 | 4 | 40 \| 6.7 \| 6.0 |
| 57 | 9.79 840 | 15 | 9.90 759 | 25 | 10.09 241 | 9.89 081 | 10 | 3 | 50 \| 8.3 \| 7.5 |
| 58 | 9.79 856 | 16 | 9.90 785 | 26 | 10.09 215 | 9.89 071 | 10 | 2 | |
| 59 | 9.79 872 | 16 | 9.90 811 | 26 | 10.09 189 | 9.89 060 | 11 | 1 | |
| **60** | 9.79 887 | 15 | 9.90 837 | 26 | 10.09 163 | 9.89 050 | 10 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 51°

## 39°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.79 887 | | 9.90 837 | | 10.09 163 | 9.89 050 | | **60** | |
| 1 | 9.79 903 | 16 | 9.90 863 | 26 | 10.09 137 | 9.89 040 | 10 | 59 | |
| 2 | 9.79 918 | 15 | 9.90 889 | 26 | 10.09 111 | 9.89 030 | 10 | 58 | |
| 3 | 9.79 934 | 16 | 9.90 914 | 25 | 10.09 086 | 9.89 020 | 10 | 57 | |
| 4 | 9.79 950 | 16 | 9.90 940 | 26 | 10.09 060 | 9.89 009 | 11 | 56 | |
| 5 | 9.79 965 | 15 | 9.90 966 | 26 | 10.09 034 | 9.88 999 | 10 | 55 | |
| 6 | 9.79 981 | 16 | 9.90 992 | 26 | 10.09 008 | 9.88 989 | 10 | 54 | |
| 7 | 9.79 996 | 15 | 9.91 018 | 26 | 10.08 982 | 9.88 978 | 11 | 53 | |
| 8 | 9.80 012 | 16 | 9.91 043 | 25 | 10.08 957 | 9.88 968 | 10 | 52 | ″ \| **26** \| **25** |
| 9 | 9.80 027 | 15 | 9.91 069 | 26 | 10.08 931 | 9.88 958 | 10 | 51 | |
| **10** | 9.80 043 | 16 | 9.91 095 | 26 | 10.08 905 | 9.88 948 | 10 | **50** | 6 \| 2.6 \| 2.5 |
| 11 | 9.80 058 | 15 | 9.91 121 | 26 | 10.08 879 | 9.88 937 | 11 | 49 | 7 \| 3.0 \| 2.9 |
| 12 | 9.80 074 | 16 | 9.91 147 | 26 | 10.08 853 | 9.88 927 | 10 | 48 | 8 \| 3.5 \| 3.3 |
| 13 | 9.80 089 | 15 | 9.91 172 | 25 | 10.08 828 | 9.88 917 | 10 | 47 | 9 \| 3.9 \| 3.8 |
| 14 | 9.80 105 | 16 | 9.91 198 | 26 | 10.08 802 | 9.88 906 | 11 | 46 | 10 \| 4.3 \| 4.2 |
| 15 | 9.80 120 | 15 | 9.91 224 | 26 | 10.08 776 | 9.88 896 | 10 | 45 | 20 \| 8.7 \| 8.3 |
| 16 | 9.80 136 | 16 | 9.91 250 | 26 | 10.08 750 | 9.88 886 | 10 | 44 | 30 \| 13.0 \| 12.5 |
| 17 | 9.80 151 | 15 | 9.91 276 | 26 | 10.08 724 | 9.88 875 | 11 | 43 | 40 \| 17.3 \| 16.7 |
| 18 | 9.80 166 | 15 | 9.91 301 | 25 | 10.08 699 | 9.88 865 | 10 | 42 | 50 \| 21.7 \| 20.8 |
| 19 | 9.80 182 | 16 | 9.91 327 | 26 | 10.08 673 | 9.88 855 | 10 | 41 | |
| **20** | 9.80 197 | 15 | 9.91 353 | 26 | 10.08 647 | 9.88 844 | 11 | **40** | |
| 21 | 9.80 213 | 16 | 9.91 379 | 26 | 10.08 621 | 9.88 834 | 10 | 39 | |
| 22 | 9.80 228 | 15 | 9.91 404 | 25 | 10.08 596 | 9.88 824 | 10 | 38 | |
| 23 | 9.80 244 | 16 | 9.91 430 | 26 | 10.08 570 | 9.88 813 | 11 | 37 | |
| 24 | 9.80 259 | 15 | 9.91 456 | 26 | 10.08 544 | 9.88 803 | 10 | 36 | |
| 25 | 9.80 274 | 15 | 9.91 482 | 26 | 10.08 518 | 9.88 793 | 10 | 35 | |
| 26 | 9.80 290 | 16 | 9.91 507 | 25 | 10.08 493 | 9.88 782 | 11 | 34 | ″ \| **16** \| **15** |
| 27 | 9.80 305 | 15 | 9.91 533 | 26 | 10.08 467 | 9.88 772 | 10 | 33 | |
| 28 | 9.80 320 | 15 | 9.91 559 | 26 | 10.08 441 | 9.88 761 | 11 | 32 | 6 \| 1.6 \| 1.5 |
| 29 | 9.80 336 | 16 | 9.91 585 | 26 | 10.08 415 | 9.88 751 | 10 | 31 | 7 \| 1.9 \| 1.8 |
| **30** | 9.80 351 | 15 | 9.91 610 | 25 | 10.08 390 | 9.88 741 | 10 | **30** | 8 \| 2.1 \| 2.0 |
| 31 | 9.80 366 | 15 | 9.91 636 | 26 | 10.08 364 | 9.88 730 | 11 | 29 | 9 \| 2.4 \| 2.2 |
| 32 | 9.80 382 | 16 | 9.91 662 | 26 | 10.08 338 | 9.88 720 | 10 | 28 | 10 \| 2.7 \| 2.5 |
| 33 | 9.80 397 | 15 | 9.91 688 | 26 | 10.08 312 | 9.88 709 | 11 | 27 | 20 \| 5.3 \| 5.0 |
| 34 | 9.80 412 | 15 | 9.91 713 | 25 | 10.08 287 | 9.88 699 | 10 | 26 | 30 \| 8.0 \| 7.5 |
| 35 | 9.80 428 | 16 | 9.91 739 | 26 | 10.08 261 | 9.88 688 | 11 | 25 | 40 \| 10.7 \| 10.0 |
| 36 | 9.80 443 | 15 | 9.91 765 | 26 | 10.08 235 | 9.88 678 | 10 | 24 | 50 \| 13.3 \| 12.5 |
| 37 | 9.80 458 | 15 | 9.91 791 | 26 | 10.08 209 | 9.88 668 | 10 | 23 | |
| 38 | 9.80 473 | 15 | 9.91 816 | 25 | 10.08 184 | 9.88 657 | 11 | 22 | |
| 39 | 9.80 489 | 16 | 9.91 842 | 26 | 10.08 158 | 9.88 647 | 10 | 21 | |
| **40** | 9.80 504 | 15 | 9.91 868 | 26 | 10.08 132 | 9.88 636 | 11 | **20** | |
| 41 | 9.80 519 | 15 | 9.91 893 | 25 | 10.08 107 | 9.88 626 | 10 | 19 | |
| 42 | 9.80 534 | 15 | 9.91 919 | 26 | 10.08 081 | 9.88 615 | 11 | 18 | |
| 43 | 9.80 550 | 16 | 9.91 945 | 26 | 10.08 055 | 9.88 605 | 10 | 17 | |
| 44 | 9.80 565 | 15 | 9.91 971 | 26 | 10.08 029 | 9.88 594 | 11 | 16 | |
| 45 | 9.80 580 | 15 | 9.91 996 | 25 | 10.08 004 | 9.88 584 | 10 | 15 | ″ \| **11** \| **10** |
| 46 | 9.80 595 | 15 | 9.92 022 | 26 | 10.07 978 | 9.88 573 | 11 | 14 | |
| 47 | 9.80 610 | 15 | 9.92 048 | 26 | 10.07 952 | 9.88 563 | 10 | 13 | 6 \| 1.1 \| 1.0 |
| 48 | 9.80 625 | 15 | 9.92 073 | 25 | 10.07 927 | 9.88 552 | 11 | 12 | 7 \| 1.3 \| 1.2 |
| 49 | 9.80 641 | 16 | 9.92 099 | 26 | 10.07 901 | 9.88 542 | 10 | 11 | 8 \| 1.5 \| 1.3 |
| **50** | 9.80 656 | 15 | 9.92 125 | 26 | 10.07 875 | 9.88 531 | 11 | **10** | 9 \| 1.6 \| 1.5 |
| 51 | 9.80 671 | 15 | 9.92 150 | 25 | 10.07 850 | 9.88 521 | 10 | 9 | 10 \| 1.8 \| 1.7 |
| 52 | 9.80 686 | 15 | 9.92 176 | 26 | 10.07 824 | 9.88 510 | 11 | 8 | 20 \| 3.7 \| 3.3 |
| 53 | 9.80 701 | 15 | 9.92 202 | 26 | 10.07 798 | 9.88 499 | 11 | 7 | 30 \| 5.5 \| 5.0 |
| 54 | 9.80 716 | 15 | 9.92 227 | 25 | 10.07 773 | 9.88 489 | 10 | 6 | 40 \| 7.3 \| 6.7 |
| 55 | 9.80 731 | 15 | 9.92 253 | 26 | 10.07 747 | 9.88 478 | 11 | 5 | 50 \| 9.2 \| 8.3 |
| 56 | 9.80 746 | 15 | 9.92 279 | 26 | 10.07 721 | 9.88 468 | 10 | 4 | |
| 57 | 9.80 762 | 16 | 9.92 304 | 25 | 10.07 696 | 9.88 457 | 11 | 3 | |
| 58 | 9.80 777 | 15 | 9.92 330 | 26 | 10.07 670 | 9.88 447 | 10 | 2 | |
| 59 | 9.80 792 | 15 | 9.92 356 | 26 | 10.07 644 | 9.88 436 | 11 | 1 | |
| **60** | 9.80 807 | 15 | 9.92 381 | 25 | 10.07 619 | 9.88 425 | 11 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 50°

## 40°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.80 807 | 15 | 9.92 381 | 26 | 10.07 619 | 9.88 425 | 10 | **60** | |
| 1 | 9.80 822 | 15 | 9.92 407 | 26 | 10.07 593 | 9.88 415 | 11 | 59 | |
| 2 | 9.80 837 | 15 | 9.92 433 | 25 | 10.07 567 | 9.88 404 | 10 | 58 | |
| 3 | 9.80 852 | 15 | 9.92 458 | 26 | 10.07 542 | 9.88 394 | 11 | 57 | |
| 4 | 9.80 867 | 15 | 9.92 484 | 26 | 10.07 516 | 9.88 383 | 11 | 56 | |
| 5 | 9.80 882 | 15 | 9.92 510 | 25 | 10.07 490 | 9.88 372 | 10 | 55 | |
| 6 | 9.80 897 | 15 | 9.92 535 | 26 | 10.07 465 | 9.88 362 | 11 | 54 | |
| 7 | 9.80 912 | 15 | 9.92 561 | 26 | 10.07 439 | 9.88 351 | 10 | 53 | ″ \| **26** \| **25** |
| 8 | 9.80 927 | 15 | 9.92 587 | 25 | 10.07 413 | 9.88 340 | 11 | 52 | |
| 9 | 9.80 942 | 15 | 9.92 612 | 26 | 10.07 388 | 9.88 330 | 11 | 51 | 6 \| 2.6 \| 2.5 |
| **10** | 9.80 957 | 15 | 9.92 638 | 25 | 10.07 362 | 9.88 319 | 11 | **50** | 7 \| 3.0 \| 2.9 |
| 11 | 9.80 972 | 15 | 9.92 663 | 26 | 10.07 337 | 9.88 308 | 11 | 49 | 8 \| 3.5 \| 3.3 |
| 12 | 9.80 987 | 15 | 9.92 689 | 26 | 10.07 311 | 9.88 298 | 10 | 48 | 9 \| 3.9 \| 3.8 |
| 13 | 9.81 002 | 15 | 9.92 715 | 25 | 10.07 285 | 9.88 287 | 11 | 47 | 10 \| 4.3 \| 4.2 |
| 14 | 9.81 017 | 15 | 9.92 740 | 26 | 10.07 260 | 9.88 276 | 11 | 46 | 20 \| 8.7 \| 8.3 |
| 15 | 9.81 032 | 15 | 9.92 766 | 26 | 10.07 234 | 9.88 266 | 10 | 45 | 30 \| 13.0 \| 12.5 |
| 16 | 9.81 047 | 14 | 9.92 792 | 25 | 10.07 208 | 9.88 255 | 11 | 44 | 40 \| 17.3 \| 16.7 |
| 17 | 9.81 061 | 15 | 9.92 817 | 26 | 10.07 183 | 9.88 244 | 11 | 43 | 50 \| 21.7 \| 20.8 |
| 18 | 9.81 076 | 15 | 9.92 843 | 25 | 10.07 157 | 9.88 234 | 10 | 42 | |
| 19 | 9.81 091 | 15 | 9.92 868 | 26 | 10.07 132 | 9.88 223 | 11 | 41 | |
| **20** | 9.81 106 | 15 | 9.92 894 | 26 | 10.07 106 | 9.88 212 | 11 | **40** | |
| 21 | 9.81 121 | 15 | 9.92 920 | 25 | 10.07 080 | 9.88 201 | 11 | 39 | |
| 22 | 9.81 136 | 15 | 9.92 945 | 26 | 10.07 055 | 9.88 191 | 10 | 38 | |
| 23 | 9.81 151 | 15 | 9.92 971 | 25 | 10.07 029 | 9.88 180 | 11 | 37 | |
| 24 | 9.81 166 | 14 | 9.92 996 | 26 | 10.07 004 | 9.88 169 | 11 | 36 | |
| 25 | 9.81 180 | 15 | 9.93 022 | 26 | 10.06 978 | 9.88 158 | 10 | 35 | |
| 26 | 9.81 195 | 15 | 9.93 048 | 25 | 10.06 952 | 9.88 148 | 11 | 34 | ″ \| **15** \| **14** |
| 27 | 9.81 210 | 15 | 9.93 073 | 26 | 10.06 927 | 9.88 137 | 11 | 33 | |
| 28 | 9.81 225 | 15 | 9.93 099 | 25 | 10.06 901 | 9.88 126 | 11 | 32 | 6 \| 1.5 \| 1.4 |
| 29 | 9.81 240 | 14 | 9.93 124 | 26 | 10.06 876 | 9.88 115 | 10 | 31 | 7 \| 1.8 \| 1.6 |
| **30** | 9.81 254 | 15 | 9.93 150 | 25 | 10.06 850 | 9.88 105 | 11 | **30** | 8 \| 2.0 \| 1.9 |
| 31 | 9.81 269 | 15 | 9.93 175 | 26 | 10.06 825 | 9.88 094 | 11 | 29 | 9 \| 2.2 \| 2.1 |
| 32 | 9.81 284 | 15 | 9.93 201 | 26 | 10.06 799 | 9.88 083 | 11 | 28 | 10 \| 2.5 \| 2.3 |
| 33 | 9.81 299 | 15 | 9.93 227 | 25 | 10.06 773 | 9.88 072 | 11 | 27 | 20 \| 5.0 \| 4.7 |
| 34 | 9.81 314 | 14 | 9.93 252 | 26 | 10.06 748 | 9.88 061 | 10 | 26 | 30 \| 7.5 \| 7.0 |
| 35 | 9.81 328 | 15 | 9.93 278 | 25 | 10.06 722 | 9.88 051 | 11 | 25 | 40 \| 10.0 \| 9.3 |
| 36 | 9.81 343 | 15 | 9.93 303 | 26 | 10.06 697 | 9.88 040 | 11 | 24 | 50 \| 12.5 \| 11.7 |
| 37 | 9.81 358 | 15 | 9.93 329 | 25 | 10.06 671 | 9.88 029 | 11 | 23 | |
| 38 | 9.81 372 | 14 | 9.93 354 | 26 | 10.06 646 | 9.88 018 | 11 | 22 | |
| 39 | 9.81 387 | 15 | 9.93 380 | 26 | 10.06 620 | 9.88 007 | 11 | 21 | |
| **40** | 9.81 402 | 15 | 9.93 406 | 25 | 10.06 594 | 9.87 996 | 11 | **20** | |
| 41 | 9.81 417 | 14 | 9.93 431 | 26 | 10.06 569 | 9.87 985 | 11 | 19 | |
| 42 | 9.81 431 | 15 | 9.93 457 | 25 | 10.06 543 | 9.87 975 | 10 | 18 | |
| 43 | 9.81 446 | 15 | 9.93 482 | 26 | 10.06 518 | 9.87 964 | 11 | 17 | |
| 44 | 9.81 461 | 14 | 9.93 508 | 25 | 10.06 492 | 9.87 953 | 11 | 16 | |
| 45 | 9.81 475 | 15 | 9.93 533 | 26 | 10.06 467 | 9.87 942 | 11 | 15 | ″ \| **11** \| **10** |
| 46 | 9.81 490 | 15 | 9.93 559 | 25 | 10.06 441 | 9.87 931 | 11 | 14 | |
| 47 | 9.81 505 | 14 | 9.93 584 | 26 | 10.06 416 | 9.87 920 | 11 | 13 | 6 \| 1.1 \| 1.0 |
| 48 | 9.81 519 | 15 | 9.93 610 | 26 | 10.06 390 | 9.87 909 | 11 | 12 | 7 \| 1.3 \| 1.2 |
| 49 | 9.81 534 | 15 | 9.93 636 | 25 | 10.06 364 | 9.87 898 | 11 | 11 | 8 \| 1.5 \| 1.3 |
| **50** | 9.81 549 | 14 | 9.93 661 | 26 | 10.06 339 | 9.87 887 | 10 | **10** | 9 \| 1.6 \| 1.5 |
| 51 | 9.81 563 | 15 | 9.93 687 | 25 | 10.06 313 | 9.87 877 | 11 | 9 | 10 \| 1.8 \| 1.7 |
| 52 | 9.81 578 | 14 | 9.93 712 | 26 | 10.06 288 | 9.87 866 | 11 | 8 | 20 \| 3.7 \| 3.3 |
| 53 | 9.81 592 | 15 | 9.93 738 | 25 | 10.06 262 | 9.87 855 | 11 | 7 | 30 \| 5.5 \| 5.0 |
| 54 | 9.81 607 | 15 | 9.93 763 | 26 | 10.06 237 | 9.87 844 | 11 | 6 | 40 \| 7.3 \| 6.7 |
| 55 | 9.81 622 | 14 | 9.93 789 | 25 | 10.06 211 | 9.87 833 | 11 | 5 | 50 \| 9.2 \| 8.3 |
| 56 | 9.81 636 | 15 | 9.93 814 | 26 | 10.06 186 | 9.87 822 | 11 | 4 | |
| 57 | 9.81 651 | 14 | 9.93 840 | 25 | 10.06 160 | 9.87 811 | 11 | 3 | |
| 58 | 9.81 665 | 15 | 9.93 865 | 26 | 10.06 135 | 9.87 800 | 11 | 2 | |
| 59 | 9.81 680 | 14 | 9.93 891 | 25 | 10.06 109 | 9.87 789 | 11 | 1 | |
| **60** | 9.81 694 | | 9.93 916 | | 10.06 084 | 9.87 778 | | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 49°

## 41°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.81 694 | | 9.93 916 | | 10.06 084 | 9.87 778 | | 60 | |
| 1 | 9.81 709 | 15 | 9.93 942 | 26 | 10.06 058 | 9.87 767 | 11 | 59 | |
| 2 | 9.81 723 | 14 | 9.93 967 | 25 | 10.06 033 | 9.87 756 | 11 | 58 | |
| 3 | 9.81 738 | 15 | 9.93 993 | 26 | 10.06 007 | 9.87 745 | 11 | 57 | |
| 4 | 9.81 752 | 14 | 9.94 018 | 25 | 10.05 982 | 9.87 734 | 11 | 56 | |
| 5 | 9.81 767 | 15 | 9.94 044 | 26 | 10.05 956 | 9.87 723 | 11 | 55 | |
| 6 | 9.81 781 | 14 | 9.94 069 | 25 | 10.05 931 | 9.87 712 | 11 | 54 | |
| 7 | 9.81 796 | 15 | 9.94 095 | 26 | 10.05 905 | 9.87 701 | 11 | 53 | |
| 8 | 9.81 810 | 14 | 9.94 120 | 25 | 10.05 880 | 9.87 690 | 11 | 52 | |
| 9 | 9.81 825 | 15 | 9.94 146 | 26 | 10.05 854 | 9.87 679 | 11 | 51 | |
| 10 | 9.81 839 | 14 | 9.94 171 | 25 | 10.05 829 | 9.87 668 | 11 | 50 | |
| 11 | 9.81 854 | 15 | 9.94 197 | 26 | 10.05 803 | 9.87 657 | 11 | 49 | |
| 12 | 9.81 868 | 14 | 9.94 222 | 25 | 10.05 778 | 9.87 646 | 11 | 48 | |
| 13 | 9.81 882 | 14 | 9.94 248 | 26 | 10.05 752 | 9.87 635 | 11 | 47 | |
| 14 | 9.81 897 | 15 | 9.94 273 | 25 | 10.05 727 | 9.87 624 | 11 | 46 | |
| 15 | 9.81 911 | 14 | 9.94 299 | 26 | 10.05 701 | 9.87 613 | 11 | 45 | |
| 16 | 9.81 926 | 15 | 9.94 324 | 25 | 10.05 676 | 9.87 601 | 12 | 44 | |
| 17 | 9.81 940 | 14 | 9.94 350 | 26 | 10.05 650 | 9.87 590 | 11 | 43 | |
| 18 | 9.81 955 | 15 | 9.94 375 | 25 | 10.05 625 | 9.87 579 | 11 | 42 | |
| 19 | 9.81 969 | 14 | 9.94 401 | 26 | 10.05 599 | 9.87 568 | 11 | 41 | |
| 20 | 9.81 983 | 14 | 9.94 426 | 25 | 10.05 574 | 9.87 557 | 11 | 40 | |
| 21 | 9.81 998 | 15 | 9.94 452 | 26 | 10.05 548 | 9.87 546 | 11 | 39 | |
| 22 | 9.82 012 | 14 | 9.94 477 | 25 | 10.05 523 | 9.87 535 | 11 | 38 | |
| 23 | 9.82 026 | 14 | 9.94 503 | 26 | 10.05 497 | 9.87 524 | 11 | 37 | |
| 24 | 9.82 041 | 15 | 9.94 528 | 25 | 10.05 472 | 9.87 513 | 11 | 36 | |
| 25 | 9.82 055 | 14 | 9.94 554 | 26 | 10.05 446 | 9.87 501 | 12 | 35 | |
| 26 | 9.82 069 | 14 | 9.94 579 | 25 | 10.05 421 | 9.87 490 | 11 | 34 | ″ 15 14 |
| 27 | 9.82 084 | 15 | 9.94 604 | 25 | 10.05 396 | 9.87 479 | 11 | 33 | |
| 28 | 9.82 098 | 14 | 9.94 630 | 26 | 10.05 370 | 9.87 468 | 11 | 32 | 6 1.5 1.4 |
| 29 | 9.82 112 | 14 | 9.94 655 | 25 | 10.05 345 | 9.87 457 | 11 | 31 | 7 1.8 1.6 |
| 30 | 9.82 126 | 14 | 9.94 681 | 26 | 10.05 319 | 9.87 446 | 11 | 30 | 8 2.0 1.9 |
| 31 | 9.82 141 | 15 | 9.94 706 | 25 | 10.05 294 | 9.87 434 | 12 | 29 | 9 2.2 2.1 |
| 32 | 9.82 155 | 14 | 9.94 732 | 26 | 10.05 268 | 9.87 423 | 11 | 28 | 10 2.5 2.3 |
| 33 | 9.82 169 | 14 | 9.94 757 | 25 | 10.05 243 | 9.87 412 | 11 | 27 | 20 5.0 4.7 |
| 34 | 9.82 184 | 15 | 9.94 783 | 26 | 10.05 217 | 9.87 401 | 11 | 26 | 30 7.5 7.0 |
| 35 | 9.82 198 | 14 | 9.94 808 | 25 | 10.05 192 | 9.87 390 | 11 | 25 | 40 10.0 9.3 |
| 36 | 9.82 212 | 14 | 9.94 834 | 26 | 10.05 166 | 9.87 378 | 12 | 24 | 50 12.5 11.7 |
| 37 | 9.82 226 | 14 | 9.94 859 | 25 | 10.05 141 | 9.87 367 | 11 | 23 | |
| 38 | 9.82 240 | 14 | 9.94 884 | 25 | 10.05 116 | 9.87 356 | 11 | 22 | |
| 39 | 9.82 255 | 15 | 9.94 910 | 26 | 10.05 090 | 9.87 345 | 11 | 21 | |
| 40 | 9.82 269 | 14 | 9.94 935 | 25 | 10.05 065 | 9.87 334 | 11 | 20 | |
| 41 | 9.82 283 | 14 | 9.94 961 | 26 | 10.05 039 | 9.87 322 | 12 | 19 | |
| 42 | 9.82 297 | 14 | 9.94 986 | 25 | 10.05 014 | 9.87 311 | 11 | 18 | |
| 43 | 9.82 311 | 14 | 9.95 012 | 26 | 10.04 988 | 9.87 300 | 11 | 17 | |
| 44 | 9.82 326 | 15 | 9.95 037 | 25 | 10.04 963 | 9.87 288 | 12 | 16 | |
| 45 | 9.82 340 | 14 | 9.95 062 | 25 | 10.04 938 | 9.87 277 | 11 | 15 | ″ 12 11 |
| 46 | 9.82 354 | 14 | 9.95 088 | 26 | 10.04 912 | 9.87 266 | 11 | 14 | |
| 47 | 9.82 368 | 14 | 9.95 113 | 25 | 10.04 887 | 9.87 255 | 11 | 13 | 6 1.2 1.1 |
| 48 | 9.82 382 | 14 | 9.95 139 | 26 | 10.04 861 | 9.87 243 | 12 | 12 | 7 1.4 1.3 |
| 49 | 9.82 396 | 14 | 9.95 164 | 25 | 10.04 836 | 9.87 232 | 11 | 11 | 8 1.6 1.5 |
| 50 | 9.82 410 | 14 | 9.95 190 | 26 | 10.04 810 | 9.87 221 | 11 | 10 | 9 1.8 1.6 |
| 51 | 9.82 424 | 14 | 9.95 215 | 25 | 10.04 785 | 9.87 209 | 12 | 9 | 10 2.0 1.8 |
| 52 | 9.82 439 | 15 | 9.95 240 | 25 | 10.04 760 | 9.87 198 | 11 | 8 | 20 4.0 3.7 |
| 53 | 9.82 453 | 14 | 9.95 266 | 26 | 10.04 734 | 9.87 187 | 11 | 7 | 30 6.0 5.5 |
| 54 | 9.82 467 | 14 | 9.95 291 | 25 | 10.04 709 | 9.87 175 | 12 | 6 | 40 8.0 7.3 |
| 55 | 9.82 481 | 14 | 9.95 317 | 26 | 10.04 683 | 9.87 164 | 11 | 5 | 50 10.0 9.2 |
| 56 | 9.82 495 | 14 | 9.95 342 | 25 | 10.04 658 | 9.87 153 | 11 | 4 | |
| 57 | 9.82 509 | 14 | 9.95 368 | 26 | 10.04 632 | 9.87 141 | 12 | 3 | |
| 58 | 9.82 523 | 14 | 9.95 393 | 25 | 10.04 607 | 9.87 130 | 11 | 2 | |
| 59 | 9.82 537 | 14 | 9.95 418 | 25 | 10.04 582 | 9.87 119 | 11 | 1 | |
| 60 | 9.82 551 | 14 | 9.95 444 | 26 | 10.04 556 | 9.87 107 | 12 | 0 | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 48°

## 42°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.82 551 | | 9.95 444 | | 10.04 556 | 9.87 107 | | **60** |
| 1 | 9.82 565 | 14 | 9.95 469 | 25 | 10.04 531 | 9.87 096 | 11 | 59 |
| 2 | 9.82 579 | 14 | 9.95 495 | 26 | 10.04 505 | 9.87 085 | 11 | 58 |
| 3 | 9.82 593 | 14 | 9.95 520 | 25 | 10.04 480 | 9.87 073 | 12 | 57 |
| 4 | 9.82 607 | 14 | 9.95 545 | 25 | 10.04 455 | 9.87 062 | 11 | 56 |
| 5 | 9.82 621 | 14 | 9.95 571 | 26 | 10.04 429 | 9.87 050 | 12 | 55 |
| 6 | 9.82 635 | 14 | 9.95 596 | 25 | 10.04 404 | 9.87 039 | 11 | 54 |
| 7 | 9.82 649 | 14 | 9.95 622 | 26 | 10.04 378 | 9.87 028 | 11 | 53 |
| 8 | 9.82 663 | 14 | 9.95 647 | 25 | 10.04 353 | 9.87 016 | 12 | 52 |
| 9 | 9.82 677 | 14 | 9.95 672 | 25 | 10.04 328 | 9.87 005 | 11 | 51 |
| **10** | 9.82 691 | 14 | 9.95 698 | 26 | 10.04 302 | 9.86 993 | 12 | **50** |
| 11 | 9.82 705 | 14 | 9.95 723 | 25 | 10.04 277 | 9.86 982 | 11 | 49 |
| 12 | 9.82 719 | 14 | 9.95 748 | 25 | 10.04 252 | 9.86 970 | 12 | 48 |
| 13 | 9.82 733 | 14 | 9.95 774 | 26 | 10.04 226 | 9.86 959 | 11 | 47 |
| 14 | 9.82 747 | 14 | 9.95 799 | 25 | 10 04 201 | 9.86 947 | 12 | 46 |
| 15 | 9.82 761 | 14 | 9.95 825 | 26 | 10.04 175 | 9.86 936 | 11 | 45 |
| 16 | 9.82 775 | 14 | 9.95 850 | 25 | 10.04 150 | 9.86 924 | 12 | 44 |
| 17 | 9.82 788 | 13 | 9.95 875 | 25 | 10.04 125 | 9.86 913 | 11 | 43 |
| 18 | 9.82 802 | 14 | 9.95 901 | 26 | 10.04 099 | 9.86 902 | 11 | 42 |
| 19 | 9.82 816 | 14 | 9.95 926 | 25 | 10.04 074 | 9.86 890 | 12 | 41 |
| **20** | 9.82 830 | 14 | 9.95 952 | 26 | 10.04 048 | 9.86 879 | 11 | **40** |
| 21 | 9.82 844 | 14 | 9.95 977 | 25 | 10.04 023 | 9.86 867 | 12 | 39 |
| 22 | 9.82 858 | 14 | 9.96 002 | 25 | 10.03 998 | 9.86 855 | 12 | 38 |
| 23 | 9.82 872 | 14 | 9.96 028 | 26 | 10.03 972 | 9.86 844 | 11 | 37 |
| 24 | 9.82 885 | 13 | 9.96 053 | 25 | 10.03 947 | 9.86 832 | 12 | 36 |
| 25 | 9.82 899 | 14 | 9.96 078 | 25 | 10.03 922 | 9.86 821 | 11 | 35 |
| 26 | 9.82 913 | 14 | 9.96 104 | 26 | 10.03 896 | 9.86 809 | 12 | 34 |
| 27 | 9.82 927 | 14 | 9.96 129 | 25 | 10.03 871 | 9.86 798 | 11 | 33 |
| 28 | 9.82 941 | 14 | 9.96 155 | 26 | 10.03 845 | 9.86 786 | 12 | 32 |
| 29 | 9.82 955 | 14 | 9.96 180 | 25 | 10.03 820 | 9.86 775 | 11 | 31 |
| **30** | 9.82 968 | 13 | 9.96 205 | 25 | 10.03 795 | 9.86 763 | 12 | **30** |
| 31 | 9.82 982 | 14 | 9.96 231 | 26 | 10.03 769 | 9.86 752 | 11 | 29 |
| 32 | 9.82 996 | 14 | 9.96 256 | 25 | 10.03 744 | 9.86 740 | 12 | 28 |
| 33 | 9.83 010 | 14 | 9.96 281 | 25 | 10.03 719 | 9.86 728 | 12 | 27 |
| 34 | 9.83 023 | 13 | 9.96 307 | 26 | 10.03 693 | 9.86 717 | 11 | 26 |
| 35 | 9.83 037 | 14 | 9.96 332 | 25 | 10.03 668 | 9.86 705 | 12 | 25 |
| 36 | 9.83 051 | 14 | 9.96 357 | 25 | 10.03 643 | 9.86 694 | 11 | 24 |
| 37 | 9.83 065 | 14 | 9.96 383 | 26 | 10.03 617 | 9.86 682 | 12 | 23 |
| 38 | 9.83 078 | 13 | 9.96 408 | 25 | 10.03 592 | 9.86 670 | 12 | 22 |
| 39 | 9.83 092 | 14 | 9.96 433 | 25 | 10.03 567 | 9.86 659 | 11 | 21 |
| **40** | 9.83 106 | 14 | 9.96 459 | 26 | 10.03 541 | 9.86 647 | 12 | **20** |
| 41 | 9.83 120 | 14 | 9.96 484 | 25 | 10.03 516 | 9.86 635 | 12 | 19 |
| 42 | 9.83 133 | 13 | 9.96 510 | 26 | 10.03 490 | 9.86 624 | 11 | 18 |
| 43 | 9.83 147 | 14 | 9.96 535 | 25 | 10.03 465 | 9.86 612 | 12 | 17 |
| 44 | 9.83 161 | 14 | 9.96 560 | 25 | 10.03 440 | 9.86 600 | 12 | 16 |
| 45 | 9.83 174 | 13 | 9.96 586 | 26 | 10.03 414 | 9.86 589 | 11 | 15 |
| 46 | 9.83 188 | 14 | 9.96 611 | 25 | 10.03 389 | 9.86 577 | 12 | 14 |
| 47 | 9.83 202 | 14 | 9.96 636 | 25 | 10.03 364 | 9.86 565 | 12 | 13 |
| 48 | 9.83 215 | 13 | 9.96 662 | 26 | 10.03 338 | 9.86 554 | 11 | 12 |
| 49 | 9.83 229 | 14 | 9.96 687 | 25 | 10.03 313 | 9.86 542 | 12 | 11 |
| **50** | 9.83 242 | 13 | 9.96 712 | 25 | 10.03 288 | 9.86 530 | 12 | **10** |
| 51 | 9.83 256 | 14 | 9.96 738 | 26 | 10.03 262 | 9.86 518 | 12 | 9 |
| 52 | 9.83 270 | 14 | 9.96 763 | 25 | 10.03 237 | 9.86 507 | 11 | 8 |
| 53 | 9.83 283 | 13 | 9.96 788 | 25 | 10.03 212 | 9.86 495 | 12 | 7 |
| 54 | 9.83 297 | 14 | 9.96 814 | 26 | 10.03 186 | 9.86 483 | 12 | 6 |
| 55 | 9.83 310 | 13 | 9.96 839 | 25 | 10.03 161 | 9.86 472 | 11 | 5 |
| 56 | 9.83 324 | 14 | 9.96 864 | 25 | 10.03 136 | 9.86 460 | 12 | 4 |
| 57 | 9.83 338 | 14 | 9.96 890 | 26 | 10.03 110 | 9.86 448 | 12 | 3 |
| 58 | 9.83 351 | 13 | 9.96 915 | 25 | 10.03 085 | 9.86 436 | 12 | 2 |
| 59 | 9.83 365 | 14 | 9.96 940 | 25 | 10.03 060 | 9.86 425 | 11 | 1 |
| **60** | 9.83 378 | 13 | 9.96 966 | 26 | 10.03 034 | 9.86 413 | 12 | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 26 | 25 |
|---|---|---|
| 6 | 2.6 | 2.5 |
| 7 | 3.0 | 2.9 |
| 8 | 3.5 | 3.3 |
| 9 | 3.9 | 3.8 |
| 10 | 4.3 | 4.2 |
| 20 | 8.7 | 8.3 |
| 30 | 13.0 | 12.5 |
| 40 | 17.3 | 16.7 |
| 50 | 21.7 | 20.8 |

| ″ | 14 | 13 |
|---|---|---|
| 6 | 1.4 | 1.3 |
| 7 | 1.6 | 1.5 |
| 8 | 1.9 | 1.7 |
| 9 | 2.1 | 2.0 |
| 10 | 2.3 | 2.2 |
| 20 | 4.7 | 4.3 |
| 30 | 7.0 | 6.5 |
| 40 | 9.3 | 8.7 |
| 50 | 11.7 | 10.8 |

| ″ | 12 | 11 |
|---|---|---|
| 6 | 1.2 | 1.1 |
| 7 | 1.4 | 1.3 |
| 8 | 1.6 | 1.5 |
| 9 | 1.8 | 1.6 |
| 10 | 2.0 | 1.8 |
| 20 | 4.0 | 3.7 |
| 30 | 6.0 | 5.5 |
| 40 | 8.0 | 7.3 |
| 50 | 10.0 | 9.2 |

Prop. Pts.

## 47°

## 43°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 9.83 378 | 14 | 9.96 966 | 25 | 10.03 034 | 9.86 413 | 12 | **60** | |
| 1 | 9.83 392 | 13 | 9.96 991 | 25 | 10.03 009 | 9.86 401 | 12 | 59 | |
| 2 | 9.83 405 | 14 | 9.97 016 | 26 | 10.02 984 | 9.86 389 | 12 | 58 | |
| 3 | 9.83 419 | 13 | 9.97 042 | 25 | 10.02 958 | 9.86 377 | 11 | 57 | |
| 4 | 9.83 432 | 14 | 9.97 067 | 25 | 10.02 933 | 9.86 366 | 12 | 56 | |
| 5 | 9.83 446 | 13 | 9.97 092 | 26 | 10.02 908 | 9.86 354 | 12 | 55 | |
| 6 | 9.83 459 | 14 | 9.97 118 | 25 | 10.02 882 | 9.86 342 | 12 | 54 | |
| 7 | 9.83 473 | 13 | 9.97 143 | 25 | 10.02 857 | 9.86 330 | 12 | 53 | ″ 26 25 |
| 8 | 9.83 486 | 14 | 9.97 168 | 25 | 10.02 832 | 9.86 318 | 12 | 52 | 6 2.6 2.5 |
| 9 | 9.83 500 | 13 | 9.97 193 | 26 | 10.02 807 | 9.86 306 | 11 | 51 | 7 3.0 2.9 |
| **10** | 9.83 513 | 14 | 9.97 219 | 25 | 10.02 781 | 9.86 295 | 12 | **50** | 8 3.5 3.3 |
| 11 | 9.83 527 | 13 | 9.97 244 | 25 | 10.02 756 | 9.86 283 | 12 | 49 | 9 3.9 3.8 |
| 12 | 9.83 540 | 14 | 9.97 269 | 26 | 10.02 731 | 9.86 271 | 12 | 48 | 10 4.3 4.2 |
| 13 | 9.83 554 | 13 | 9.97 295 | 25 | 10.02 705 | 9.86 259 | 12 | 47 | 20 8.7 8.3 |
| 14 | 9.83 567 | 14 | 9.97 320 | 25 | 10.02 680 | 9.86 247 | 12 | 46 | 30 13.0 12.5 |
| 15 | 9.83 581 | 13 | 9.97 345 | 26 | 10.02 655 | 9.86 235 | 12 | 45 | 40 17.3 16.7 |
| 16 | 9.83 594 | 14 | 9.97 371 | 25 | 10.02 629 | 9.86 223 | 12 | 44 | 50 21.7 20.8 |
| 17 | 9.83 608 | 13 | 9.97 396 | 25 | 10.02 604 | 9.86 211 | 12 | 43 | |
| 18 | 9.83 621 | 13 | 9.97 421 | 26 | 10.02 579 | 9.86 200 | 11 | 42 | |
| 19 | 9.83 634 | 14 | 9.97 447 | 25 | 10.02 553 | 9.86 188 | 12 | 41 | |
| **20** | 9.83 648 | 13 | 9.97 472 | 25 | 10.02 528 | 9.86 176 | 12 | **40** | |
| 21 | 9.83 661 | 13 | 9.97 497 | 26 | 10.02 503 | 9.86 164 | 12 | 39 | |
| 22 | 9.83 674 | 14 | 9.97 523 | 25 | 10.02 477 | 9.86 152 | 12 | 38 | |
| 23 | 9.83 688 | 13 | 9.97 548 | 25 | 10.02 452 | 9.86 140 | 12 | 37 | |
| 24 | 9.83 701 | 14 | 9.97 573 | 25 | 10.02 427 | 9.86 128 | 12 | 36 | |
| 25 | 9.83 715 | 13 | 9.97 598 | 26 | 10.02 402 | 9.86 116 | 12 | 35 | |
| 26 | 9.83 728 | 13 | 9.97 624 | 25 | 10.02 376 | 9.86 104 | 12 | 34 | ″ 14 13 |
| 27 | 9.83 741 | 14 | 9.97 649 | 25 | 10.02 351 | 9.86 092 | 12 | 33 | 6 1.4 1.3 |
| 28 | 9.83 755 | 13 | 9.97 674 | 26 | 10.02 326 | 9.86 080 | 12 | 32 | 7 1.6 1.5 |
| 29 | 9.83 768 | 13 | 9.97 700 | 25 | 10.02 300 | 9.86 068 | 12 | 31 | 8 1.9 1.7 |
| **30** | 9.83 781 | 14 | 9.97 725 | 25 | 10.02 275 | 9.86 056 | 12 | **30** | 9 2.1 2.0 |
| 31 | 9.83 795 | 13 | 9.97 750 | 26 | 10.02 250 | 9.86 044 | 12 | 29 | 10 2.3 2.2 |
| 32 | 9.83 808 | 13 | 9.97 776 | 25 | 10.02 224 | 9.86 032 | 12 | 28 | 20 4.7 4.3 |
| 33 | 9.83 821 | 13 | 9.97 801 | 25 | 10.02 199 | 9.86 020 | 12 | 27 | 30 7.0 6.5 |
| 34 | 9.83 834 | 14 | 9.97 826 | 25 | 10.02 174 | 9.86 008 | 12 | 26 | 40 9.3 8.7 |
| 35 | 9.83 848 | 13 | 9.97 851 | 26 | 10.02 149 | 9.85 996 | 12 | 25 | 50 11.7 10.8 |
| 36 | 9.83 861 | 13 | 9.97 877 | 25 | 10.02 123 | 9.85 984 | 12 | 24 | |
| 37 | 9.83 874 | 13 | 9.97 902 | 25 | 10.02 098 | 9.85 972 | 12 | 23 | |
| 38 | 9.83 887 | 14 | 9.97 927 | 26 | 10.02 073 | 9.85 960 | 12 | 22 | |
| 39 | 9.83 901 | 13 | 9.97 953 | 25 | 10.02 047 | 9.85 948 | 12 | 21 | |
| **40** | 9.83 914 | 13 | 9.97 978 | 25 | 10.02 022 | 9.85 936 | 12 | **20** | |
| 41 | 9.83 927 | 13 | 9.98 003 | 26 | 10.01 997 | 9.85 924 | 12 | 19 | |
| 42 | 9.83 940 | 14 | 9.98 029 | 25 | 10.01 971 | 9.85 912 | 12 | 18 | |
| 43 | 9.83 954 | 13 | 9.98 054 | 25 | 10.01 946 | 9.85 900 | 12 | 17 | |
| 44 | 9.83 967 | 13 | 9.98 079 | 25 | 10.01 921 | 9.85 883 | 12 | 16 | |
| 45 | 9.83 980 | 13 | 9.98 104 | 26 | 10.01 896 | 9.85 876 | 12 | 15 | ″ 12 11 |
| 46 | 9.83 993 | 13 | 9.98 130 | 25 | 10.01 870 | 9.85 864 | 12 | 14 | 6 1.2 1.1 |
| 47 | 9.84 006 | 14 | 9.98 155 | 25 | 10.01 845 | 9.85 851 | 13 | 13 | 7 1.4 1.3 |
| 48 | 9.84 020 | 13 | 9.98 180 | 26 | 10.01 820 | 9.85 839 | 12 | 12 | 8 1.6 1.5 |
| 49 | 9.84 033 | 13 | 9.98 206 | 25 | 10.01 794 | 9.85 827 | 12 | 11 | 9 1.8 1.6 |
| **50** | 9.84 046 | 13 | 9.98 231 | 25 | 10.01 769 | 9.85 815 | 12 | **10** | 10 2.0 1.8 |
| 51 | 9.84 059 | 13 | 9.98 256 | 25 | 10.01 744 | 9.85 803 | 12 | 9 | 20 4.0 3.7 |
| 52 | 9.84 072 | 13 | 9.98 281 | 26 | 10.01 719 | 9.85 791 | 12 | 8 | 30 6.0 5.5 |
| 53 | 9.84 085 | 13 | 9.98 307 | 25 | 10.01 693 | 9.85 779 | 12 | 7 | 40 8.0 7.3 |
| 54 | 9.84 098 | 14 | 9.98 332 | 25 | 10.01 668 | 9.85 766 | 13 | 6 | 50 10.0 9.2 |
| 55 | 9.84 112 | 13 | 9.98 357 | 26 | 10.01 643 | 9.85 754 | 12 | 5 | |
| 56 | 9.84 125 | 13 | 9.98 383 | 25 | 10.01 617 | 9.85 742 | 12 | 4 | |
| 57 | 9.84 138 | 13 | 9.98 408 | 25 | 10.01 592 | 9.85 730 | 12 | 3 | |
| 58 | 9.84 151 | 13 | 9.98 433 | 25 | 10.01 567 | 9.85 718 | 12 | 2 | |
| 59 | 9.84 164 | 13 | 9.98 458 | 26 | 10.01 542 | 9.85 706 | 12 | 1 | |
| **60** | 9.84 177 | | 9.98 484 | | 10.01 516 | 9.85 693 | 13 | **0** | |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ | Prop. Pts. |

## 46°

## 44°

| ′ | L Sin | d | L Tan | c d | L Cot | L Cos | d | |
|---|---|---|---|---|---|---|---|---|
| **0** | 9.84 177 | 13 | 9.98 484 | 25 | 10.01 516 | 9.85 693 | 12 | **60** |
| 1 | 9.84 190 | 13 | 9.98 509 | 25 | 10.01 491 | 9.85 681 | 12 | 59 |
| 2 | 9.84 203 | 13 | 9.98 534 | 26 | 10.01 466 | 9.85 669 | 12 | 58 |
| 3 | 9.84 216 | 13 | 9.98 560 | 25 | 10.01 440 | 9.85 657 | 12 | 57 |
| 4 | 9.84 229 | 13 | 9.98 585 | 25 | 10.01 415 | 9.85 645 | 13 | 56 |
| 5 | 9.84 242 | 13 | 9.98 610 | 25 | 10.01 390 | 9.85 632 | 12 | 55 |
| 6 | 9.84 255 | 14 | 9.98 635 | 26 | 10.01 365 | 9.85 620 | 12 | 54 |
| 7 | 9.84 269 | 13 | 9.98 661 | 25 | 10.01 339 | 9.85 608 | 12 | 53 |
| 8 | 9.84 282 | 13 | 9.98 686 | 25 | 10.01 314 | 9.85 596 | 13 | 52 |
| 9 | 9.84 295 | 13 | 9.98 711 | 26 | 10.01 289 | 9.85 583 | 12 | 51 |
| **10** | 9.84 308 | 13 | 9.98 737 | 25 | 10.01 263 | 9.85 571 | 12 | **50** |
| 11 | 9.84 321 | 13 | 9.98 762 | 25 | 10.01 238 | 9.85 559 | 12 | 49 |
| 12 | 9.84 334 | 13 | 9.98 787 | 25 | 10.01 213 | 9.85 547 | 13 | 48 |
| 13 | 9.84 347 | 13 | 9.98 812 | 26 | 10.01 188 | 9.85 534 | 12 | 47 |
| 14 | 9.84 360 | 13 | 9.98 838 | 25 | 10.01 162 | 9.85 522 | 12 | 46 |
| 15 | 9.84 373 | 12 | 9.98 863 | 25 | 10.01 137 | 9.85 510 | 13 | 45 |
| 16 | 9.84 385 | 13 | 9.98 888 | 25 | 10.01 112 | 9.85 497 | 12 | 44 |
| 17 | 9.84 398 | 13 | 9.98 913 | 26 | 10.01 087 | 9.85 485 | 12 | 43 |
| 18 | 9.84 411 | 13 | 9.98 939 | 25 | 10.01 061 | 9.85 473 | 13 | 42 |
| 19 | 9.84 424 | 13 | 9.98 964 | 25 | 10.01 036 | 9.85 460 | 12 | 41 |
| **20** | 9.84 437 | 13 | 9.98 989 | 26 | 10.01 011 | 9.85 448 | 12 | **40** |
| 21 | 9.84 450 | 13 | 9.99 015 | 25 | 10.00 985 | 9.85 436 | 13 | 39 |
| 22 | 9.84 463 | 13 | 9.99 040 | 25 | 10.00 960 | 9.85 423 | 12 | 38 |
| 23 | 9.84 476 | 13 | 9.99 065 | 25 | 10.00 935 | 9.85 411 | 12 | 37 |
| 24 | 9.84 489 | 13 | 9.99 090 | 26 | 10.00 910 | 9.85 399 | 13 | 36 |
| 25 | 9.84 502 | 13 | 9.99 116 | 25 | 10.00 884 | 9.85 386 | 12 | 35 |
| 26 | 9.84 515 | 13 | 9.99 141 | 25 | 10.00 859 | 9.85 374 | 13 | 34 |
| 27 | 9.84 528 | 12 | 9.99 166 | 25 | 10.00 834 | 9.85 361 | 12 | 33 |
| 28 | 9.84 540 | 13 | 9.99 191 | 26 | 10.00 809 | 9.85 349 | 12 | 32 |
| 29 | 9.84 553 | 13 | 9.99 217 | 25 | 10.00 783 | 9.85 337 | 13 | 31 |
| **30** | 9.84 566 | 13 | 9.99 242 | 25 | 10.00 758 | 9.85 324 | 12 | **30** |
| 31 | 9.84 579 | 13 | 9.99 267 | 26 | 10.00 733 | 9.85 312 | 13 | 29 |
| 32 | 9.84 592 | 13 | 9.99 293 | 25 | 10.00 707 | 9.85 299 | 12 | 28 |
| 33 | 9.84 605 | 13 | 9.99 318 | 25 | 10.00 682 | 9.85 287 | 13 | 27 |
| 34 | 9.84 618 | 12 | 9.99 343 | 25 | 10.00 657 | 9.85 274 | 12 | 26 |
| 35 | 9.84 630 | 13 | 9.99 368 | 26 | 10.00 632 | 9.85 262 | 12 | 25 |
| 36 | 9.84 643 | 13 | 9.99 394 | 25 | 10.00 606 | 9.85 250 | 13 | 24 |
| 37 | 9.84 656 | 13 | 9.99 419 | 25 | 10.00 581 | 9.85 237 | 12 | 23 |
| 38 | 9.84 669 | 13 | 9.99 444 | 25 | 10.00 556 | 9.85 225 | 13 | 22 |
| 39 | 9.84 682 | 12 | 9.99 469 | 26 | 10.00 531 | 9.85 212 | 12 | 21 |
| **40** | 9.84 694 | 13 | 9.99 495 | 25 | 10.00 505 | 9.85 200 | 13 | **20** |
| 41 | 9.84 707 | 13 | 9.99 520 | 25 | 10.00 480 | 9.85 187 | 12 | 19 |
| 42 | 9.84 720 | 13 | 9.99 545 | 25 | 10.00 455 | 9.85 175 | 13 | 18 |
| 43 | 9.84 733 | 12 | 9.99 570 | 26 | 10.00 430 | 9.85 162 | 12 | 17 |
| 44 | 9.84 745 | 13 | 9.99 596 | 25 | 10.00 404 | 9.85 150 | 13 | 16 |
| 45 | 9.84 758 | 13 | 9.99 621 | 25 | 10.00 379 | 9.85 137 | 12 | 15 |
| 46 | 9.84 771 | 13 | 9.99 646 | 25 | 10.00 354 | 9.85 125 | 13 | 14 |
| 47 | 9.84 784 | 12 | 9.99 672 | 25 | 10.00 328 | 9.85 112 | 12 | 13 |
| 48 | 9.84 796 | 13 | 9.99 697 | 25 | 10.00 303 | 9.85 100 | 13 | 12 |
| 49 | 9.84 809 | 13 | 9.99 722 | 25 | 10.00 278 | 9.85 087 | 13 | 11 |
| **50** | 9.84 822 | 13 | 9.99 747 | 26 | 10.00 253 | 9.85 074 | 12 | **10** |
| 51 | 9.84 835 | 12 | 9.99 773 | 25 | 10.00 227 | 9.85 062 | 13 | 9 |
| 52 | 9.84 847 | 13 | 9.99 798 | 25 | 10.00 202 | 9.85 049 | 12 | 8 |
| 53 | 9.84 860 | 13 | 9.99 823 | 25 | 10.00 177 | 9.85 037 | 13 | 7 |
| 54 | 9.84 873 | 12 | 9.99 848 | 26 | 10.00 152 | 9.85 024 | 12 | 6 |
| 55 | 9.84 885 | 13 | 9.99 874 | 25 | 10.00 126 | 9.85 012 | 13 | 5 |
| 56 | 9.84 898 | 13 | 9.99 899 | 25 | 10.00 101 | 9.84 999 | 13 | 4 |
| 57 | 9.84 911 | 12 | 9.99 924 | 25 | 10.00 076 | 9.84 986 | 12 | 3 |
| 58 | 9.84 923 | 13 | 9.99 949 | 26 | 10.00 051 | 9.84 974 | 13 | 2 |
| 59 | 9.84 936 | 13 | 9.99 975 | 25 | 10.00 025 | 9.84 961 | 12 | 1 |
| **60** | 9.84 949 | | 0.00 000 | | 10.00 000 | 9.84 949 | | **0** |
| | L Cos | d | L Cot | c d | L Tan | L Sin | d | ′ |

Prop. Pts.

| ″ | 26 | 25 | 14 |
|---|---|---|---|
| 6 | 2.6 | 2.5 | 1.4 |
| 7 | 3.0 | 2.9 | 1.6 |
| 8 | 3.5 | 3.3 | 1.9 |
| 9 | 3.9 | 3.8 | 2.1 |
| 10 | 4.3 | 4.2 | 2.3 |
| 20 | 8.7 | 8.3 | 4.7 |
| 30 | 13.0 | 12.5 | 7.0 |
| 40 | 17.3 | 16.7 | 9.3 |
| 50 | 21.7 | 20.8 | 11.7 |

| ″ | 13 | 12 |
|---|---|---|
| 6 | 1.3 | 1.2 |
| 7 | 1.5 | 1.4 |
| 8 | 1.7 | 1.6 |
| 9 | 2.0 | 1.8 |
| 10 | 2.2 | 2.0 |
| 20 | 4.3 | 4.0 |
| 30 | 6.5 | 6.0 |
| 40 | 8.7 | 8.0 |
| 50 | 10.8 | 10.0 |

## 45°

TABLE VII

COMMON LOGARITHMS
OF NUMBERS
FROM
**1** TO **10000**
TO
FIVE DECIMAL PLACES

## 1–100

| N | Log | N | Log | N | Log | N | Log | N | Log |
|---|---|---|---|---|---|---|---|---|---|
| **0** | — | **20** | 1. 30 103 | **40** | 1. 60 206 | **60** | 1. 77 815 | **80** | 1. 90 309 |
| 1 | 0. 00 000 | 21 | 1. 32 222 | 41 | 1. 61 278 | 61 | 1. 78 533 | 81 | 1. 90 849 |
| 2 | 0. 30 103 | 22 | 1. 34 242 | 42 | 1. 62 325 | 62 | 1. 79 239 | 82 | 1. 91 381 |
| 3 | 0. 47 712 | 23 | 1. 36 173 | 43 | 1. 63 347 | 63 | 1. 79 934 | 83 | 1. 91 908 |
| 4 | 0. 60 206 | 24 | 1. 38 021 | 44 | 1. 64 345 | 64 | 1. 80 618 | 84 | 1. 92 428 |
| 5 | 0. 69 897 | 25 | 1. 39 794 | 45 | 1. 65 321 | 65 | 1. 81 291 | 85 | 1. 92 942 |
| 6 | 0. 77 815 | 26 | 1. 41 497 | 46 | 1. 66 276 | 66 | 1. 81 954 | 86 | 1. 93 450 |
| 7 | 0. 84 510 | 27 | 1. 43 136 | 47 | 1. 67 210 | 67 | 1. 82 607 | 87 | 1. 93 952 |
| 8 | 0. 90 309 | 28 | 1. 44 716 | 48 | 1. 68 124 | 68 | 1. 83 251 | 88 | 1. 94 448 |
| 9 | 0. 95 424 | 29 | 1. 46 240 | 49 | 1. 69 020 | 69 | 1. 83 885 | 89 | 1. 94 939 |
| **10** | 1. 00 000 | **30** | 1. 47 712 | **50** | 1. 69 897 | **70** | 1. 84 510 | **90** | 1. 95 424 |
| 11 | 1. 04 139 | 31 | 1. 49 136 | 51 | 1. 70 757 | 71 | 1. 85 126 | 91 | 1. 95 904 |
| 12 | 1. 07 918 | 32 | 1. 50 515 | 52 | 1. 71 600 | 72 | 1. 85 733 | 92 | 1. 96 379 |
| 13 | 1. 11 394 | 33 | 1. 51 851 | 53 | 1. 72 428 | 73 | 1. 86 332 | 93 | 1. 96 848 |
| 14 | 1. 14 613 | 34 | 1. 53 148 | 54 | 1. 73 239 | 74 | 1. 86 923 | 94 | 1. 97 313 |
| 15 | 1. 17 609 | 35 | 1. 54 407 | 55 | 1. 74 036 | 75 | 1. 87 506 | 95 | 1. 97 772 |
| 16 | 1. 20 412 | 36 | 1. 55 630 | 56 | 1. 74 819 | 76 | 1. 88 081 | 96 | 1. 98 227 |
| 17 | 1. 23 045 | 37 | 1. 56 820 | 57 | 1. 75 587 | 77 | 1. 88 649 | 97 | 1. 98 677 |
| 18 | 1. 25 527 | 38 | 1. 57 978 | 58 | 1. 76 343 | 78 | 1. 89 209 | 98 | 1. 99 123 |
| 19 | 1. 27 875 | 39 | 1. 59 106 | 59 | 1. 77 085 | 79 | 1. 89 763 | 99 | 1. 99 564 |
| **20** | 1. 30 103 | **40** | 1. 60 206 | **60** | 1. 77 815 | **80** | 1. 90 309 | **100** | 2. 00 000 |

## 100–150

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **100** | 00 000 | 043 | 087 | 130 | 173 | 217 | 260 | 303 | 346 | 389 |
| 101 | 432 | 475 | 518 | 561 | 604 | 647 | 689 | 732 | 775 | 817 |
| 102 | 860 | 903 | 945 | 988 | *030 | *072 | *115 | *157 | *199 | *242 |
| 103 | 01 284 | 326 | 368 | 410 | 452 | 494 | 536 | 578 | 620 | 662 |
| 104 | 703 | 745 | 787 | 828 | 870 | 912 | 953 | 995 | *036 | *078 |
| 105 | 02 119 | 160 | 202 | 243 | 284 | 325 | 366 | 407 | 449 | 490 |
| 106 | 531 | 572 | 612 | 653 | 694 | 735 | 776 | 816 | 857 | 898 |
| 107 | 938 | 979 | *019 | *060 | *100 | *141 | *181 | *222 | *262 | *302 |
| 108 | 03 342 | 383 | 423 | 463 | 503 | 543 | 583 | 623 | 663 | 703 |
| 109 | 743 | 782 | 822 | 862 | 902 | 941 | 981 | *021 | *060 | *100 |
| **110** | 04 139 | 179 | 218 | 258 | 297 | 336 | 376 | 415 | 454 | 493 |
| 111 | 532 | 571 | 610 | 650 | 689 | 727 | 766 | 805 | 844 | 883 |
| 112 | 922 | 961 | 999 | *038 | *077 | *115 | *154 | *192 | *231 | *269 |
| 113 | 05 308 | 346 | 385 | 423 | 461 | 500 | 538 | 576 | 614 | 652 |
| 114 | 690 | 729 | 767 | 805 | 843 | 881 | 918 | 956 | 994 | *032 |
| 115 | 06 070 | 108 | 145 | 183 | 221 | 258 | 296 | 333 | 371 | 408 |
| 116 | 446 | 483 | 521 | 558 | 595 | 633 | 670 | 707 | 744 | 781 |
| 117 | 819 | 856 | 893 | 930 | 967 | *004 | *041 | *078 | *115 | *151 |
| 118 | 07 188 | 225 | 262 | 298 | 335 | 372 | 408 | 445 | 482 | 518 |
| 119 | 555 | 591 | 628 | 664 | 700 | 737 | 773 | 809 | 846 | 882 |
| **120** | 918 | 954 | 990 | *027 | *063 | *099 | *135 | *171 | *207 | *243 |
| 121 | 08 279 | 314 | 350 | 386 | 422 | 458 | 493 | 529 | 565 | 600 |
| 122 | 636 | 672 | 707 | 743 | 778 | 814 | 849 | 884 | 920 | 955 |
| 123 | 991 | *026 | *061 | *096 | *132 | *167 | *202 | *237 | *272 | *307 |
| 124 | 09 342 | 377 | 412 | 447 | 482 | 517 | 552 | 587 | 621 | 656 |
| 125 | 691 | 726 | 760 | 795 | 830 | 864 | 899 | 934 | 968 | *003 |
| 126 | 10 037 | 072 | 106 | 140 | 175 | 209 | 243 | 278 | 312 | 346 |
| 127 | 380 | 415 | 449 | 483 | 517 | 551 | 585 | 619 | 653 | 687 |
| 128 | 721 | 755 | 789 | 823 | 857 | 890 | 924 | 958 | 992 | *025 |
| 129 | 11 059 | 093 | 126 | 160 | 193 | 227 | 261 | 294 | 327 | 361 |
| **130** | 394 | 428 | 461 | 494 | 528 | 561 | 594 | 628 | 661 | 694 |
| 131 | 727 | 760 | 793 | 826 | 860 | 893 | 926 | 959 | 992 | *024 |
| 132 | 12 057 | 090 | 123 | 156 | 189 | 222 | 254 | 287 | 320 | 352 |
| 133 | 385 | 418 | 450 | 483 | 516 | 548 | 581 | 613 | 646 | 678 |
| 134 | 710 | 743 | 775 | 808 | 840 | 872 | 905 | 937 | 969 | *001 |
| 135 | 13 033 | 066 | 098 | 130 | 162 | 194 | 226 | 258 | 290 | 322 |
| 136 | 354 | 386 | 418 | 450 | 481 | 513 | 545 | 577 | 609 | 640 |
| 137 | 672 | 704 | 735 | 767 | 799 | 830 | 862 | 893 | 925 | 956 |
| 138 | 988 | *019 | *051 | *082 | *114 | *145 | *176 | *208 | *239 | *270 |
| 139 | 14 301 | 333 | 364 | 395 | 426 | 457 | 489 | 520 | 551 | 582 |
| **140** | 613 | 644 | 675 | 706 | 737 | 768 | 799 | 829 | 860 | 891 |
| 141 | 922 | 953 | 983 | *014 | *045 | *076 | *106 | *137 | *168 | *198 |
| 142 | 15 229 | 259 | 290 | 320 | 351 | 381 | 412 | 442 | 473 | 503 |
| 143 | 534 | 564 | 594 | 625 | 655 | 685 | 715 | 746 | 776 | 806 |
| 144 | 836 | 866 | 897 | 927 | 957 | 987 | *017 | *047 | *077 | *107 |
| 145 | 16 137 | 167 | 197 | 227 | 256 | 286 | 316 | 346 | 376 | 406 |
| 146 | 435 | 465 | 495 | 524 | 554 | 584 | 613 | 643 | 673 | 702 |
| 147 | 732 | 761 | 791 | 820 | 850 | 879 | 909 | 938 | 967 | 997 |
| 148 | 17 026 | 056 | 085 | 114 | 143 | 173 | 202 | 231 | 260 | 289 |
| 149 | 319 | 348 | 377 | 406 | 435 | 464 | 493 | 522 | 551 | 580 |
| **150** | 609 | 638 | 667 | 696 | 725 | 754 | 782 | 811 | 840 | 869 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Prop. Pts.**

| | 44 | 43 | 42 |
|---|---|---|---|
| 1 | 4.4 | 4.3 | 4.2 |
| 2 | 8.8 | 8.6 | 8.4 |
| 3 | 13.2 | 12.9 | 12.6 |
| 4 | 17.6 | 17.2 | 16.8 |
| 5 | 22.0 | 21.5 | 21.0 |
| 6 | 26.4 | 25.8 | 25.2 |
| 7 | 30.8 | 30.1 | 29.4 |
| 8 | 35.2 | 34.4 | 33.6 |
| 9 | 39.6 | 38.7 | 37.8 |

| | 41 | 40 | 39 |
|---|---|---|---|
| 1 | 4.1 | 4.0 | 3.9 |
| 2 | 8.2 | 8.0 | 7.8 |
| 3 | 12.3 | 12.0 | 11.7 |
| 4 | 16.4 | 16.0 | 15.6 |
| 5 | 20.5 | 20.0 | 19.5 |
| 6 | 24.6 | 24.0 | 23.4 |
| 7 | 28.7 | 28.0 | 27.3 |
| 8 | 32.8 | 32.0 | 31.2 |
| 9 | 36.9 | 36.0 | 35.1 |

| | 38 | 37 | 36 |
|---|---|---|---|
| 1 | 3.8 | 3.7 | 3.6 |
| 2 | 7.6 | 7.4 | 7.2 |
| 3 | 11.4 | 11.1 | 10.8 |
| 4 | 15.2 | 14.8 | 14.4 |
| 5 | 19.0 | 18.5 | 18.0 |
| 6 | 22.8 | 22.2 | 21.6 |
| 7 | 26.6 | 25.9 | 25.2 |
| 8 | 30.4 | 29.6 | 28.8 |
| 9 | 34.2 | 33.3 | 32.4 |

| | 35 | 34 | 33 |
|---|---|---|---|
| 1 | 3.5 | 3.4 | 3.3 |
| 2 | 7.0 | 6.8 | 6.6 |
| 3 | 10.5 | 10.2 | 9.9 |
| 4 | 14.0 | 13.6 | 13.2 |
| 5 | 17.5 | 17.0 | 16.5 |
| 6 | 21.0 | 20.4 | 19.8 |
| 7 | 24.5 | 23.8 | 23.1 |
| 8 | 28.0 | 27.2 | 26.4 |
| 9 | 31.5 | 30.6 | 29.7 |

| | 32 | 31 | 30 |
|---|---|---|---|
| 1 | 3.2 | 3.1 | 3.0 |
| 2 | 6.4 | 6.2 | 6.0 |
| 3 | 9.6 | 9.3 | 9.0 |
| 4 | 12.8 | 12.4 | 12.0 |
| 5 | 16.0 | 15.5 | 15.0 |
| 6 | 19.2 | 18.6 | 18.0 |
| 7 | 22.4 | 21.7 | 21.0 |
| 8 | 25.6 | 24.8 | 24.0 |
| 9 | 28.8 | 27.9 | 27.0 |

## 150–200

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **150** | 17 609 | 638 | 667 | 696 | 725 | 754 | 782 | 811 | 840 | 869 |
| 151 | 898 | 926 | 955 | 984 | *013 | *041 | *070 | *099 | *127 | *156 |
| 152 | 18 184 | 213 | 241 | 270 | 298 | 327 | 355 | 384 | 412 | 441 |
| 153 | 469 | 498 | 526 | 554 | 583 | 611 | 639 | 667 | 696 | 724 |
| 154 | 752 | 780 | 808 | 837 | 865 | 893 | 921 | 949 | 977 | *005 |
| 155 | 19 033 | 061 | 089 | 117 | 145 | 173 | 201 | 229 | 257 | 285 |
| 156 | 312 | 340 | 368 | 396 | 424 | 451 | 479 | 507 | 535 | 562 |
| 157 | 590 | 618 | 645 | 673 | 700 | 728 | 756 | 783 | 811 | 838 |
| 158 | 866 | 893 | 921 | 948 | 976 | *003 | *030 | *058 | *085 | *112 |
| 159 | 20 140 | 167 | 194 | 222 | 249 | 276 | 303 | 330 | 358 | 385 |
| **160** | 412 | 439 | 466 | 493 | 520 | 548 | 575 | 602 | 629 | 656 |
| 161 | 683 | 710 | 737 | 763 | 790 | 817 | 844 | 871 | 898 | 925 |
| 162 | 952 | 978 | *005 | *032 | *059 | *085 | *112 | *139 | *165 | *192 |
| 163 | 21 219 | 245 | 272 | 299 | 325 | 352 | 378 | 405 | 431 | 458 |
| 164 | 484 | 511 | 537 | 564 | 590 | 617 | 643 | 669 | 696 | 722 |
| 165 | 748 | 775 | 801 | 827 | 854 | 880 | 906 | 932 | 958 | 985 |
| 166 | 22 011 | 037 | 063 | 089 | 115 | 141 | 167 | 194 | 220 | 246 |
| 167 | 272 | 298 | 324 | 350 | 376 | 401 | 427 | 453 | 479 | 505 |
| 168 | 531 | 557 | 583 | 608 | 634 | 660 | 686 | 712 | 737 | 763 |
| 169 | 789 | 814 | 840 | 866 | 891 | 917 | 943 | 968 | 994 | *019 |
| **170** | 23 045 | 070 | 096 | 121 | 147 | 172 | 198 | 223 | 249 | 274 |
| 171 | 300 | 325 | 350 | 376 | 401 | 426 | 452 | 477 | 502 | 528 |
| 172 | 553 | 578 | 603 | 629 | 654 | 679 | 704 | 729 | 754 | 779 |
| 173 | 805 | 830 | 855 | 880 | 905 | 930 | 955 | 980 | *005 | *030 |
| 174 | 24 055 | 080 | 105 | 130 | 155 | 180 | 204 | 229 | 254 | 279 |
| 175 | 304 | 329 | 353 | 378 | 403 | 428 | 452 | 477 | 502 | 527 |
| 176 | 551 | 576 | 601 | 625 | 650 | 674 | 699 | 724 | 748 | 773 |
| 177 | 797 | 822 | 846 | 871 | 895 | 920 | 944 | 969 | 993 | *018 |
| 178 | 25 042 | 066 | 091 | 115 | 139 | 164 | 188 | 212 | 237 | 261 |
| 179 | 285 | 310 | 334 | 358 | 382 | 406 | 431 | 455 | 479 | 503 |
| **180** | 527 | 551 | 575 | 600 | 624 | 648 | 672 | 696 | 720 | 744 |
| 181 | 768 | 792 | 816 | 840 | 864 | 888 | 912 | 935 | 959 | 983 |
| 182 | 26 007 | 031 | 055 | 079 | 102 | 126 | 150 | 174 | 198 | 221 |
| 183 | 245 | 269 | 293 | 316 | 340 | 364 | 387 | 411 | 435 | 458 |
| 184 | 482 | 505 | 529 | 553 | 576 | 600 | 623 | 647 | 670 | 694 |
| 185 | 717 | 741 | 764 | 788 | 811 | 834 | 858 | 881 | 905 | 928 |
| 186 | 951 | 975 | 998 | *021 | *045 | *068 | *091 | *114 | *138 | *161 |
| 187 | 27 184 | 207 | 231 | 254 | 277 | 300 | 323 | 346 | 370 | 393 |
| 188 | 416 | 439 | 462 | 485 | 508 | 531 | 554 | 577 | 600 | 623 |
| 189 | 646 | 669 | 692 | 715 | 738 | 761 | 784 | 807 | 830 | 852 |
| **190** | 875 | 898 | 921 | 944 | 967 | 989 | *012 | *035 | *058 | *081 |
| 191 | 28 103 | 126 | 149 | 171 | 194 | 217 | 240 | 262 | 285 | 307 |
| 192 | 330 | 353 | 375 | 398 | 421 | 443 | 466 | 488 | 511 | 533 |
| 193 | 556 | 578 | 601 | 623 | 646 | 668 | 691 | 713 | 735 | 758 |
| 194 | 780 | 803 | 825 | 847 | 870 | 892 | 914 | 937 | 959 | 981 |
| 195 | 29 003 | 026 | 048 | 070 | 092 | 115 | 137 | 159 | 181 | 203 |
| 196 | 226 | 248 | 270 | 292 | 314 | 336 | 358 | 380 | 403 | 425 |
| 197 | 447 | 469 | 491 | 513 | 535 | 557 | 579 | 601 | 623 | 645 |
| 198 | 667 | 688 | 710 | 732 | 754 | 776 | 798 | 820 | 842 | 863 |
| 199 | 885 | 907 | 929 | 951 | 973 | 994 | *016 | *038 | *060 | *081 |
| **200** | 30 103 | 125 | 146 | 168 | 190 | 211 | 233 | 255 | 276 | 298 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Prop. Pts.**

| | 29 | 28 |
|---|---|---|
| 1 | 2.9 | 2.8 |
| 2 | 5.8 | 5.6 |
| 3 | 8.7 | 8.4 |
| 4 | 11.6 | 11.2 |
| 5 | 14.5 | 14.0 |
| 6 | 17.4 | 16.8 |
| 7 | 20.3 | 19.6 |
| 8 | 23.2 | 22.4 |
| 9 | 26.1 | 25.2 |

| | 27 | 26 |
|---|---|---|
| 1 | 2.7 | 2.6 |
| 2 | 5.4 | 5.2 |
| 3 | 8.1 | 7.8 |
| 4 | 10.8 | 10.4 |
| 5 | 13.5 | 13.0 |
| 6 | 16.2 | 15.6 |
| 7 | 18.9 | 18.2 |
| 8 | 21.6 | 20.8 |
| 9 | 24.3 | 23.4 |

| | 25 |
|---|---|
| 1 | 2.5 |
| 2 | 5.0 |
| 3 | 7.5 |
| 4 | 10.0 |
| 5 | 12.5 |
| 6 | 15.0 |
| 7 | 17.5 |
| 8 | 20.0 |
| 9 | 22.5 |

| | 24 | 23 |
|---|---|---|
| 1 | 2.4 | 2.3 |
| 2 | 4.8 | 4.6 |
| 3 | 7.2 | 6.9 |
| 4 | 9.6 | 9.2 |
| 5 | 12.0 | 11.5 |
| 6 | 14.4 | 13.8 |
| 7 | 16.8 | 16.1 |
| 8 | 19.2 | 18.4 |
| 9 | 21.6 | 20.7 |

| | 22 | 21 |
|---|---|---|
| 1 | 2.2 | 2.1 |
| 2 | 4.4 | 4.2 |
| 3 | 6.6 | 6.3 |
| 4 | 8.8 | 8.4 |
| 5 | 11.0 | 10.5 |
| 6 | 13.2 | 12.6 |
| 7 | 15.4 | 14.7 |
| 8 | 17.6 | 16.8 |
| 9 | 19.8 | 18.9 |

## 200–250

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **200** | 30 103 | 125 | 146 | 168 | 190 | 211 | 233 | 255 | 276 | 298 |
| 201 | 320 | 341 | 363 | 384 | 406 | 428 | 449 | 471 | 492 | 514 |
| 202 | 535 | 557 | 578 | 600 | 621 | 643 | 664 | 685 | 707 | 728 |
| 203 | 750 | 771 | 792 | 814 | 835 | 856 | 878 | 899 | 920 | 942 |
| 204 | 963 | 984 | *006 | *027 | *048 | *069 | *091 | *112 | *133 | *154 |
| 205 | 31 175 | 197 | 218 | 239 | 260 | 281 | 302 | 323 | 345 | 366 |
| 206 | 387 | 408 | 429 | 450 | 471 | 492 | 513 | 534 | 555 | 576 |
| 207 | 597 | 618 | 639 | 660 | 681 | 702 | 723 | 744 | 765 | 785 |
| 208 | 806 | 827 | 848 | 869 | 890 | 911 | 931 | 952 | 973 | 994 |
| 209 | 32 015 | 035 | 056 | 077 | 098 | 118 | 139 | 160 | 181 | 201 |
| **210** | 222 | 243 | 263 | 284 | 305 | 325 | 346 | 366 | 387 | 408 |
| 211 | 428 | 449 | 469 | 490 | 510 | 531 | 552 | 572 | 593 | 613 |
| 212 | 634 | 654 | 675 | 695 | 715 | 736 | 756 | 777 | 797 | 818 |
| 213 | 838 | 858 | 879 | 899 | 919 | 940 | 960 | 980 | *001 | *021 |
| 214 | 33 041 | 062 | 082 | 102 | 122 | 143 | 163 | 183 | 203 | 224 |
| 215 | 244 | 264 | 284 | 304 | 325 | 345 | 365 | 385 | 405 | 425 |
| 216 | 445 | 465 | 486 | 506 | 526 | 546 | 566 | 586 | 606 | 626 |
| 217 | 646 | 666 | 686 | 706 | 726 | 746 | 766 | 786 | 806 | 826 |
| 218 | 846 | 866 | 885 | 905 | 925 | 945 | 965 | 985 | *005 | *025 |
| 219 | 34 044 | 064 | 084 | 104 | 124 | 143 | 163 | 183 | 203 | 223 |
| **220** | 242 | 262 | 282 | 301 | 321 | 341 | 361 | 380 | 400 | 420 |
| 221 | 439 | 459 | 479 | 498 | 518 | 537 | 557 | 577 | 596 | 616 |
| 222 | 635 | 655 | 674 | 694 | 713 | 733 | 753 | 772 | 792 | 811 |
| 223 | 830 | 850 | 869 | 889 | 908 | 928 | 947 | 967 | 986 | *005 |
| 224 | 35 025 | 044 | 064 | 083 | 102 | 122 | 141 | 160 | 180 | 199 |
| 225 | 218 | 238 | 257 | 276 | 295 | 315 | 334 | 353 | 372 | 392 |
| 226 | 411 | 430 | 449 | 468 | 488 | 507 | 526 | 545 | 564 | 583 |
| 227 | 603 | 622 | 641 | 660 | 679 | 698 | 717 | 736 | 755 | 774 |
| 228 | 793 | 813 | 832 | 851 | 870 | 889 | 908 | 927 | 946 | 965 |
| 229 | 984 | *003 | *021 | *040 | *059 | *078 | *097 | *116 | *135 | *154 |
| **230** | 36 173 | 192 | 211 | 229 | 248 | 267 | 286 | 305 | 324 | 342 |
| 231 | 361 | 380 | 399 | 418 | 436 | 455 | 474 | 493 | 511 | 530 |
| 232 | 549 | 568 | 586 | 605 | 624 | 642 | 661 | 680 | 698 | 717 |
| 233 | 736 | 754 | 773 | 791 | 810 | 829 | 847 | 866 | 884 | 903 |
| 234 | 922 | 940 | 959 | 977 | 996 | *014 | *033 | *051 | *070 | *088 |
| 235 | 37 107 | 125 | 144 | 162 | 181 | 199 | 218 | 236 | 254 | 273 |
| 236 | 291 | 310 | 328 | 346 | 365 | 383 | 401 | 420 | 438 | 457 |
| 237 | 475 | 493 | 511 | 530 | 548 | 566 | 585 | 603 | 621 | 639 |
| 238 | 658 | 676 | 694 | 712 | 731 | 749 | 767 | 785 | 803 | 822 |
| 239 | 840 | 858 | 876 | 894 | 912 | 931 | 949 | 967 | 985 | *003 |
| **240** | 38 021 | 039 | 057 | 075 | 093 | 112 | 130 | 148 | 166 | 184 |
| 241 | 202 | 220 | 238 | 256 | 274 | 292 | 310 | 328 | 346 | 364 |
| 242 | 382 | 399 | 417 | 435 | 453 | 471 | 489 | 507 | 525 | 543 |
| 243 | 561 | 578 | 596 | 614 | 632 | 650 | 668 | 686 | 703 | 721 |
| 244 | 739 | 757 | 775 | 792 | 810 | 828 | 846 | 863 | 881 | 899 |
| 245 | 917 | 934 | 952 | 970 | 987 | *005 | *023 | *041 | *058 | *076 |
| 246 | 39 094 | 111 | 129 | 146 | 164 | 182 | 199 | 217 | 235 | 252 |
| 247 | 270 | 287 | 305 | 322 | 340 | 358 | 375 | 393 | 410 | 428 |
| 248 | 445 | 463 | 480 | 498 | 515 | 533 | 550 | 568 | 585 | 602 |
| 249 | 620 | 637 | 655 | 672 | 690 | 707 | 724 | 742 | 759 | 777 |
| **250** | 794 | 811 | 829 | 846 | 863 | 881 | 898 | 915 | 933 | 950 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 21 | |
|---|---|---|
| 1 | 2.2 | 2.1 |
| 2 | 4.4 | 4.2 |
| 3 | 6.6 | 6.3 |
| 4 | 8.8 | 8.4 |
| 5 | 11.0 | 10.5 |
| 6 | 13.2 | 12.6 |
| 7 | 15.4 | 14.7 |
| 8 | 17.6 | 16.8 |
| 9 | 19.8 | 18.9 |

| | 20 |
|---|---|
| 1 | 2.0 |
| 2 | 4.0 |
| 3 | 6.0 |
| 4 | 8.0 |
| 5 | 10.0 |
| 6 | 12.0 |
| 7 | 14.0 |
| 8 | 16.0 |
| 9 | 18.0 |

| | 19 |
|---|---|
| 1 | 1.9 |
| 2 | 3.8 |
| 3 | 5.7 |
| 4 | 7.6 |
| 5 | 9.5 |
| 6 | 11.4 |
| 7 | 13.3 |
| 8 | 15.2 |
| 9 | 17.1 |

| | 18 |
|---|---|
| 1 | 1.8 |
| 2 | 3.6 |
| 3 | 5.4 |
| 4 | 7.2 |
| 5 | 9.0 |
| 6 | 10.8 |
| 7 | 12.6 |
| 8 | 14.4 |
| 9 | 16.2 |

| | 17 |
|---|---|
| 1 | 1.7 |
| 2 | 3.4 |
| 3 | 5.1 |
| 4 | 6.8 |
| 5 | 8.5 |
| 6 | 10.2 |
| 7 | 11.9 |
| 8 | 13.6 |
| 9 | 15.3 |

Prop. Pts.

## 250–300

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Prop. Pts. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **250** | 39 794 | 811 | 829 | 846 | 863 | 881 | 898 | 915 | 933 | 950 | |
| 251 | 967 | 985 | *002 | *019 | *037 | *054 | *071 | *088 | *106 | *123 | |
| 252 | 40 140 | 157 | 175 | 192 | 209 | 226 | 243 | 261 | 278 | 295 | **18** |
| 253 | 312 | 329 | 346 | 364 | 381 | 398 | 415 | 432 | 449 | 466 | |
| 254 | 483 | 500 | 518 | 535 | 552 | 569 | 586 | 603 | 620 | 637 | 1 1.8 |
| 255 | 654 | 671 | 688 | 705 | 722 | 739 | 756 | 773 | 790 | 807 | 2 3.6 |
| 256 | 824 | 841 | 858 | 875 | 892 | 909 | 926 | 943 | 960 | 976 | 3 5.4 |
| 257 | 993 | *010 | *027 | *044 | *061 | *078 | *095 | *111 | *128 | *145 | 4 7.2 |
| 258 | 41 162 | 179 | 196 | 212 | 229 | 246 | 263 | 280 | 296 | 313 | 5 9.0 |
| 259 | 330 | 347 | 363 | 380 | 397 | 414 | 430 | 447 | 464 | 481 | 6 10.8 |
| **260** | 497 | 514 | 531 | 547 | 564 | 581 | 597 | 614 | 631 | 647 | 7 12.6 |
| 261 | 664 | 681 | 697 | 714 | 731 | 747 | 764 | 780 | 797 | 814 | 8 14.4 |
| 262 | 830 | 847 | 863 | 880 | 896 | 913 | 929 | 946 | 963 | 979 | 9 16.2 |
| 263 | 996 | *012 | *029 | *045 | *062 | *078 | *095 | *111 | *127 | *144 | **17** |
| 264 | 42 160 | 177 | 193 | 210 | 226 | 243 | 259 | 275 | 292 | 308 | 1 1.7 |
| 265 | 325 | 341 | 357 | 374 | 390 | 406 | 423 | 439 | 455 | 472 | 2 3.4 |
| 266 | 488 | 504 | 521 | 537 | 553 | 570 | 586 | 602 | 619 | 635 | 3 5.1 |
| 267 | 651 | 667 | 684 | 700 | 716 | 732 | 749 | 765 | 781 | 797 | 4 6.8 |
| 268 | 813 | 830 | 846 | 862 | 878 | 894 | 911 | 927 | 943 | 959 | 5 8.5 |
| 269 | 975 | 991 | *008 | *024 | *040 | *056 | *072 | *088 | *104 | *120 | 6 10.2 |
| **270** | 43 136 | 152 | 169 | 185 | 201 | 217 | 233 | 249 | 265 | 281 | 7 11.9 |
| 271 | 297 | 313 | 329 | 345 | 361 | 377 | 393 | 409 | 425 | 441 | 8 13.6 |
| 272 | 457 | 473 | 489 | 505 | 521 | 537 | 553 | 569 | 584 | 600 | 9 15.3 |
| 273 | 616 | 632 | 648 | 664 | 680 | 696 | 712 | 727 | 743 | 759 | **15** |
| 274 | 775 | 791 | 807 | 823 | 838 | 854 | 870 | 886 | 902 | 917 | 1 1.6 |
| 275 | 933 | 949 | 965 | 981 | 996 | *012 | *028 | *044 | *059 | *075 | 2 3.2 |
| 276 | 44 091 | 107 | 122 | 138 | 154 | 170 | 185 | 201 | 217 | 232 | 3 4.8 |
| 277 | 248 | 264 | 279 | 295 | 311 | 326 | 342 | 358 | 373 | 389 | 4 6.4 |
| 278 | 404 | 420 | 436 | 451 | 467 | 483 | 498 | 514 | 529 | 545 | 5 8.0 |
| 279 | 560 | 576 | 592 | 607 | 623 | 638 | 654 | 669 | 685 | 700 | 6 9.6 |
| **280** | 716 | 731 | 747 | 762 | 778 | 793 | 809 | 824 | 840 | 855 | 7 11.2 |
| 281 | 871 | 886 | 902 | 917 | 932 | 948 | 963 | 979 | 994 | *010 | 8 12.8 |
| 282 | 45 025 | 040 | 056 | 071 | 086 | 102 | 117 | 133 | 148 | 163 | 9 14.4 |
| 283 | 179 | 194 | 209 | 225 | 240 | 255 | 271 | 286 | 301 | 317 | **15** |
| 284 | 332 | 347 | 362 | 378 | 393 | 408 | 423 | 439 | 454 | 469 | 1 1.5 |
| 285 | 484 | 500 | 515 | 530 | 545 | 561 | 576 | 591 | 606 | 621 | 2 3.0 |
| 286 | 637 | 652 | 667 | 682 | 697 | 712 | 728 | 743 | 758 | 773 | 3 4.5 |
| 287 | 788 | 803 | 818 | 834 | 849 | 864 | 879 | 894 | 909 | 924 | 4 6.0 |
| 288 | 939 | 954 | 969 | 984 | *000 | *015 | *030 | *045 | *060 | *075 | 5 7.5 |
| 289 | 46 090 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 | 6 9.0 |
| **290** | 240 | 255 | 270 | 285 | 300 | 315 | 330 | 345 | 359 | 374 | 7 10.5 |
| 291 | 389 | 404 | 419 | 434 | 449 | 464 | 479 | 494 | 509 | 523 | 8 12.0 |
| 292 | 538 | 553 | 568 | 583 | 598 | 613 | 627 | 642 | 657 | 672 | 9 13.5 |
| 293 | 687 | 702 | 716 | 731 | 746 | 761 | 776 | 790 | 805 | 820 | **14** |
| 294 | 835 | 850 | 864 | 879 | 894 | 909 | 923 | 938 | 953 | 967 | 1 1.4 |
| 295 | 982 | 997 | *012 | *026 | *041 | *056 | *070 | *085 | *100 | *114 | 2 2.8 |
| 296 | 47 129 | 144 | 159 | 173 | 188 | 202 | 217 | 232 | 246 | 261 | 3 4.2 |
| 297 | 276 | 290 | 305 | 319 | 334 | 349 | 363 | 378 | 392 | 407 | 4 5.6 |
| 298 | 422 | 436 | 451 | 465 | 480 | 494 | 509 | 524 | 538 | 553 | 5 7.0 |
| 299 | 567 | 582 | 596 | 611 | 625 | 640 | 654 | 669 | 683 | 698 | 6 8.4 |
| **300** | 712 | 727 | 741 | 756 | 770 | 784 | 799 | 813 | 828 | 842 | 7 9.8 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 8 11.2 9 12.6 Prop. Pts. |

log $e$ = .43429

## 300–350

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **300** | 47 712 | 727 | 741 | 756 | 770 | 784 | 799 | 813 | 828 | 842 |
| 301 | 857 | 871 | 885 | 900 | 914 | 929 | 943 | 958 | 972 | 986 |
| 302 | 48 001 | 015 | 029 | 044 | 058 | 073 | 087 | 101 | 116 | 130 |
| 303 | 144 | 159 | 173 | 187 | 202 | 216 | 230 | 244 | 259 | 273 |
| 304 | 287 | 302 | 316 | 330 | 344 | 359 | 373 | 387 | 401 | 416 |
| 305 | 430 | 444 | 458 | 473 | 487 | 501 | 515 | 530 | 544 | 558 |
| 306 | 572 | 586 | 601 | 615 | 629 | 643 | 657 | 671 | 686 | 700 |
| 307 | 714 | 728 | 742 | 756 | 770 | 785 | 799 | 813 | 827 | 841 |
| 308 | 855 | 869 | 883 | 897 | 911 | 926 | 940 | 954 | 968 | 982 |
| 309 | 996 | *010 | *024 | *038 | *052 | *066 | *080 | *094 | *108 | *122 |
| **310** | 49 136 | 150 | 164 | 178 | 192 | 206 | 220 | 234 | 248 | 262 |
| 311 | 276 | 290 | 304 | 318 | 332 | 346 | 360 | 374 | 388 | 402 |
| 312 | 415 | 429 | 443 | 457 | 471 | 485 | 499 | 513 | 527 | 541 |
| 313 | 554 | 568 | 582 | 596 | 610 | 624 | 638 | 651 | 665 | 679 |
| 314 | 693 | 707 | 721 | 734 | 748 | 762 | 776 | 790 | 803 | 817 |
| 315 | 831 | 845 | 859 | 872 | 886 | 900 | 914 | 927 | 941 | 955 |
| 316 | 969 | 982 | 996 | *010 | *024 | *037 | *051 | *065 | *079 | *092 |
| 317 | 50 106 | 120 | 133 | 147 | 161 | 174 | 188 | 202 | 215 | 229 |
| 318 | 243 | 256 | 270 | 284 | 297 | 311 | 325 | 338 | 352 | 365 |
| 319 | 379 | 393 | 406 | 420 | 433 | 447 | 461 | 474 | 488 | 501 |
| **320** | 515 | 529 | 542 | 556 | 569 | 583 | 596 | 610 | 623 | 637 |
| 321 | 651 | 664 | 678 | 691 | 705 | 718 | 732 | 745 | 759 | 772 |
| 322 | 786 | 799 | 813 | 826 | 840 | 853 | 866 | 880 | 893 | 907 |
| 323 | 920 | 934 | 947 | 961 | 974 | 987 | *001 | *014 | *028 | *041 |
| 324 | 51 055 | 068 | 081 | 095 | 108 | 121 | 135 | 148 | 162 | 175 |
| 325 | 188 | 202 | 215 | 228 | 242 | 255 | 268 | 282 | 295 | 308 |
| 326 | 322 | 335 | 348 | 362 | 375 | 388 | 402 | 415 | 428 | 441 |
| 327 | 455 | 468 | 481 | 495 | 508 | 521 | 534 | 548 | 561 | 574 |
| 328 | 587 | 601 | 614 | 627 | 640 | 654 | 667 | 680 | 693 | 706 |
| 329 | 720 | 733 | 746 | 759 | 772 | 786 | 799 | 812 | 825 | 838 |
| **330** | 851 | 865 | 878 | 891 | 904 | 917 | 930 | 943 | 957 | 970 |
| 331 | 983 | 996 | *009 | *022 | *035 | *048 | *061 | *075 | *088 | *101 |
| 332 | 52 114 | 127 | 140 | 153 | 166 | 179 | 192 | 205 | 218 | 231 |
| 333 | 244 | 257 | 270 | 284 | 297 | 310 | 323 | 336 | 349 | 362 |
| 334 | 375 | 388 | 401 | 414 | 427 | 440 | 453 | 466 | 479 | 492 |
| 335 | 504 | 517 | 530 | 543 | 556 | 569 | 582 | 595 | 608 | 621 |
| 336 | 634 | 647 | 660 | 673 | 686 | 699 | 711 | 724 | 737 | 750 |
| 337 | 763 | 776 | 789 | 802 | 815 | 827 | 840 | 853 | 866 | 879 |
| 338 | 892 | 905 | 917 | 930 | 943 | 956 | 969 | 982 | 994 | *007 |
| 339 | 53 020 | 033 | 046 | 058 | 071 | 084 | 097 | 110 | 122 | 135 |
| **340** | 148 | 161 | 173 | 186 | 199 | 212 | 224 | 237 | 250 | 263 |
| 341 | 275 | 288 | 301 | 314 | 326 | 339 | 352 | 364 | 377 | 390 |
| 342 | 403 | 415 | 428 | 441 | 453 | 466 | 479 | 491 | 504 | 517 |
| 343 | 529 | 542 | 555 | 567 | 580 | 593 | 605 | 618 | 631 | 643 |
| 344 | 656 | 668 | 681 | 694 | 706 | 719 | 732 | 744 | 757 | 769 |
| 345 | 782 | 794 | 807 | 820 | 832 | 845 | 857 | 870 | 882 | 895 |
| 346 | 908 | 920 | 933 | 945 | 958 | 970 | 983 | 995 | *008 | *020 |
| 347 | 54 033 | 045 | 058 | 070 | 083 | 095 | 108 | 120 | 133 | 145 |
| 348 | 158 | 170 | 183 | 195 | 208 | 220 | 233 | 245 | 258 | 270 |
| 349 | 283 | 295 | 307 | 320 | 332 | 345 | 357 | 370 | 382 | 394 |
| **350** | 407 | 419 | 432 | 444 | 456 | 469 | 481 | 494 | 506 | 518 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 15 | 14 | 13 | 12 |
|---|---|---|---|---|
| 1 | 1.5 | 1.4 | 1.3 | 1.2 |
| 2 | 3.0 | 2.8 | 2.6 | 2.4 |
| 3 | 4.5 | 4.2 | 3.9 | 3.6 |
| 4 | 6.0 | 5.6 | 5.2 | 4.8 |
| 5 | 7.5 | 7.0 | 6.5 | 6.0 |
| 6 | 9.0 | 8.4 | 7.8 | 7.2 |
| 7 | 10.5 | 9.8 | 9.1 | 8.4 |
| 8 | 12.0 | 11.2 | 10.4 | 9.6 |
| 9 | 13.5 | 12.6 | 11.7 | 10.8 |

Prop. Pts.

$\log \pi = .49715$

## 350–400

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **350** | 54 407 | 419 | 432 | 444 | 456 | 469 | 481 | 494 | 506 | 518 |
| 351 | 531 | 543 | 555 | 568 | 580 | 593 | 605 | 617 | 630 | 642 |
| 352 | 654 | 667 | 679 | 691 | 704 | 716 | 728 | 741 | 753 | 765 |
| 353 | 777 | 790 | 802 | 814 | 827 | 839 | 851 | 864 | 876 | 888 |
| 354 | 900 | 913 | 925 | 937 | 949 | 962 | 974 | 986 | 998 | *011 |
| 355 | 55 023 | 035 | 047 | 060 | 072 | 084 | 096 | 108 | 121 | 133 |
| 356 | 145 | 157 | 169 | 182 | 194 | 206 | 218 | 230 | 242 | 255 |
| 357 | 267 | 279 | 291 | 303 | 315 | 328 | 340 | 352 | 364 | 376 |
| 358 | 388 | 400 | 413 | 425 | 437 | 449 | 461 | 473 | 485 | 497 |
| 359 | 509 | 522 | 534 | 546 | 558 | 570 | 582 | 594 | 606 | 618 |
| **360** | 630 | 642 | 654 | 666 | 678 | 691 | 703 | 715 | 727 | 739 |
| 361 | 751 | 763 | 775 | 787 | 799 | 811 | 823 | 835 | 847 | 859 |
| 362 | 871 | 883 | 895 | 907 | 919 | 931 | 943 | 955 | 967 | 979 |
| 363 | 991 | *003 | *015 | *027 | *038 | *050 | *062 | *074 | *086 | *098 |
| 364 | 56 110 | 122 | 134 | 146 | 158 | 170 | 182 | 194 | 205 | 217 |
| 365 | 229 | 241 | 253 | 265 | 277 | 289 | 301 | 312 | 324 | 336 |
| 366 | 348 | 360 | 372 | 384 | 396 | 407 | 419 | 431 | 443 | 455 |
| 367 | 467 | 478 | 490 | 502 | 514 | 526 | 538 | 549 | 561 | 573 |
| 368 | 585 | 597 | 608 | 620 | 632 | 644 | 656 | 667 | 679 | 691 |
| 369 | 703 | 714 | 726 | 738 | 750 | 761 | 773 | 785 | 797 | 808 |
| **370** | 820 | 832 | 844 | 855 | 867 | 879 | 891 | 902 | 914 | 926 |
| 371 | 937 | 949 | 961 | 972 | 984 | 996 | *008 | *019 | *031 | *043 |
| 372 | 57 054 | 066 | 078 | 089 | 101 | 113 | 124 | 136 | 148 | 159 |
| 373 | 171 | 183 | 194 | 206 | 217 | 229 | 241 | 252 | 264 | 276 |
| 374 | 287 | 299 | 310 | 322 | 334 | 345 | 357 | 368 | 380 | 392 |
| 375 | 403 | 415 | 426 | 438 | 449 | 461 | 473 | 484 | 496 | 507 |
| 376 | 519 | 530 | 542 | 553 | 565 | 576 | 588 | 600 | 611 | 623 |
| 377 | 634 | 646 | 657 | 669 | 680 | 692 | 703 | 715 | 726 | 738 |
| 378 | 749 | 761 | 772 | 784 | 795 | 807 | 818 | 830 | 841 | 852 |
| 379 | 864 | 875 | 887 | 898 | 910 | 921 | 933 | 944 | 955 | 967 |
| **380** | 978 | 990 | *001 | *013 | *024 | *035 | *047 | *058 | *070 | *081 |
| 381 | 58 092 | 104 | 115 | 127 | 138 | 149 | 161 | 172 | 184 | 195 |
| 382 | 206 | 218 | 229 | 240 | 252 | 263 | 274 | 286 | 297 | 309 |
| 383 | 320 | 331 | 343 | 354 | 365 | 377 | 388 | 399 | 410 | 422 |
| 384 | 433 | 444 | 456 | 467 | 478 | 490 | 501 | 512 | 524 | 535 |
| 385 | 546 | 557 | 569 | 580 | 591 | 602 | 614 | 625 | 636 | 647 |
| 386 | 659 | 670 | 681 | 692 | 704 | 715 | 726 | 737 | 749 | 760 |
| 387 | 771 | 782 | 794 | 805 | 816 | 827 | 838 | 850 | 861 | 872 |
| 388 | 883 | 894 | 906 | 917 | 928 | 939 | 950 | 961 | 973 | 984 |
| 389 | 995 | *006 | *017 | *028 | *040 | *051 | *062 | *073 | *084 | *095 |
| **390** | 59 106 | 118 | 129 | 140 | 151 | 162 | 173 | 184 | 195 | 207 |
| 391 | 218 | 229 | 240 | 251 | 262 | 273 | 284 | 295 | 306 | 318 |
| 392 | 329 | 340 | 351 | 362 | 373 | 384 | 395 | 406 | 417 | 428 |
| 393 | 439 | 450 | 461 | 472 | 483 | 494 | 506 | 517 | 528 | 539 |
| 394 | 550 | 561 | 572 | 583 | 594 | 605 | 616 | 627 | 638 | 649 |
| 395 | 660 | 671 | 682 | 693 | 704 | 715 | 726 | 737 | 748 | 759 |
| 396 | 770 | 780 | 791 | 802 | 813 | 824 | 835 | 846 | 857 | 868 |
| 397 | 879 | 890 | 901 | 912 | 923 | 934 | 945 | 956 | 966 | 977 |
| 398 | 988 | 999 | *010 | *021 | *032 | *043 | *054 | *065 | *076 | *086 |
| 399 | 60 097 | 108 | 119 | 130 | 141 | 152 | 163 | 173 | 184 | 195 |
| **400** | 206 | 217 | 228 | 239 | 249 | 260 | 271 | 282 | 293 | 304 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 13 | 12 | 11 | 10 |
|---|---|---|---|---|
| 1 | 1.3 | 1.2 | 1.1 | 1.0 |
| 2 | 2.6 | 2.4 | 2.2 | 2.0 |
| 3 | 3.9 | 3.6 | 3.3 | 3.0 |
| 4 | 5.2 | 4.8 | 4.4 | 4.0 |
| 5 | 6.5 | 6.0 | 5.5 | 5.0 |
| 6 | 7.8 | 7.2 | 6.6 | 6.0 |
| 7 | 9.1 | 8.4 | 7.7 | 7.0 |
| 8 | 10.4 | 9.6 | 8.8 | 8.0 |
| 9 | 11.7 | 10.8 | 9.9 | 9.0 |

Prop. Pts.

## 400–450

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **400** | 60 206 | 217 | 228 | 239 | 249 | 260 | 271 | 282 | 293 | 304 |
| 401 | 314 | 325 | 336 | 347 | 358 | 369 | 379 | 390 | 401 | 412 |
| 402 | 423 | 433 | 444 | 455 | 466 | 477 | 487 | 498 | 509 | 520 |
| 403 | 531 | 541 | 552 | 563 | 574 | 584 | 595 | 606 | 617 | 627 |
| 404 | 638 | 649 | 660 | 670 | 681 | 692 | 703 | 713 | 724 | 735 |
| 405 | 746 | 756 | 767 | 778 | 788 | 799 | 810 | 821 | 831 | 842 |
| 406 | 853 | 863 | 874 | 885 | 895 | 906 | 917 | 927 | 938 | 949 |
| 407 | 959 | 970 | 981 | 991 | *002 | *013 | *023 | *034 | *045 | *055 |
| 408 | 61 066 | 077 | 087 | 098 | 109 | 119 | 130 | 140 | 151 | 162 |
| 409 | 172 | 183 | 194 | 204 | 215 | 225 | 236 | 247 | 257 | 268 |
| **410** | 278 | 289 | 300 | 310 | 321 | 331 | 342 | 352 | 363 | 374 |
| 411 | 384 | 395 | 405 | 416 | 426 | 437 | 448 | 458 | 469 | 479 |
| 412 | 490 | 500 | 511 | 521 | 532 | 542 | 553 | 563 | 574 | 584 |
| 413 | 595 | 606 | 616 | 627 | 637 | 648 | 658 | 669 | 679 | 690 |
| 414 | 700 | 711 | 721 | 731 | 742 | 752 | 763 | 773 | 784 | 794 |
| 415 | 805 | 815 | 826 | 836 | 847 | 857 | 868 | 878 | 888 | 899 |
| 416 | 909 | 920 | 930 | 941 | 951 | 962 | 972 | 982 | 993 | *003 |
| 417 | 62 014 | 024 | 034 | 045 | 055 | 066 | 076 | 086 | 097 | 107 |
| 418 | 118 | 128 | 138 | 149 | 159 | 170 | 180 | 190 | 201 | 211 |
| 419 | 221 | 232 | 242 | 252 | 263 | 273 | 284 | 294 | 304 | 315 |
| **420** | 325 | 335 | 346 | 356 | 366 | 377 | 387 | 397 | 408 | 418 |
| 421 | 428 | 439 | 449 | 459 | 469 | 480 | 490 | 500 | 511 | 521 |
| 422 | 531 | 542 | 552 | 562 | 572 | 583 | 593 | 603 | 613 | 624 |
| 423 | 634 | 644 | 655 | 665 | 675 | 685 | 696 | 706 | 716 | 726 |
| 424 | 737 | 747 | 757 | 767 | 778 | 788 | 798 | 808 | 818 | 829 |
| 425 | 839 | 849 | 859 | 870 | 880 | 890 | 900 | 910 | 921 | 931 |
| 426 | 941 | 951 | 961 | 972 | 982 | 992 | *002 | *012 | *022 | *033 |
| 427 | 63 043 | 053 | 063 | 073 | 083 | 094 | 104 | 114 | 124 | 134 |
| 428 | 144 | 155 | 165 | 175 | 185 | 195 | 205 | 215 | 225 | 236 |
| 429 | 246 | 256 | 266 | 276 | 286 | 296 | 306 | 317 | 327 | 337 |
| **430** | 347 | 357 | 367 | 377 | 387 | 397 | 407 | 417 | 428 | 438 |
| 431 | 448 | 458 | 468 | 478 | 488 | 498 | 508 | 518 | 528 | 538 |
| 432 | 548 | 558 | 568 | 579 | 589 | 599 | 609 | 619 | 629 | 639 |
| 433 | 649 | 659 | 669 | 679 | 689 | 699 | 709 | 719 | 729 | 739 |
| 434 | 749 | 759 | 769 | 779 | 789 | 799 | 809 | 819 | 829 | 839 |
| 435 | 849 | 859 | 869 | 879 | 889 | 899 | 909 | 919 | 929 | 939 |
| 436 | 949 | 959 | 969 | 979 | 988 | 998 | *008 | *018 | *028 | *038 |
| 437 | 64 048 | 058 | 068 | 078 | 088 | 098 | 108 | 118 | 128 | 137 |
| 438 | 147 | 157 | 167 | 177 | 187 | 197 | 207 | 217 | 227 | 237 |
| 439 | 246 | 256 | 266 | 276 | 286 | 296 | 306 | 316 | 326 | 335 |
| **440** | 345 | 355 | 365 | 375 | 385 | 395 | 404 | 414 | 424 | 434 |
| 441 | 444 | 454 | 464 | 473 | 483 | 493 | 503 | 513 | 523 | 532 |
| 442 | 542 | 552 | 562 | 572 | 582 | 591 | 601 | 611 | 621 | 631 |
| 443 | 640 | 650 | 660 | 670 | 680 | 689 | 699 | 709 | 719 | 729 |
| 444 | 738 | 748 | 758 | 768 | 777 | 787 | 797 | 807 | 816 | 826 |
| 445 | 836 | 846 | 856 | 865 | 875 | 885 | 895 | 904 | 914 | 924 |
| 446 | 933 | 943 | 953 | 963 | 972 | 982 | 992 | *002 | *011 | *021 |
| 447 | 65 031 | 040 | 050 | 060 | 070 | 079 | 089 | 099 | 108 | 118 |
| 448 | 128 | 137 | 147 | 157 | 167 | 176 | 186 | 196 | 205 | 215 |
| 449 | 225 | 234 | 244 | 254 | 263 | 273 | 283 | 292 | 302 | 312 |
| **450** | 321 | 331 | 341 | 350 | 360 | 369 | 379 | 389 | 398 | 408 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Prop. Pts.**

| | 11 | 10 | 9 |
|---|---|---|---|
| 1 | 1.1 | 1.0 | 0.9 |
| 2 | 2.2 | 2.0 | 1.8 |
| 3 | 3.3 | 3.0 | 2.7 |
| 4 | 4.4 | 4.0 | 3.6 |
| 5 | 5.5 | 5.0 | 4.5 |
| 6 | 6.6 | 6.0 | 5.4 |
| 7 | 7.7 | 7.0 | 6.3 |
| 8 | 8.8 | 8.0 | 7.2 |
| 9 | 9.9 | 9.0 | 8.1 |

Prop. Pts.

## 450–500

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **450** | 65 321 | 331 | 341 | 350 | 360 | 369 | 379 | 389 | 398 | 408 |
| 451 | 418 | 427 | 437 | 447 | 456 | 466 | 475 | 485 | 495 | 504 |
| 452 | 514 | 523 | 533 | 543 | 552 | 562 | 571 | 581 | 591 | 600 |
| 453 | 610 | 619 | 629 | 639 | 648 | 658 | 667 | 677 | 686 | 696 |
| 454 | 706 | 715 | 725 | 734 | 744 | 753 | 763 | 772 | 782 | 792 |
| 455 | 801 | 811 | 820 | 830 | 839 | 849 | 858 | 868 | 877 | 887 |
| 456 | 896 | 906 | 916 | 925 | 935 | 944 | 954 | 963 | 973 | 982 |
| 457 | 992 | *001 | *011 | *020 | *030 | *039 | *049 | *058 | *068 | *077 |
| 458 | 66 087 | 096 | 106 | 115 | 124 | 134 | 143 | 153 | 162 | 172 |
| 459 | 181 | 191 | 200 | 210 | 219 | 229 | 238 | 247 | 257 | 266 |
| **460** | 276 | 285 | 295 | 304 | 314 | 323 | 332 | 342 | 351 | 361 |
| 461 | 370 | 380 | 389 | 398 | 408 | 417 | 427 | 436 | 445 | 455 |
| 462 | 464 | 474 | 483 | 492 | 502 | 511 | 521 | 530 | 539 | 549 |
| 463 | 558 | 567 | 577 | 586 | 596 | 605 | 614 | 624 | 633 | 642 |
| 464 | 652 | 661 | 671 | 680 | 689 | 699 | 708 | 717 | 727 | 736 |
| 465 | 745 | 755 | 764 | 773 | 783 | 792 | 801 | 811 | 820 | 829 |
| 466 | 839 | 848 | 857 | 867 | 876 | 885 | 894 | 904 | 913 | 922 |
| 467 | 932 | 941 | 950 | 960 | 969 | 978 | 987 | 997 | *006 | *015 |
| 468 | 67 025 | 034 | 043 | 052 | 062 | 071 | 080 | 089 | 099 | 108 |
| 469 | 117 | 127 | 136 | 145 | 154 | 164 | 173 | 182 | 191 | 201 |
| **470** | 210 | 219 | 228 | 237 | 247 | 256 | 265 | 274 | 284 | 293 |
| 471 | 302 | 311 | 321 | 330 | 339 | 348 | 357 | 367 | 376 | 385 |
| 472 | 394 | 403 | 413 | 422 | 431 | 440 | 449 | 459 | 468 | 477 |
| 473 | 486 | 495 | 504 | 514 | 523 | 532 | 541 | 550 | 560 | 569 |
| 474 | 578 | 587 | 596 | 605 | 614 | 624 | 633 | 642 | 651 | 660 |
| 475 | 669 | 679 | 688 | 697 | 706 | 715 | 724 | 733 | 742 | 752 |
| 476 | 761 | 770 | 779 | 788 | 797 | 806 | 815 | 825 | 834 | 843 |
| 477 | 852 | 861 | 870 | 879 | 888 | 897 | 906 | 916 | 925 | 934 |
| 478 | 943 | 952 | 961 | 970 | 979 | 988 | 997 | *006 | *015 | *024 |
| 479 | 68 034 | 043 | 052 | 061 | 070 | 079 | 088 | 097 | 106 | 115 |
| **480** | 124 | 133 | 142 | 151 | 160 | 169 | 178 | 187 | 196 | 205 |
| 481 | 215 | 224 | 233 | 242 | 251 | 260 | 269 | 278 | 287 | 296 |
| 482 | 305 | 314 | 323 | 332 | 341 | 350 | 359 | 368 | 377 | 386 |
| 483 | 395 | 404 | 413 | 422 | 431 | 440 | 449 | 458 | 467 | 476 |
| 484 | 485 | 494 | 502 | 511 | 520 | 529 | 538 | 547 | 556 | 565 |
| 485 | 574 | 583 | 592 | 601 | 610 | 619 | 628 | 637 | 646 | 655 |
| 486 | 664 | 673 | 681 | 690 | 699 | 708 | 717 | 726 | 735 | 744 |
| 487 | 753 | 762 | 771 | 780 | 789 | 797 | 806 | 815 | 824 | 833 |
| 488 | 842 | 851 | 860 | 869 | 878 | 886 | 895 | 904 | 913 | 922 |
| 489 | 931 | 940 | 949 | 958 | 966 | 975 | 984 | 993 | *002 | *011 |
| **490** | 69 020 | 028 | 037 | 046 | 055 | 064 | 073 | 082 | 090 | 099 |
| 491 | 108 | 117 | 126 | 135 | 144 | 152 | 161 | 170 | 179 | 188 |
| 492 | 197 | 205 | 214 | 223 | 232 | 241 | 249 | 258 | 267 | 276 |
| 493 | 285 | 294 | 302 | 311 | 320 | 329 | 338 | 346 | 355 | 364 |
| 494 | 373 | 381 | 390 | 399 | 408 | 417 | 425 | 434 | 443 | 452 |
| 495 | 461 | 469 | 478 | 487 | 496 | 504 | 513 | 522 | 531 | 539 |
| 496 | 548 | 557 | 566 | 574 | 583 | 592 | 601 | 609 | 618 | 627 |
| 497 | 636 | 644 | 653 | 662 | 671 | 679 | 688 | 697 | 705 | 714 |
| 498 | 723 | 732 | 740 | 749 | 758 | 767 | 775 | 784 | 793 | 801 |
| 499 | 810 | 819 | 827 | 836 | 845 | 854 | 862 | 871 | 880 | 888 |
| **500** | 897 | 906 | 914 | 923 | 932 | 940 | 949 | 958 | 966 | 975 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 10 | 9 | 8 |
|---|---|---|---|
| 1 | 1.0 | 0.9 | 0.8 |
| 2 | 2.0 | 1.8 | 1.6 |
| 3 | 3.0 | 2.7 | 2.4 |
| 4 | 4.0 | 3.6 | 3.2 |
| 5 | 5.0 | 4.5 | 4.0 |
| 6 | 6.0 | 5.4 | 4.8 |
| 7 | 7.0 | 6.3 | 5.6 |
| 8 | 8.0 | 7.2 | 6.4 |
| 9 | 9.0 | 8.1 | 7.2 |

Prop. Pts.

## 500–550

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **500** | 69 897 | 906 | 914 | 923 | 932 | 940 | 949 | 958 | 966 | 975 |
| 501 | 984 | 992 | *001 | *010 | *018 | *027 | *036 | *044 | *053 | *062 |
| 502 | 70 070 | 079 | 088 | 096 | 105 | 114 | 122 | 131 | 140 | 148 |
| 503 | 157 | 165 | 174 | 183 | 191 | 200 | 209 | 217 | 226 | 234 |
| 504 | 243 | 252 | 260 | 269 | 278 | 286 | 295 | 303 | 312 | 321 |
| 505 | 329 | 338 | 346 | 355 | 364 | 372 | 381 | 389 | 398 | 406 |
| 506 | 415 | 424 | 432 | 441 | 449 | 458 | 467 | 475 | 484 | 492 |
| 507 | 501 | 509 | 518 | 526 | 535 | 544 | 552 | 561 | 569 | 578 |
| 508 | 586 | 595 | 603 | 612 | 621 | 629 | 638 | 646 | 655 | 663 |
| 509 | 672 | 680 | 689 | 697 | 706 | 714 | 723 | 731 | 740 | 749 |
| **510** | 757 | 766 | 774 | 783 | 791 | 800 | 808 | 817 | 825 | 834 |
| 511 | 842 | 851 | 859 | 868 | 876 | 885 | 893 | 902 | 910 | 919 |
| 512 | 927 | 935 | 944 | 952 | 961 | 969 | 978 | 986 | 995 | *003 |
| 513 | 71 012 | 020 | 029 | 037 | 046 | 054 | 063 | 071 | 079 | 088 |
| 514 | 096 | 105 | 113 | 122 | 130 | 139 | 147 | 155 | 164 | 172 |
| 515 | 181 | 189 | 198 | 206 | 214 | 223 | 231 | 240 | 248 | 257 |
| 516 | 265 | 273 | 282 | 290 | 299 | 307 | 315 | 324 | 332 | 341 |
| 517 | 349 | 357 | 366 | 374 | 383 | 391 | 399 | 408 | 416 | 425 |
| 518 | 433 | 441 | 450 | 458 | 466 | 475 | 483 | 492 | 500 | 508 |
| 519 | 517 | 525 | 533 | 542 | 550 | 559 | 567 | 575 | 584 | 592 |
| **520** | 600 | 609 | 617 | 625 | 634 | 642 | 650 | 659 | 667 | 675 |
| 521 | 684 | 692 | 700 | 709 | 717 | 725 | 734 | 742 | 750 | 759 |
| 522 | 767 | 775 | 784 | 792 | 800 | 809 | 817 | 825 | 834 | 842 |
| 523 | 850 | 858 | 867 | 875 | 883 | 892 | 900 | 908 | 917 | 925 |
| 524 | 933 | 941 | 950 | 958 | 966 | 975 | 983 | 991 | 999 | *008 |
| 525 | 72 016 | 024 | 032 | 041 | 049 | 057 | 066 | 074 | 082 | 090 |
| 526 | 099 | 107 | 115 | 123 | 132 | 140 | 148 | 156 | 165 | 173 |
| 527 | 181 | 189 | 198 | 206 | 214 | 222 | 230 | 239 | 247 | 255 |
| 528 | 263 | 272 | 280 | 288 | 296 | 304 | 313 | 321 | 329 | 337 |
| 529 | 346 | 354 | 362 | 370 | 378 | 387 | 395 | 403 | 411 | 419 |
| **530** | 428 | 436 | 444 | 452 | 460 | 469 | 477 | 485 | 493 | 501 |
| 531 | 509 | 518 | 526 | 534 | 542 | 550 | 558 | 567 | 575 | 583 |
| 532 | 591 | 599 | 607 | 616 | 624 | 632 | 640 | 648 | 656 | 665 |
| 533 | 673 | 681 | 689 | 697 | 705 | 713 | 722 | 730 | 738 | 746 |
| 534 | 754 | 762 | 770 | 779 | 787 | 795 | 803 | 811 | 819 | 827 |
| 535 | 835 | 843 | 852 | 860 | 868 | 876 | 884 | 892 | 900 | 908 |
| 536 | 916 | 925 | 933 | 941 | 949 | 957 | 965 | 973 | 981 | 989 |
| 537 | 997 | *006 | *014 | *022 | *030 | *038 | *046 | *054 | *062 | *070 |
| 538 | 73 078 | 086 | 094 | 102 | 111 | 119 | 127 | 135 | 143 | 151 |
| 539 | 159 | 167 | 175 | 183 | 191 | 199 | 207 | 215 | 223 | 231 |
| **540** | 239 | 247 | 255 | 263 | 272 | 280 | 288 | 296 | 304 | 312 |
| 541 | 320 | 328 | 336 | 344 | 352 | 360 | 368 | 376 | 384 | 392 |
| 542 | 400 | 408 | 416 | 424 | 432 | 440 | 448 | 456 | 464 | 472 |
| 543 | 480 | 488 | 496 | 504 | 512 | 520 | 528 | 536 | 544 | 552 |
| 544 | 560 | 568 | 576 | 584 | 592 | 600 | 608 | 616 | 624 | 632 |
| 545 | 640 | 648 | 656 | 664 | 672 | 679 | 687 | 695 | 703 | 711 |
| 546 | 719 | 727 | 735 | 743 | 751 | 759 | 767 | 775 | 783 | 791 |
| 547 | 799 | 807 | 815 | 823 | 830 | 838 | 846 | 854 | 862 | 870 |
| 548 | 878 | 886 | 894 | 902 | 910 | 918 | 926 | 933 | 941 | 949 |
| 549 | 957 | 965 | 973 | 981 | 989 | 997 | *005 | *013 | *020 | *028 |
| **550** | 74 036 | 044 | 052 | 060 | 068 | 076 | 084 | 092 | 099 | 107 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Prop. Pts.**

| | 9 | 8 | 7 |
|---|---|---|---|
| 1 | 0.9 | 0.8 | 0.7 |
| 2 | 1.8 | 1.6 | 1.4 |
| 3 | 2.7 | 2.4 | 2.1 |
| 4 | 3.6 | 3.2 | 2.8 |
| 5 | 4.5 | 4.0 | 3.5 |
| 6 | 5.4 | 4.8 | 4.2 |
| 7 | 6.3 | 5.6 | 4.9 |
| 8 | 7.2 | 6.4 | 5.6 |
| 9 | 8.1 | 7.2 | 6.3 |

Prop. Pts.

## 550–600

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **550** | 74 036 | 044 | 052 | 060 | 068 | 076 | 084 | 092 | 099 | 107 |
| 551 | 115 | 123 | 131 | 139 | 147 | 155 | 162 | 170 | 178 | 186 |
| 552 | 194 | 202 | 210 | 218 | 225 | 233 | 241 | 249 | 257 | 265 |
| 553 | 273 | 280 | 288 | 296 | 304 | 312 | 320 | 327 | 335 | 343 |
| 554 | 351 | 359 | 367 | 374 | 382 | 390 | 398 | 406 | 414 | 421 |
| 555 | 429 | 437 | 445 | 453 | 461 | 468 | 476 | 484 | 492 | 500 |
| 556 | 507 | 515 | 523 | 531 | 539 | 547 | 554 | 562 | 570 | 578 |
| 557 | 586 | 593 | 601 | 609 | 617 | 624 | 632 | 640 | 648 | 656 |
| 558 | 663 | 671 | 679 | 687 | 695 | 702 | 710 | 718 | 726 | 733 |
| 559 | 741 | 749 | 757 | 764 | 772 | 780 | 788 | 796 | 803 | 811 |
| **560** | 819 | 827 | 834 | 842 | 850 | 858 | 865 | 873 | 881 | 889 |
| 561 | 896 | 904 | 912 | 920 | 927 | 935 | 943 | 950 | 958 | 966 |
| 562 | 974 | 981 | 989 | 997 | *005 | *012 | *020 | *028 | *035 | *043 |
| 563 | 75 051 | 059 | 066 | 074 | 082 | 089 | 097 | 105 | 113 | 120 |
| 564 | 128 | 136 | 143 | 151 | 159 | 166 | 174 | 182 | 189 | 197 |
| 565 | 205 | 213 | 220 | 228 | 236 | 243 | 251 | 259 | 266 | 274 |
| 566 | 282 | 289 | 297 | 305 | 312 | 320 | 328 | 335 | 343 | 351 |
| 567 | 358 | 366 | 374 | 381 | 389 | 397 | 404 | 412 | 420 | 427 |
| 568 | 435 | 442 | 450 | 458 | 465 | 473 | 481 | 488 | 496 | 504 |
| 569 | 511 | 519 | 526 | 534 | 542 | 549 | 557 | 565 | 572 | 580 |
| **570** | 587 | 595 | 603 | 610 | 618 | 626 | 633 | 641 | 648 | 656 |
| 571 | 664 | 671 | 679 | 686 | 694 | 702 | 709 | 717 | 724 | 732 |
| 572 | 740 | 747 | 755 | 762 | 770 | 778 | 785 | 793 | 800 | 808 |
| 573 | 815 | 823 | 831 | 838 | 846 | 853 | 861 | 868 | 876 | 884 |
| 574 | 891 | 899 | 906 | 914 | 921 | 929 | 937 | 944 | 952 | 959 |
| 575 | 967 | 974 | 982 | 989 | 997 | *005 | *012 | *020 | *027 | *035 |
| 576 | 76 042 | 050 | 057 | 065 | 072 | 080 | 087 | 095 | 103 | 110 |
| 577 | 118 | 125 | 133 | 140 | 148 | 155 | 163 | 170 | 178 | 185 |
| 578 | 193 | 200 | 208 | 215 | 223 | 230 | 238 | 245 | 253 | 260 |
| 579 | 268 | 275 | 283 | 290 | 298 | 305 | 313 | 320 | 328 | 335 |
| **580** | 343 | 350 | 358 | 365 | 373 | 380 | 388 | 395 | 403 | 410 |
| 581 | 418 | 425 | 433 | 440 | 448 | 455 | 462 | 470 | 477 | 485 |
| 582 | 492 | 500 | 507 | 515 | 522 | 530 | 537 | 545 | 552 | 559 |
| 583 | 567 | 574 | 582 | 589 | 597 | 604 | 612 | 619 | 626 | 634 |
| 584 | 641 | 649 | 656 | 664 | 671 | 678 | 686 | 693 | 701 | 708 |
| 585 | 716 | 723 | 730 | 738 | 745 | 753 | 760 | 768 | 775 | 782 |
| 586 | 790 | 797 | 805 | 812 | 819 | 827 | 834 | 842 | 849 | 856 |
| 587 | 864 | 871 | 879 | 886 | 893 | 901 | 908 | 916 | 923 | 930 |
| 588 | 938 | 945 | 953 | 960 | 867 | 975 | 982 | 989 | 997 | *004 |
| 589 | 77 012 | 019 | 026 | 034 | 041 | 048 | 056 | 063 | 070 | 078 |
| **590** | 085 | 093 | 100 | 107 | 115 | 122 | 129 | 137 | 144 | 151 |
| 591 | 159 | 166 | 173 | 181 | 188 | 195 | 203 | 210 | 217 | 225 |
| 592 | 232 | 240 | 247 | 254 | 262 | 269 | 276 | 283 | 291 | 298 |
| 593 | 305 | 313 | 320 | 327 | 335 | 342 | 349 | 357 | 364 | 371 |
| 594 | 379 | 386 | 393 | 401 | 408 | 415 | 422 | 430 | 437 | 444 |
| 595 | 452 | 459 | 466 | 474 | 481 | 488 | 495 | 503 | 510 | 517 |
| 596 | 525 | 532 | 539 | 546 | 554 | 561 | 568 | 576 | 583 | 590 |
| 597 | 597 | 605 | 612 | 619 | 627 | 634 | 641 | 648 | 656 | 663 |
| 598 | 670 | 677 | 685 | 692 | 699 | 706 | 714 | 721 | 728 | 735 |
| 599 | 743 | 750 | 757 | 764 | 772 | 779 | 786 | 793 | 801 | 808 |
| **600** | 815 | 822 | 830 | 837 | 844 | 851 | 859 | 866 | 873 | 880 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 8 |
|---|---|
| 1 | 0.8 |
| 2 | 1.6 |
| 3 | 2.4 |
| 4 | 3.2 |
| 5 | 4.0 |
| 6 | 4.8 |
| 7 | 5.6 |
| 8 | 6.4 |
| 9 | 7.2 |

| | 7 |
|---|---|
| 1 | 0.7 |
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

Prop. Pts.

## 600–650

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **600** | 77 815 | 822 | 830 | 837 | 844 | 851 | 859 | 866 | 873 | 880 |
| 601 | 887 | 895 | 902 | 909 | 916 | 924 | 931 | 938 | 945 | 952 |
| 602 | 960 | 967 | 974 | 981 | 988 | 996 | *003 | *010 | *017 | *025 |
| 603 | 78 032 | 039 | 046 | 053 | 061 | 068 | 075 | 082 | 089 | 097 |
| 604 | 104 | 111 | 118 | 125 | 132 | 140 | 147 | 154 | 161 | 168 |
| 605 | 176 | 183 | 190 | 197 | 204 | 211 | 219 | 226 | 233 | 240 |
| 606 | 247 | 254 | 262 | 269 | 276 | 283 | 290 | 297 | 305 | 312 |
| 607 | 319 | 326 | 333 | 340 | 347 | 355 | 362 | 369 | 376 | 383 |
| 608 | 390 | 398 | 405 | 412 | 419 | 426 | 433 | 440 | 447 | 455 |
| 609 | 462 | 469 | 476 | 483 | 490 | 497 | 504 | 512 | 519 | 526 |
| **610** | 533 | 540 | 547 | 554 | 561 | 569 | 576 | 583 | 590 | 597 |
| 611 | 604 | 611 | 618 | 625 | 633 | 640 | 647 | 654 | 661 | 668 |
| 612 | 675 | 682 | 689 | 696 | 704 | 711 | 718 | 725 | 732 | 739 |
| 613 | 746 | 753 | 760 | 767 | 774 | 781 | 789 | 796 | 803 | 810 |
| 614 | 817 | 824 | 831 | 838 | 845 | 852 | 859 | 866 | 873 | 880 |
| 615 | 888 | 895 | 902 | 909 | 916 | 923 | 930 | 937 | 944 | 951 |
| 616 | 958 | 965 | 972 | 979 | 986 | 993 | *000 | *007 | *014 | *021 |
| 617 | 79 029 | 036 | 043 | 050 | 057 | 064 | 071 | 078 | 085 | 092 |
| 618 | 099 | 106 | 113 | 120 | 127 | 134 | 141 | 148 | 155 | 162 |
| 619 | 169 | 176 | 183 | 190 | 197 | 204 | 211 | 218 | 225 | 232 |
| **620** | 239 | 246 | 253 | 260 | 267 | 274 | 281 | 288 | 295 | 302 |
| 621 | 309 | 316 | 323 | 330 | 337 | 344 | 351 | 358 | 365 | 372 |
| 622 | 379 | 386 | 393 | 400 | 407 | 414 | 421 | 428 | 435 | 442 |
| 623 | 449 | 456 | 463 | 470 | 477 | 484 | 491 | 498 | 505 | 511 |
| 624 | 518 | 525 | 532 | 539 | 546 | 553 | 560 | 567 | 574 | 581 |
| 625 | 588 | 595 | 602 | 609 | 616 | 623 | 630 | 637 | 644 | 650 |
| 626 | 657 | 664 | 671 | 678 | 685 | 692 | 699 | 706 | 713 | 720 |
| 627 | 727 | 734 | 741 | 748 | 754 | 761 | 768 | 775 | 782 | 789 |
| 628 | 796 | 803 | 810 | 817 | 824 | 831 | 837 | 844 | 851 | 858 |
| 629 | 865 | 872 | 879 | 886 | 893 | 900 | 906 | 913 | 920 | 927 |
| **630** | 934 | 941 | 948 | 955 | 962 | 969 | 975 | 982 | 989 | 996 |
| 631 | 80 003 | 010 | 017 | 024 | 030 | 037 | 044 | 051 | 058 | 065 |
| 632 | 072 | 079 | 085 | 092 | 099 | 106 | 113 | 120 | 127 | 134 |
| 633 | 140 | 147 | 154 | 161 | 168 | 175 | 182 | 188 | 195 | 202 |
| 634 | 209 | 216 | 223 | 229 | 236 | 243 | 250 | 257 | 264 | 271 |
| 635 | 277 | 284 | 291 | 298 | 305 | 312 | 318 | 325 | 332 | 339 |
| 636 | 346 | 353 | 359 | 366 | 373 | 380 | 387 | 393 | 400 | 407 |
| 637 | 414 | 421 | 428 | 434 | 441 | 448 | 455 | 462 | 468 | 475 |
| 638 | 482 | 489 | 496 | 502 | 509 | 516 | 523 | 530 | 536 | 543 |
| 639 | 550 | 557 | 564 | 570 | 577 | 584 | 591 | 598 | 604 | 611 |
| **640** | 618 | 625 | 632 | 638 | 645 | 652 | 659 | 665 | 672 | 679 |
| 641 | 686 | 693 | 699 | 706 | 713 | 720 | 726 | 733 | 740 | 747 |
| 642 | 754 | 760 | 767 | 774 | 781 | 787 | 794 | 801 | 808 | 814 |
| 643 | 821 | 828 | 835 | 841 | 848 | 855 | 862 | 868 | 875 | 882 |
| 644 | 889 | 895 | 902 | 909 | 916 | 922 | 929 | 936 | 943 | 949 |
| 645 | 956 | 963 | 969 | 976 | 983 | 990 | 996 | *003 | *010 | *017 |
| 646 | 81 023 | 030 | 037 | 043 | 050 | 057 | 064 | 070 | 077 | 084 |
| 647 | 090 | 097 | 104 | 111 | 117 | 124 | 131 | 137 | 144 | 151 |
| 648 | 158 | 164 | 171 | 178 | 184 | 191 | 198 | 204 | 211 | 218 |
| 649 | 224 | 231 | 238 | 245 | 251 | 258 | 265 | 271 | 278 | 285 |
| **650** | 291 | 298 | 305 | 311 | 318 | 325 | 331 | 338 | 345 | 351 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 8 |
|---|---|
| 1 | 0.8 |
| 2 | 1.6 |
| 3 | 2.4 |
| 4 | 3.2 |
| 5 | 4.0 |
| 6 | 4.8 |
| 7 | 5.6 |
| 8 | 6.4 |
| 9 | 7.2 |

| | 7 |
|---|---|
| 1 | 0.7 |
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

| | 6 |
|---|---|
| 1 | 0.6 |
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

Prop. Pts.

## 650–700

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **650** | 81 291 | 298 | 305 | 311 | 318 | 325 | 331 | 338 | 345 | 351 |
| 651 | 358 | 365 | 371 | 378 | 385 | 391 | 398 | 405 | 411 | 418 |
| 652 | 425 | 431 | 438 | 445 | 451 | 458 | 465 | 471 | 478 | 485 |
| 653 | 491 | 498 | 505 | 511 | 518 | 525 | 531 | 538 | 544 | 551 |
| 654 | 558 | 564 | 571 | 578 | 584 | 591 | 598 | 604 | 611 | 617 |
| 655 | 624 | 631 | 637 | 644 | 651 | 657 | 664 | 671 | 677 | 684 |
| 656 | 690 | 697 | 704 | 710 | 717 | 723 | 730 | 737 | 743 | 750 |
| 657 | 757 | 763 | 770 | 776 | 783 | 790 | 796 | 803 | 809 | 816 |
| 658 | 823 | 829 | 836 | 842 | 849 | 856 | 862 | 869 | 875 | 882 |
| 659 | 889 | 895 | 902 | 908 | 915 | 921 | 928 | 935 | 941 | 948 |
| **660** | 954 | 961 | 968 | 974 | 981 | 987 | 994 | *000 | *007 | *014 |
| 661 | 82 020 | 027 | 033 | 040 | 046 | 053 | 060 | 066 | 073 | 079 |
| 662 | 086 | 092 | 099 | 105 | 112 | 119 | 125 | 132 | 138 | 145 |
| 663 | 151 | 158 | 164 | 171 | 178 | 184 | 191 | 197 | 204 | 210 |
| 664 | 217 | 223 | 230 | 236 | 243 | 249 | 256 | 263 | 269 | 276 |
| 665 | 282 | 289 | 295 | 302 | 308 | 315 | 321 | 328 | 334 | 341 |
| 666 | 347 | 354 | 360 | 367 | 373 | 380 | 387 | 393 | 400 | 406 |
| 667 | 413 | 419 | 426 | 432 | 439 | 445 | 452 | 458 | 465 | 471 |
| 668 | 478 | 484 | 491 | 497 | 504 | 510 | 517 | 523 | 530 | 536 |
| 669 | 543 | 549 | 556 | 562 | 569 | 575 | 582 | 588 | 595 | 601 |
| **670** | 607 | 614 | 620 | 627 | 633 | 640 | 646 | 653 | 659 | 666 |
| 671 | 672 | 679 | 685 | 692 | 698 | 705 | 711 | 718 | 724 | 730 |
| 672 | 737 | 743 | 750 | 756 | 763 | 769 | 776 | 782 | 789 | 795 |
| 673 | 802 | 808 | 814 | 821 | 827 | 834 | 840 | 847 | 853 | 860 |
| 674 | 866 | 872 | 879 | 885 | 892 | 898 | 905 | 911 | 918 | 924 |
| 675 | 930 | 937 | 943 | 950 | 956 | 963 | 969 | 975 | 982 | 988 |
| 676 | 995 | *001 | *008 | *014 | *020 | *027 | *033 | *040 | *046 | *052 |
| 677 | 83 059 | 065 | 072 | 078 | 085 | 091 | 097 | 104 | 110 | 117 |
| 678 | 123 | 129 | 136 | 142 | 149 | 155 | 161 | 168 | 174 | 181 |
| 679 | 187 | 193 | 200 | 206 | 213 | 219 | 225 | 232 | 238 | 245 |
| **680** | 251 | 257 | 264 | 270 | 276 | 283 | 289 | 296 | 302 | 308 |
| 681 | 315 | 321 | 327 | 334 | 340 | 347 | 353 | 359 | 366 | 372 |
| 682 | 378 | 385 | 391 | 398 | 404 | 410 | 417 | 423 | 429 | 436 |
| 683 | 442 | 448 | 455 | 461 | 467 | 474 | 480 | 487 | 493 | 499 |
| 684 | 506 | 512 | 518 | 525 | 531 | 537 | 544 | 550 | 556 | 563 |
| 685 | 569 | 575 | 582 | 588 | 594 | 601 | 607 | 613 | 620 | 626 |
| 686 | 632 | 639 | 645 | 651 | 658 | 664 | 670 | 677 | 683 | 689 |
| 687 | 696 | 702 | 708 | 715 | 721 | 727 | 734 | 740 | 746 | 753 |
| 688 | 759 | 765 | 771 | 778 | 784 | 790 | 797 | 803 | 809 | 816 |
| 689 | 822 | 828 | 835 | 841 | 847 | 853 | 860 | 866 | 872 | 879 |
| **690** | 885 | 891 | 897 | 904 | 910 | 916 | 923 | 929 | 935 | 942 |
| 691 | 948 | 954 | 960 | 967 | 973 | 979 | 985 | 992 | 998 | *004 |
| 692 | 84 011 | 017 | 023 | 029 | 036 | 042 | 048 | 055 | 061 | 067 |
| 693 | 073 | 080 | 086 | 092 | 098 | 105 | 111 | 117 | 123 | 130 |
| 694 | 136 | 142 | 148 | 155 | 161 | 167 | 173 | 180 | 186 | 192 |
| 695 | 198 | 205 | 211 | 217 | 223 | 230 | 236 | 242 | 248 | 255 |
| 696 | 261 | 267 | 273 | 280 | 286 | 292 | 298 | 305 | 311 | 317 |
| 697 | 323 | 330 | 336 | 342 | 348 | 354 | 361 | 367 | 373 | 379 |
| 698 | 386 | 392 | 398 | 404 | 410 | 417 | 423 | 429 | 435 | 442 |
| 699 | 448 | 454 | 460 | 466 | 473 | 479 | 485 | 491 | 497 | 504 |
| **700** | 510 | 516 | 522 | 528 | 535 | 541 | 547 | 553 | 559 | 566 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 7 |
|---|---|
| 1 | 0.7 |
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

| | 6 |
|---|---|
| 1 | 0.6 |
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

Prop. Pts.

## 700–750

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **700** | 84 510 | 516 | 522 | 528 | 535 | 541 | 547 | 553 | 559 | 566 |
| 701 | 572 | 578 | 584 | 590 | 597 | 603 | 609 | 615 | 621 | 628 |
| 702 | 634 | 640 | 646 | 652 | 658 | 665 | 671 | 677 | 683 | 689 |
| 703 | 696 | 702 | 708 | 714 | 720 | 726 | 733 | 739 | 745 | 751 |
| 704 | 757 | 763 | 770 | 776 | 782 | 788 | 794 | 800 | 807 | 813 |
| 705 | 819 | 825 | 831 | 837 | 844 | 850 | 856 | 862 | 868 | 874 |
| 706 | 880 | 887 | 893 | 899 | 905 | 911 | 917 | 924 | 930 | 936 |
| 707 | 942 | 948 | 954 | 960 | 967 | 973 | 979 | 985 | 991 | 997 |
| 708 | 85 003 | 009 | 016 | 022 | 028 | 034 | 040 | 046 | 052 | 058 |
| 709 | 065 | 071 | 077 | 083 | 089 | 095 | 101 | 107 | 114 | 120 |
| **710** | 126 | 132 | 138 | 144 | 150 | 156 | 163 | 169 | 175 | 181 |
| 711 | 187 | 193 | 199 | 205 | 211 | 217 | 224 | 230 | 236 | 242 |
| 712 | 248 | 254 | 260 | 266 | 272 | 278 | 285 | 291 | 297 | 303 |
| 713 | 309 | 315 | 321 | 327 | 333 | 339 | 345 | 352 | 358 | 364 |
| 714 | 370 | 376 | 382 | 388 | 394 | 400 | 406 | 412 | 418 | 425 |
| 715 | 431 | 437 | 443 | 449 | 455 | 461 | 467 | 473 | 479 | 485 |
| 716 | 491 | 497 | 503 | 509 | 516 | 522 | 528 | 534 | 540 | 546 |
| 717 | 552 | 558 | 564 | 570 | 576 | 582 | 588 | 594 | 600 | 606 |
| 718 | 612 | 618 | 625 | 631 | 637 | 643 | 649 | 655 | 661 | 667 |
| 719 | 673 | 679 | 685 | 691 | 697 | 703 | 709 | 715 | 721 | 727 |
| **720** | 733 | 739 | 745 | 751 | 757 | 763 | 769 | 775 | 781 | 788 |
| 721 | 794 | 800 | 806 | 812 | 818 | 824 | 830 | 836 | 842 | 848 |
| 722 | 854 | 860 | 866 | 872 | 878 | 884 | 890 | 896 | 902 | 908 |
| 723 | 914 | 920 | 926 | 932 | 938 | 944 | 950 | 956 | 962 | 968 |
| 724 | 974 | 980 | 986 | 992 | 998 | *004 | *010 | *016 | *022 | *028 |
| 725 | 86 034 | 040 | 046 | 052 | 058 | 064 | 070 | 076 | 082 | 088 |
| 726 | 094 | 100 | 106 | 112 | 118 | 124 | 130 | 136 | 141 | 147 |
| 727 | 153 | 159 | 165 | 171 | 177 | 183 | 189 | 195 | 201 | 207 |
| 728 | 213 | 219 | 225 | 231 | 237 | 243 | 249 | 255 | 261 | 267 |
| 729 | 273 | 279 | 285 | 291 | 297 | 303 | 308 | 314 | 320 | 326 |
| **730** | 332 | 338 | 344 | 350 | 356 | 362 | 368 | 374 | 380 | 386 |
| 731 | 392 | 398 | 404 | 410 | 415 | 421 | 427 | 433 | 439 | 445 |
| 732 | 451 | 457 | 463 | 469 | 475 | 481 | 487 | 493 | 499 | 504 |
| 733 | 510 | 516 | 522 | 528 | 534 | 540 | 546 | 552 | 558 | 564 |
| 734 | 570 | 576 | 581 | 587 | 593 | 599 | 605 | 611 | 617 | 623 |
| 735 | 629 | 635 | 641 | 646 | 652 | 658 | 664 | 670 | 676 | 682 |
| 736 | 688 | 694 | 700 | 705 | 711 | 717 | 723 | 729 | 735 | 741 |
| 737 | 747 | 753 | 759 | 764 | 770 | 776 | 782 | 788 | 794 | 800 |
| 738 | 806 | 812 | 817 | 823 | 829 | 835 | 841 | 847 | 853 | 859 |
| 739 | 864 | 870 | 876 | 882 | 888 | 894 | 900 | 906 | 911 | 917 |
| **740** | 923 | 929 | 935 | 941 | 947 | 953 | 958 | 964 | 970 | 976 |
| 741 | 982 | 988 | 994 | 999 | *005 | *011 | *017 | *023 | *029 | *035 |
| 742 | 87 040 | 046 | 052 | 058 | 064 | 070 | 075 | 081 | 087 | 093 |
| 743 | 099 | 105 | 111 | 116 | 122 | 128 | 134 | 140 | 146 | 151 |
| 744 | ·157 | 163 | 169 | 175 | 181 | 186 | 192 | 198 | 204 | 210 |
| 745 | 216 | 221 | 227 | 233 | 239 | 245 | 251 | 256 | 262 | 268 |
| 746 | 274 | 280 | 286 | 291 | 297 | 303 | 309 | 315 | 320 | 326 |
| 747 | 332 | 338 | 344 | 349 | 355 | 361 | 367 | 373 | 379 | 384 |
| 748 | 390 | 396 | 402 | 408 | 413 | 419 | 425 | 431 | 437 | 442 |
| 749 | 448 | 454 | 460 | 466 | 471 | 477 | 483 | 489 | 495 | 500 |
| **750** | 506 | 512 | 518 | 523 | 529 | 535 | 541 | 547 | 552 | 558 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 7 | 6 | 5 |
|---|---|---|---|
| 1 | 0.7 | 0.6 | 0.5 |
| 2 | 1.4 | 1.2 | 1.0 |
| 3 | 2.1 | 1.8 | 1.5 |
| 4 | 2.8 | 2.4 | 2.0 |
| 5 | 3.5 | 3.0 | 2.5 |
| 6 | 4.2 | 3.6 | 3.0 |
| 7 | 4.9 | 4.2 | 3.5 |
| 8 | 5.6 | 4.8 | 4.0 |
| 9 | 6.3 | 5.4 | 4.5 |

## 750–800

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **750** | 87 506 | 512 | 518 | 523 | 529 | 535 | 541 | 547 | 552 | 558 |
| 751 | 564 | 570 | 576 | 581 | 587 | 593 | 599 | 604 | 610 | 616 |
| 752 | 622 | 628 | 633 | 639 | 645 | 651 | 656 | 662 | 668 | 674 |
| 753 | 679 | 685 | 691 | 697 | 703 | 708 | 714 | 720 | 726 | 731 |
| 754 | 737 | 743 | 749 | 754 | 760 | 766 | 772 | 777 | 783 | 789 |
| 755 | 795 | 800 | 806 | 812 | 818 | 823 | 829 | 835 | 841 | 846 |
| 756 | 852 | 858 | 864 | 869 | 875 | 881 | 887 | 892 | 898 | 904 |
| 757 | 910 | 915 | 921 | 927 | 933 | 938 | 944 | 950 | 955 | 961 |
| 758 | 967 | 973 | 978 | 984 | 990 | 996 | *001 | *007 | *013 | *018 |
| 759 | 88 024 | 030 | 036 | 041 | 047 | 053 | 058 | 064 | 070 | 076 |
| **760** | 081 | 087 | 093 | 098 | 104 | 110 | 116 | 121 | 127 | 133 |
| 761 | 138 | 144 | 150 | 156 | 161 | 167 | 173 | 178 | 184 | 190 |
| 762 | 195 | 201 | 207 | 213 | 218 | 224 | 230 | 235 | 241 | 247 |
| 763 | 252 | 258 | 264 | 270 | 275 | 281 | 287 | 292 | 298 | 304 |
| 764 | 309 | 315 | 321 | 326 | 332 | 338 | 343 | 349 | 355 | 360 |
| 765 | 366 | 372 | 377 | 383 | 389 | 395 | 400 | 406 | 412 | 417 |
| 766 | 423 | 429 | 434 | 440 | 446 | 451 | 457 | 463 | 468 | 474 |
| 767 | 480 | 485 | 491 | 497 | 502 | 508 | 513 | 519 | 525 | 530 |
| 768 | 536 | 542 | 547 | 553 | 559 | 564 | 570 | 576 | 581 | 587 |
| 769 | 593 | 598 | 604 | 610 | 615 | 621 | 627 | 632 | 638 | 643 |
| **770** | 649 | 655 | 660 | 666 | 672 | 677 | 683 | 689 | 694 | 700 |
| 771 | 705 | 711 | 717 | 722 | 728 | 734 | 739 | 745 | 750 | 756 |
| 772 | 762 | 767 | 773 | 779 | 784 | 790 | 795 | 801 | 807 | 812 |
| 773 | 818 | 824 | 829 | 835 | 840 | 846 | 852 | 857 | 863 | 868 |
| 774 | 874 | 880 | 885 | 891 | 897 | 902 | 908 | 913 | 919 | 925 |
| 775 | 930 | 936 | 941 | 947 | 953 | 958 | 964 | 969 | 975 | 981 |
| 776 | 986 | 992 | 997 | *003 | *009 | *014 | *020 | *025 | *031 | *037 |
| 777 | 89 042 | 048 | 053 | 059 | 064 | 070 | 076 | 081 | 087 | 092 |
| 778 | 098 | 104 | 109 | 115 | 120 | 126 | 131 | 137 | 143 | 148 |
| 779 | 154 | 159 | 165 | 170 | 176 | 182 | 187 | 193 | 198 | 204 |
| **780** | 209 | 215 | 221 | 226 | 232 | 237 | 243 | 248 | 254 | 260 |
| 781 | 265 | 271 | 276 | 282 | 287 | 293 | 298 | 304 | 310 | 315 |
| 782 | 321 | 326 | 332 | 337 | 343 | 348 | 354 | 360 | 365 | 371 |
| 783 | 376 | 382 | 387 | 393 | 398 | 404 | 409 | 415 | 421 | 426 |
| 784 | 432 | 437 | 443 | 448 | 454 | 459 | 465 | 470 | 476 | 481 |
| 785 | 487 | 492 | 498 | 504 | 509 | 515 | 520 | 526 | 531 | 537 |
| 786 | 542 | 548 | 553 | 559 | 564 | 570 | 575 | 581 | 586 | 592 |
| 787 | 597 | 603 | 609 | 614 | 620 | 625 | 631 | 636 | 642 | 647 |
| 788 | 653 | 658 | 664 | 669 | 675 | 680 | 686 | 691 | 697 | 702 |
| 789 | 708 | 713 | 719 | 724 | 730 | 735 | 741 | 746 | 752 | 757 |
| **790** | 763 | 768 | 774 | 779 | 785 | 790 | 796 | 801 | 807 | 812 |
| 791 | 818 | 823 | 829 | 834 | 840 | 845 | 851 | 856 | 862 | 867 |
| 792 | 873 | 878 | 883 | 889 | 894 | 900 | 905 | 911 | 916 | 922 |
| 793 | 927 | 933 | 938 | 944 | 949 | 955 | 960 | 966 | 971 | 977 |
| 794 | 982 | 988 | 993 | 998 | *004 | *009 | *015 | *020 | *026 | *031 |
| 795 | 90 037 | 042 | 048 | 053 | 059 | 064 | 069 | 075 | 080 | 086 |
| 796 | 091 | 097 | 102 | 108 | 113 | 119 | 124 | 129 | 135 | 140 |
| 797 | 146 | 151 | 157 | 162 | 168 | 173 | 179 | 184 | 189 | 195 |
| 798 | 200 | 206 | 211 | 217 | 222 | 227 | 233 | 238 | 244 | 249 |
| 799 | 255 | 260 | 266 | 271 | 276 | 282 | 287 | 293 | 298 | 304 |
| **800** | 309 | 314 | 320 | 325 | 331 | 336 | 342 | 347 | 352 | 358 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 6 |
|---|---|
| 1 | 0.6 |
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

| | 5 |
|---|---|
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

Prop. Pts.

## 800–850

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **800** | 90 309 | 314 | 320 | 325 | 331 | 336 | 342 | 347 | 352 | 358 |
| 801 | 363 | 369 | 374 | 380 | 385 | 390 | 396 | 401 | 407 | 412 |
| 802 | 417 | 423 | 428 | 434 | 439 | 445 | 450 | 455 | 461 | 466 |
| 803 | 472 | 477 | 482 | 488 | 493 | 499 | 504 | 509 | 515 | 520 |
| 804 | 526 | 531 | 536 | 542 | 547 | 553 | 558 | 563 | 569 | 574 |
| 805 | 580 | 585 | 590 | 596 | 601 | 607 | 612 | 617 | 623 | 628 |
| 806 | 634 | 639 | 644 | 650 | 655 | 660 | 666 | 671 | 677 | 682 |
| 807 | 687 | 693 | 698 | 703 | 709 | 714 | 720 | 725 | 730 | 736 |
| 808 | 741 | 747 | 752 | 757 | 763 | 768 | 773 | 779 | 784 | 789 |
| 809 | 795 | 800 | 806 | 811 | 816 | 822 | 827 | 832 | 838 | 843 |
| **810** | 849 | 854 | 859 | 865 | 870 | 875 | 881 | 886 | 891 | 897 |
| 811 | 902 | 907 | 913 | 918 | 924 | 929 | 934 | 940 | 945 | 950 |
| 812 | 956 | 961 | 966 | 972 | 977 | 982 | 988 | 993 | 998 | *004 |
| 813 | 91 009 | 014 | 020 | 025 | 030 | 036 | 041 | 046 | 052 | 057 |
| 814 | 062 | 068 | 073 | 078 | 084 | 089 | 094 | 100 | 105 | 110 |
| 815 | 116 | 121 | 126 | 132 | 137 | 142 | 148 | 153 | 158 | 164 |
| 816 | 169 | 174 | 180 | 185 | 190 | 196 | 201 | 206 | 212 | 217 |
| 817 | 222 | 228 | 233 | 238 | 243 | 249 | 254 | 259 | 265 | 270 |
| 818 | 275 | 281 | 286 | 291 | 297 | 302 | 307 | 312 | 318 | 323 |
| 819 | 328 | 334 | 339 | 344 | 350 | 355 | 360 | 365 | 371 | 376 |
| **820** | 381 | 387 | 392 | 397 | 403 | 408 | 413 | 418 | 424 | 429 |
| 821 | 434 | 440 | 445 | 450 | 455 | 461 | 466 | 471 | 477 | 482 |
| 822 | 487 | 492 | 498 | 503 | 508 | 514 | 519 | 524 | 529 | 535 |
| 823 | 540 | 545 | 551 | 556 | 561 | 566 | 572 | 577 | 582 | 587 |
| 824 | 593 | 598 | 603 | 609 | 614 | 619 | 624 | 630 | 635 | 640 |
| 825 | 645 | 651 | 656 | 661 | 666 | 672 | 677 | 682 | 687 | 693 |
| 826 | 698 | 703 | 709 | 714 | 719 | 724 | 730 | 735 | 740 | 745 |
| 827 | 751 | 756 | 761 | 766 | 772 | 777 | 782 | 787 | 793 | 798 |
| 828 | 803 | 808 | 814 | 819 | 824 | 829 | 834 | 840 | 845 | 850 |
| 829 | 855 | 861 | 866 | 871 | 876 | 882 | 887 | 892 | 897 | 903 |
| **830** | 908 | 913 | 918 | 924 | 929 | 934 | 939 | 944 | 950 | 955 |
| 831 | 960 | 965 | 971 | 976 | 981 | 986 | 991 | 997 | *002 | *007 |
| 832 | 92 012 | 018 | 023 | 028 | 033 | 038 | 044 | 049 | 054 | 059 |
| 833 | 065 | 070 | 075 | 080 | 085 | 091 | 096 | 101 | 106 | 111 |
| 834 | 117 | 122 | 127 | 132 | 137 | 143 | 148 | 153 | 158 | 163 |
| 835 | 169 | 174 | 179 | 184 | 189 | 195 | 200 | 205 | 210 | 215 |
| 836 | 221 | 226 | 231 | 236 | 241 | 247 | 252 | 257 | 262 | 267 |
| 837 | 273 | 278 | 283 | 288 | 293 | 298 | 304 | 309 | 314 | 319 |
| 838 | 324 | 330 | 335 | 340 | 345 | 350 | 355 | 361 | 366 | 371 |
| 839 | 376 | 381 | 387 | 392 | 397 | 402 | 407 | 412 | 418 | 423 |
| **840** | 428 | 433 | 438 | 443 | 449 | 454 | 459 | 464 | 469 | 474 |
| 841 | 480 | 485 | 490 | 495 | 500 | 505 | 511 | 516 | 521 | 526 |
| 842 | 531 | 536 | 542 | 547 | 552 | 557 | 562 | 567 | 572 | 578 |
| 843 | 583 | 588 | 593 | 598 | 603 | 609 | 614 | 619 | 624 | 629 |
| 844 | 634 | 639 | 645 | 650 | 655 | 660 | 665 | 670 | 675 | 681 |
| 845 | 686 | 691 | 696 | 701 | 706 | 711 | 716 | 722 | 727 | 732 |
| 846 | 737 | 742 | 747 | 752 | 758 | 763 | 768 | 773 | 778 | 783 |
| 847 | 788 | 793 | 799 | 804 | 809 | 814 | 819 | 824 | 829 | 834 |
| 848 | 840 | 845 | 850 | 855 | 860 | 865 | 870 | 875 | 881 | 886 |
| 849 | 891 | 896 | 901 | 906 | 911 | 916 | 921 | 927 | 932 | 937 |
| **850** | 942 | 947 | 952 | 957 | 962 | 967 | 973 | 978 | 983 | 988 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 6 |
|---|---|
| 1 | 0.6 |
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

| | 5 |
|---|---|
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

Prop. Pts.

## 850–900

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **850** | 92 942 | 947 | 952 | 957 | 962 | 967 | 973 | 978 | 983 | 988 |
| 851 | 993 | 998 | *003 | *008 | *013 | *018 | *024 | *029 | *034 | *039 |
| 852 | 93 044 | 049 | 054 | 059 | 064 | 069 | 075 | 080 | 085 | 090 |
| 853 | 095 | 100 | 105 | 110 | 115 | 120 | 125 | 131 | 181 | 141 |
| 854 | 146 | 151 | 156 | 161 | 166 | 171 | 176 | 181 | 186 | 192 |
| 855 | 197 | 202 | 207 | 212 | 217 | 222 | 227 | 232 | 237 | 242 |
| 856 | 247 | 252 | 258 | 263 | 268 | 273 | 278 | 283 | 288 | 293 |
| 857 | 298 | 303 | 308 | 313 | 318 | 323 | 328 | 334 | 339 | 344 |
| 858 | 349 | 354 | 359 | 364 | 369 | 374 | 379 | 384 | 389 | 394 |
| 859 | 399 | 404 | 409 | 414 | 420 | 425 | 430 | 435 | 440 | 445 |
| **860** | 450 | 455 | 460 | 465 | 470 | 475 | 480 | 485 | 490 | 495 |
| 861 | 500 | 505 | 510 | 515 | 520 | 526 | 531 | 536 | 541 | 546 |
| 862 | 551 | 556 | 561 | 566 | 571 | 576 | 581 | 586 | 591 | 596 |
| 863 | 601 | 606 | 611 | 616 | 621 | 626 | 631 | 636 | 641 | 646 |
| 864 | 651 | 656 | 661 | 666 | 671 | 676 | 682 | 687 | 692 | 697 |
| 865 | 702 | 707 | 712 | 717 | 722 | 727 | 732 | 737 | 742 | 747 |
| 866 | 752 | 757 | 762 | 767 | 772 | 777 | 782 | 787 | 792 | 797 |
| 867 | 802 | 807 | 812 | 817 | 822 | 827 | 832 | 837 | 842 | 847 |
| 868 | 852 | 857 | 862 | 867 | 872 | 877 | 882 | 887 | 892 | 897 |
| 869 | 902 | 907 | 912 | 917 | 922 | 927 | 932 | 937 | 942 | 947 |
| **870** | 952 | 957 | 962 | 967 | 972 | 977 | 982 | 987 | 992 | 997 |
| 871 | 94 002 | 007 | 012 | 017 | 022 | 027 | 032 | 037 | 042 | 047 |
| 872 | 052 | 057 | 062 | 067 | 072 | 077 | 082 | 086 | 091 | 096 |
| 873 | 101 | 106 | 111 | 116 | 121 | 126 | 131 | 136 | 141 | 146 |
| 874 | 151 | 156 | 161 | 166 | 171 | 176 | 181 | 186 | 191 | 196 |
| 875 | 201 | 206 | 211 | 216 | 221 | 226 | 231 | 236 | 240 | 245 |
| 876 | 250 | 255 | 260 | 265 | 270 | 275 | 280 | 285 | 290 | 295 |
| 877 | 300 | 305 | 310 | 315 | 320 | 325 | 330 | 335 | 340 | 345 |
| 878 | 349 | 354 | 359 | 364 | 369 | 374 | 379 | 384 | 389 | 394 |
| 879 | 399 | 404 | 409 | 414 | 419 | 424 | 429 | 433 | 438 | 443 |
| **880** | 448 | 453 | 458 | 463 | 468 | 473 | 478 | 483 | 488 | 493 |
| 881 | 498 | 503 | 507 | 512 | 517 | 522 | 527 | 532 | 537 | 542 |
| 882 | 547 | 552 | 557 | 562 | 567 | 571 | 576 | 581 | 586 | 591 |
| 883 | 596 | 601 | 606 | 611 | 616 | 621 | 626 | 630 | 635 | 640 |
| 884 | 645 | 650 | 655 | 660 | 665 | 670 | 675 | 680 | 685 | 689 |
| 885 | 694 | 699 | 704 | 709 | 714 | 719 | 724 | 729 | 734 | 738 |
| 886 | 743 | 748 | 753 | 758 | 763 | 768 | 773 | 778 | 783 | 787 |
| 887 | 792 | 797 | 802 | 807 | 812 | 817 | 822 | 827 | 832 | 836 |
| 888 | 841 | 846 | 851 | 856 | 861 | 866 | 871 | 876 | 880 | 885 |
| 889 | 890 | 895 | 900 | 905 | 910 | 915 | 919 | 924 | 929 | 934 |
| **890** | 939 | 944 | 949 | 954 | 959 | 963 | 968 | 973 | 978 | 983 |
| 891 | 988 | 993 | 998 | *002 | *007 | *012 | *017 | *022 | *027 | *032 |
| 892 | 95 036 | 041 | 046 | 051 | 056 | 061 | 066 | 071 | 075 | 080 |
| 893 | 085 | 090 | 095 | 100 | 105 | 109 | 114 | 119 | 124 | 129 |
| 894 | 134 | 139 | 143 | 148 | 153 | 158 | 163 | 168 | 173 | 177 |
| 895 | 182 | 187 | 192 | 197 | 202 | 207 | 211 | 216 | 221 | 226 |
| 896 | 231 | 236 | 240 | 245 | 250 | 255 | 260 | 265 | 270 | 274 |
| 897 | 279 | 284 | 289 | 294 | 299 | 303 | 308 | 313 | 318 | 323 |
| 898 | 328 | 332 | 337 | 342 | 347 | 352 | 357 | 361 | 366 | 371 |
| 899 | 376 | 381 | 386 | 390 | 395 | 400 | 405 | 410 | 415 | 419 |
| **900** | 424 | 429 | 434 | 439 | 444 | 448 | 453 | 458 | 463 | 468 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 6 | 5 | 4 |
|---|---|---|---|
| 1 | 0.6 | 0.5 | 0.4 |
| 2 | 1.2 | 1.0 | 0.8 |
| 3 | 1.8 | 1.5 | 1.2 |
| 4 | 2.4 | 2.0 | 1.6 |
| 5 | 3.0 | 2.5 | 2.0 |
| 6 | 3.6 | 3.0 | 2.4 |
| 7 | 4.2 | 3.5 | 2.8 |
| 8 | 4.8 | 4.0 | 3.2 |
| 9 | 5.4 | 4.5 | 3.6 |

Prop. Pts.

## 900–950

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **900** | 95 424 | 429 | 434 | 439 | 444 | 448 | 453 | 458 | 463 | 468 |
| 901 | 472 | 477 | 482 | 487 | 492 | 497 | 501 | 506 | 511 | 516 |
| 902 | 521 | 525 | 530 | 535 | 540 | 545 | 550 | 554 | 559 | 564 |
| 903 | 569 | 574 | 578 | 583 | 588 | 593 | 598 | 602 | 607 | 612 |
| 904 | 617 | 622 | 626 | 631 | 636 | 641 | 646 | 650 | 655 | 660 |
| 905 | 665 | 670 | 674 | 679 | 684 | 689 | 694 | 698 | 703 | 708 |
| 906 | 713 | 718 | 722 | 727 | 732 | 737 | 742 | 746 | 751 | 756 |
| 907 | 761 | 766 | 770 | 775 | 780 | 785 | 789 | 794 | 799 | 804 |
| 908 | 809 | 813 | 818 | 823 | 828 | 832 | 837 | 842 | 847 | 852 |
| 909 | 856 | 861 | 866 | 871 | 875 | 880 | 885 | 890 | 895 | 899 |
| **910** | 904 | 909 | 914 | 018 | 923 | 928 | 933 | 938 | 942 | 947 |
| 911 | 952 | 957 | 961 | 966 | 971 | 976 | 980 | 985 | 990 | 995 |
| 912 | 999 | *004 | *009 | *014 | *019 | *023 | *028 | *033 | *038 | *042 |
| 913 | 96 047 | 052 | 057 | 061 | 066 | 071 | 076 | 080 | 085 | 090 |
| 914 | 095 | 099 | 104 | 109 | 114 | 118 | 123 | 128 | 133 | 137 |
| 915 | 142 | 147 | 152 | 156 | 161 | 166 | 171 | 175 | 180 | 185 |
| 916 | 190 | 194 | 199 | 204 | 209 | 213 | 218 | 223 | 227 | 232 |
| 917 | 237 | 242 | 246 | 251 | 256 | 261 | 265 | 270 | 275 | 280 |
| 918 | 284 | 289 | 294 | 298 | 303 | 308 | 313 | 317 | 322 | 327 |
| 919 | 332 | 336 | 341 | 346 | 350 | 355 | 360 | 365 | 369 | 374 |
| **920** | 379 | 384 | 388 | 393 | 398 | 402 | 407 | 412 | 417 | 421 |
| 921 | 426 | 431 | 435 | 440 | 445 | 450 | 454 | 459 | 464 | 468 |
| 922 | 473 | 478 | 483 | 487 | 492 | 497 | 501 | 506 | 511 | 515 |
| 923 | 520 | 525 | 530 | 534 | 539 | 544 | 548 | 553 | 558 | 562 |
| 924 | 567 | 572 | 577 | 581 | 586 | 591 | 595 | 600 | 605 | 609 |
| 925 | 614 | 619 | 624 | 628 | 633 | 638 | 642 | 647 | 652 | 656 |
| 926 | 661 | 666 | 670 | 675 | 680 | 685 | 689 | 694 | 699 | 703 |
| 927 | 708 | 713 | 717 | 722 | 727 | 731 | 736 | 741 | 745 | 750 |
| 928 | 755 | 759 | 764 | 769 | 774 | 778 | 783 | 788 | 792 | 797 |
| 929 | 802 | 806 | 811 | 816 | 820 | 825 | 830 | 834 | 839 | 844 |
| **930** | 848 | 853 | 858 | 862 | 876 | 872 | 876 | 881 | 886 | 890 |
| 931 | 895 | 900 | 904 | 909 | 914 | 918 | 923 | 928 | 932 | 937 |
| 932 | 942 | 946 | 951 | 956 | 960 | 965 | 970 | 974 | 979 | 984 |
| 933 | 988 | 993 | 997 | *002 | *007 | *011 | *016 | *021 | *025 | *030 |
| 934 | 97 035 | 039 | 044 | 049 | 053 | 058 | 063 | 067 | 072 | 077 |
| 935 | 081 | 086 | 090 | 095 | 100 | 104 | 109 | 114 | 118 | 123 |
| 936 | 128 | 132 | 137 | 142 | 146 | 151 | 155 | 160 | 165 | 169 |
| 937 | 174 | 179 | 183 | 188 | 192 | 197 | 202 | 206 | 211 | 216 |
| 938 | 220 | 225 | 230 | 234 | 239 | 243 | 248 | 253 | 257 | 262 |
| 939 | 267 | 271 | 276 | 280 | 285 | 290 | 294 | 299 | 304 | 308 |
| **940** | 313 | 317 | 322 | 327 | 331 | 336 | 340 | 345 | 350 | 354 |
| 941 | 359 | 364 | 368 | 373 | 377 | 382 | 387 | 391 | 396 | 400 |
| 942 | 405 | 410 | 414 | 419 | 424 | 428 | 433 | 437 | 442 | 447 |
| 943 | 451 | 456 | 460 | 465 | 470 | 474 | 479 | 483 | 488 | 493 |
| 944 | 497 | 502 | 506 | 511 | 516 | 520 | 525 | 529 | 534 | 539 |
| 945 | 543 | 548 | 552 | 557 | 562 | 566 | 571 | 575 | 580 | 585 |
| 946 | 589 | 594 | 598 | 603 | 607 | 612 | 617 | 621 | 626 | 630 |
| 947 | 635 | 640 | 644 | 649 | 653 | 658 | 663 | 667 | 672 | 676 |
| 948 | 681 | 685 | 690 | 695 | 699 | 704 | 708 | 713 | 717 | 722 |
| 949 | 727 | 731 | 736 | 740 | 745 | 749 | 754 | 759 | 763 | 768 |
| **950** | 772 | 777 | 782 | 786 | 791 | 795 | 800 | 804 | 809 | 813 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | 5 |
|---|---|
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

| | 4 |
|---|---|
| 1 | 0.4 |
| 2 | 0.8 |
| 3 | 1.2 |
| 4 | 1.6 |
| 5 | 2.0 |
| 6 | 2.4 |
| 7 | 2.8 |
| 8 | 3.2 |
| 9 | 3.6 |

Prop. Pts.

## 950–1000

| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **950** | 97 772 | 777 | 782 | 786 | 791 | 795 | 800 | 804 | 809 | 813 |
| 951 | 818 | 823 | 827 | 832 | 836 | 841 | 845 | 850 | 855 | 859 |
| 952 | 864 | 868 | 873 | 877 | 882 | 886 | 891 | 896 | 900 | 905 |
| 953 | 909 | 914 | 918 | 923 | 928 | 932 | 937 | 941 | 946 | 950 |
| 954 | 955 | 959 | 964 | 968 | 973 | 978 | 982 | 987 | 991 | 996 |
| 955 | 98 000 | 005 | 009 | 014 | 019 | 023 | 028 | 032 | 037 | 041 |
| 956 | 046 | 050 | 055 | 059 | 064 | 068 | 073 | 078 | 082 | 087 |
| 957 | 091 | 096 | 100 | 105 | 109 | 114 | 118 | 123 | 127 | 132 |
| 958 | 137 | 141 | 146 | 150 | 155 | 159 | 164 | 168 | 173 | 177 |
| 959 | 182 | 186 | 191 | 195 | 200 | 204 | 209 | 214 | 218 | 223 |
| **960** | 227 | 232 | 236 | 241 | 245 | 250 | 254 | 259 | 263 | 268 |
| 961 | 272 | 277 | 281 | 286 | 290 | 295 | 299 | 304 | 308 | 313 |
| 962 | 318 | 322 | 327 | 331 | 336 | 340 | 345 | 349 | 354 | 358 |
| 963 | 363 | 367 | 372 | 376 | 381 | 385 | 390 | 394 | 399 | 403 |
| 964 | 408 | 412 | 417 | 421 | 426 | 430 | 435 | 439 | 444 | 448 |
| 965 | 453 | 457 | 462 | 466 | 471 | 475 | 480 | 484 | 489 | 493 |
| 966 | 498 | 502 | 507 | 511 | 516 | 520 | 525 | 529 | 534 | 538 |
| 967 | 543 | 547 | 552 | 556 | 561 | 565 | 570 | 574 | 579 | 583 |
| 968 | 588 | 592 | 597 | 601 | 605 | 610 | 614 | 619 | 623 | 628 |
| 969 | 632 | 637 | 641 | 646 | 650 | 655 | 659 | 664 | 668 | 673 |
| **970** | 677 | 682 | 686 | 691 | 695 | 700 | 704 | 709 | 713 | 717 |
| 971 | 722 | 726 | 731 | 735 | 740 | 744 | 749 | 753 | 758 | 762 |
| 972 | 767 | 771 | 776 | 780 | 784 | 789 | 793 | 798 | 802 | 807 |
| 973 | 811 | 816 | 820 | 825 | 829 | 834 | 838 | 843 | 847 | 851 |
| 974 | 856 | 860 | 865 | 869 | 874 | 878 | 883 | 887 | 892 | 896 |
| 975 | 900 | 905 | 909 | 914 | 918 | 923 | 927 | 932 | 936 | 941 |
| 976 | 945 | 949 | 954 | 958 | 963 | 967 | 972 | 976 | 981 | 985 |
| 977 | 989 | 994 | 998 | *003 | *007 | *012 | *016 | *021 | *025 | *029 |
| 978 | 99 034 | 038 | 043 | 047 | 052 | 056 | 061 | 065 | 069 | 074 |
| 979 | 078 | 083 | 087 | 092 | 096 | 100 | 105 | 109 | 114 | 118 |
| **980** | 123 | 127 | 131 | 136 | 140 | 145 | 149 | 154 | 158 | 162 |
| 981 | 167 | 171 | 176 | 180 | 185 | 189 | 193 | 198 | 202 | 207 |
| 982 | 211 | 216 | 220 | 224 | 229 | 233 | 238 | 242 | 247 | 251 |
| 983 | 255 | 260 | 264 | 269 | 273 | 277 | 282 | 286 | 291 | 295 |
| 984 | 300 | 304 | 308 | 313 | 317 | 322 | 326 | 330 | 335 | 339 |
| 985 | 344 | 348 | 352 | 357 | 361 | 366 | 370 | 374 | 379 | 383 |
| 986 | 388 | 392 | 396 | 401 | 405 | 410 | 414 | 419 | 423 | 427 |
| 987 | 432 | 436 | 441 | 445 | 449 | 454 | 458 | 463 | 467 | 471 |
| 988 | 476 | 480 | 484 | 489 | 493 | 498 | 502 | 506 | 511 | 515 |
| 989 | 520 | 524 | 528 | 533 | 537 | 542 | 546 | 550 | 555 | 559 |
| **990** | 564 | 568 | 572 | 577 | 581 | 585 | 590 | 594 | 599 | 603 |
| 991 | 607 | 612 | 616 | 621 | 625 | 629 | 634 | 638 | 642 | 647 |
| 992 | 651 | 656 | 660 | 664 | 669 | 673 | 677 | 682 | 686 | 691 |
| 993 | 695 | 699 | 704 | 708 | 712 | 717 | 721 | 726 | 730 | 734 |
| 994 | 739 | 743 | 747 | 752 | 756 | 760 | 765 | 769 | 774 | 778 |
| 995 | 782 | 787 | 791 | 795 | 800 | 804 | 808 | 813 | 817 | 822 |
| 996 | 826 | 830 | 835 | 839 | 843 | 848 | 852 | 856 | 861 | 865 |
| 997 | 870 | 874 | 878 | 883 | 887 | 891 | 896 | 900 | 904 | 909 |
| 998 | 913 | 917 | 922 | 926 | 930 | 935 | 939 | 944 | 948 | 952 |
| 999 | 957 | 961 | 965 | 970 | 974 | 978 | 983 | 987 | 991 | 996 |
| **1000** | 00 000 | 004 | 009 | 013 | 017 | 022 | 026 | 030 | 035 | 039 |
| N | L 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Prop. Pts.

| | **5** |
|---|---|
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

| | **4** |
|---|---|
| 1 | 0.4 |
| 2 | 0.8 |
| 3 | 1.2 |
| 4 | 1.6 |
| 5 | 2.0 |
| 6 | 2.4 |
| 7 | 2.8 |
| 8 | 3.2 |
| 9 | 3.6 |

Prop. Pts.

| | | Number | Logarithm |
|---|---|---|---|
| Base of Naperian logarithms | $e$ | = 2.71828183 | 0.4342945 |
| Modulus of common logarithms | $u$ | = 0.43429448 | 9.6377843–10 |
| Reciprocal of modulus | $\frac{1}{u}$ | = 2.30258509 | |
| Circumference of a circle in degrees | | = 360 | 2.5563025 |
| Circumference of a circle in minutes | | = 21600 | 4.3344538 |
| Circumference of a circle in seconds | | =1296000 | 6.1126050 |
| Radian expressed in degrees | | = 57.29578 | 1.7581226 |
| Radian expressed in minutes | | = 3437.7468 | 3.5362739 |
| Radian expressed in seconds | | = 206264.806 | 5.3144251 |
| Ratio of a circumference to diameter | $\pi$ | = 3.14159265 | 0.4971499 |

$\pi$ = 3.14159 26535 89793 23846 26433 8328

| Number | Logarithm | | | Logarithm |
|---|---|---|---|---|
| $2\pi$ = 6.28318531 | 0.7981799 | $\pi^2$ | = 9.86960440 | 0.9942997 |
| $4\pi$ = 12.56637061 | 1.0992099 | $\frac{1}{\pi^2}$ | = 0.10132118 | 9.0057003–10 |
| $\frac{\pi}{2}$ = 1.57079633 | 0.1961199 | $\sqrt{\pi}$ | = 1.77245385 | 0.2485749 |
| $\frac{\pi}{3}$ = 1.04719755 | 0.0200286 | $\frac{1}{\sqrt{\pi}}$ | = 0.56418958 | 9.7514251–10 |
| $\frac{4\pi}{3}$ = 4.18879020 | 0.6220886 | $\sqrt{\frac{3}{\pi}}$ | = 0.97720502 | 9.9899857–10 |
| $\frac{\pi}{4}$ = 0.78539816 | 9.8950899–10 | $\sqrt{\frac{4}{\pi}}$ | = 1.12837917 | 0.0524551 |
| $\frac{\pi}{6}$ = 0.52359878 | 9.7189986–10 | $\sqrt[3]{\pi}$ | = 1.46459189 | 0.1657166 |
| $\frac{1}{\pi}$ = 0.31830989 | 9.5028501–10 | $\frac{1}{\sqrt[3]{\pi}}$ | = 0.68278406 | 9.8342834–10 |
| $\frac{1}{2\pi}$ = 0.15915494 | 9.2018201–10 | $\sqrt[3]{\pi^2}$ | = 2.14502940 | 0.3314332 |
| $\frac{3}{\pi}$ = 0.95492966 | 9.9799714–10 | $\sqrt{\frac{3}{4\pi}}$ | = 0.62035049 | 9.7926371–10 |
| $\frac{4}{\pi}$ = 1.27323954 | 0.1049101 | $\sqrt[3]{\frac{\pi}{6}}$ | = 0.80599598 | 9.9063329–10 |

If the radius $r = 1$, the length of the arc is

| | | |
|---|---|---|
| for 1 degree $= \frac{\pi}{180}$ | = 0.01745329 | 8.2418774–10 |
| for 1 minute $= \frac{\pi}{10800}$ | = 0.00029089 | 6.4637261–10 |
| for 1 second $= \frac{\pi}{648000}$ | = 0.00000485 | 4.6855749–10 |
| $\sin 1''$ | = 0.00000485 | 4.6855749–10 |

# TABLE IX

## NATURAL LOGARITHMS OF NUMBERS

BASE $e = 2.71828...$

NOTE. — $\text{Log}_e 10N = \text{Log}_e N + \text{Log}_e 10$

$$\text{Log}_e \frac{N}{10} = \text{Log}_e N - \text{Log}_e 10$$

$$\text{Log}_e 10 = 2.30259$$

For example: $\text{Log}_e 27 = \text{Log}_e 2.7 + \text{Log}_e 10$

$$= 0.99325 + 2.30259 = 3.29584$$

$$\text{Log}_e .27 = \text{Log}_e 2.7 - \text{Log}_e 10$$

$$= 0.99325 - 2.30259 = 8.69066 - 10$$

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1.0** | 0.0 0000 | 0995 | 1980 | 2956 | 3922 | 4879 | 5827 | 6766 | 7696 | 8618 |
| 1.1 | 9531 | *0436 | *1333 | *2222 | *3103 | *3976 | *4842 | *5700 | *6551 | *7395 |
| 1.2 | 0.1 8232 | 9062 | 9885 | *0701 | *1511 | *2314 | *3111 | *3902 | *4686 | *5464 |
| 1.3 | 0.2 6236 | 7003 | 7763 | 8518 | 9267 | *0010 | *0748 | *1481 | *2208 | *2930 |
| 1.4 | 0.3 3647 | 4359 | 5066 | 5767 | 6464 | 7156 | 7844 | 8526 | 9204 | 9878 |
| 1.5 | 0.4 0547 | 1211 | 1871 | 2527 | 3178 | 3825 | 4469 | 5108 | 5742 | 6373 |
| 1.6 | 7000 | 7623 | 8243 | 8858 | 9470 | *0078 | *0682 | *1282 | *1879 | *2473 |
| 1.7 | 0.5 3063 | 3649 | 4232 | 4812 | 5389 | 5962 | 6531 | 7098 | 7661 | 8222 |
| 1.8 | 8779 | 9333 | 9884 | *0432 | *0977 | *1519 | *2078 | *2594 | *3127 | *3658 |
| 1.9 | 0.6 4185 | 4710 | 5233 | 5752 | 6269 | 6783 | 7294 | 7803 | 8310 | 8813 |
| **2.0** | 9315 | 9813 | *0310 | *0804 | *1295 | *1784 | *2271 | *2755 | *3237 | *3716 |
| 2.1 | 0.7 4194 | 4669 | 5142 | 5612 | 6081 | 6547 | 7011 | 7473 | 7932 | 8390 |
| 2.2 | 8846 | 9299 | 9751 | *0200 | *0648 | *1093 | *1536 | *1978 | *2418 | *2855 |
| 2.3 | 0.8 3291 | 3725 | 4157 | 4587 | 5015 | 5442 | 5866 | 6289 | 6710 | 7129 |
| 2.4 | 7547 | 7963 | 8377 | 8789 | 9200 | 9609 | *0016 | *0422 | *0826 | *1228 |
| 2.5 | 0.9 1629 | 2028 | 2426 | 2822 | 3216 | 3609 | 4001 | 4391 | 4779 | 5166 |
| 2.6 | 5551 | 5935 | 6317 | 6698 | 7078 | 7456 | 7833 | 8208 | 8582 | 8954 |
| 2.7 | 9325 | 9695 | *0063 | *0430 | *0796 | *1160 | *1523 | *1885 | *2245 | *2604 |
| 2.8 | 1.0 2962 | 3318 | 3674 | 4028 | 4380 | 4732 | 5082 | 5431 | 5779 | 6126 |
| 2.9 | 6471 | 6815 | 7158 | 7500 | 7841 | 8181 | 8519 | 8856 | 9192 | 9527 |
| **3.0** | 9861 | *0194 | *0526 | *0856 | *1186 | *1514 | *1841 | *2168 | *2493 | *2817 |
| 3.1 | 1.1 3140 | 3462 | 3783 | 4103 | 4422 | 4740 | 5057 | 5373 | 5688 | 6002 |
| 3.2 | 6315 | 6627 | 6938 | 7248 | 7557 | 7865 | 8173 | 8479 | 8784 | 9089 |
| 3.3 | 9392 | 9695 | 9996 | *0297 | *0597 | *0896 | *1194 | *1491 | *1788 | *2083 |
| 3.4 | 1.2 2378 | 2671 | 2964 | 3256 | 3547 | 3837 | 4127 | 4415 | 4703 | 4990 |
| 3.5 | 5276 | 5562 | 5846 | 6130 | 6413 | 6695 | 6976 | 7257 | 7536 | 7815 |
| 3.6 | 8093 | 8371 | 8647 | 8923 | 9198 | 9473 | 9746 | *0019 | *0291 | *0563 |
| 3.7 | 1.3 0833 | 1103 | 1372 | 1641 | 1909 | 2176 | 2442 | 2708 | 2972 | 3237 |
| 3.8 | 3500 | 3763 | 4025 | 4286 | 4547 | 4807 | 5067 | 5325 | 5584 | 5841 |
| 3.9 | 6098 | 6354 | 6609 | 6864 | 7118 | 7372 | 7624 | 7877 | 8128 | 8379 |
| **4.0** | 8629 | 8879 | 9128 | 9377 | 9624 | 9872 | *0118 | *0364 | *0610 | *0854 |
| 4.1 | 1.4 1099 | 1342 | 1585 | 1828 | 2070 | 2311 | 2552 | 2792 | 3031 | 3270 |
| 4.2 | 3508 | 3746 | 3984 | 4220 | 4456 | 4692 | 4927 | 5161 | 5395 | 5629 |
| 4.3 | 5862 | 6094 | 6326 | 6557 | 6787 | 7018 | 7247 | 7476 | 7705 | 7933 |
| 4.4 | 8160 | 8387 | 8614 | 8840 | 9065 | 9290 | 9515 | 9739 | 9962 | *0185 |
| 4.5 | 1.5 0408 | 0630 | 0851 | 1072 | 1293 | 1513 | 1732 | 1951 | 2170 | 2388 |
| 4.6 | 2606 | 2823 | 3039 | 3256 | 3471 | 3687 | 3902 | 4116 | 4330 | 4543 |
| 4.7 | 4756 | 4969 | 5181 | 5393 | 5604 | 5814 | 6025 | 6235 | 6444 | 6653 |
| 4.8 | 6862 | 7070 | 7277 | 7485 | 7691 | 7898 | 8104 | 8309 | 8515 | 8719 |
| 4.9 | 8924 | 9127 | 9331 | 9534 | 9737 | 9939 | *0141 | *0342 | *0543 | *0744 |
| **5.0** | 1.6 0944 | 1144 | 1343 | 1542 | 1741 | 1939 | 2137 | 2334 | 2531 | 2728 |
| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **5.0** | 1.6 0944 | 1144 | 1343 | 1542 | 1741 | 1939 | 2137 | 2334 | 2531 | 2728 |
| 5.1 | 2924 | 3120 | 3315 | 3511 | 3705 | 3900 | 4094 | 4287 | 4481 | 4673 |
| 5.2 | 4866 | 5058 | 5250 | 5441 | 5632 | 5823 | 6013 | 6203 | 6393 | 6582 |
| 5.3 | 6771 | 6959 | 7147 | 7335 | 7523 | 7710 | 7896 | 8083 | 8269 | 8455 |
| 5.4 | 8640 | 8825 | 9010 | 9194 | 9378 | 9562 | 9745 | 9928 | *0111 | *0293 |
| 5.5 | 1.7 0475 | 0656 | 0838 | 1019 | 1199 | 1380 | 1560 | 1740 | 1919 | 2098 |
| 5.6 | 2277 | 2455 | 2633 | 2811 | 2988 | 3166 | 3342 | 3519 | 3695 | 3871 |
| 5.7 | 4047 | 4222 | 4397 | 4572 | 4746 | 4920 | 5094 | 5267 | 5440 | 5613 |
| 5.8 | 5786 | 5958 | 6130 | 6302 | 6473 | 6644 | 6815 | 6985 | 7156 | 7326 |
| 5.9 | 7495 | 7665 | 7834 | 8002 | 8171 | 8339 | 8507 | 8675 | 8842 | 9009 |
| **6.0** | 9176 | 9342 | 9509 | 9675 | 9840 | *0006 | *0171 | *0336 | *0500 | *0665 |
| 6.1 | 1.8 0829 | 0993 | 1156 | 1319 | 1482 | 1645 | 1808 | 1970 | 2132 | 2294 |
| 6.2 | 2455 | 2616 | 2777 | 2938 | 3098 | 3258 | 3418 | 3578 | 3737 | 3896 |
| 6.3 | 4055 | 4214 | 4372 | 4530 | 4688 | 4845 | 5003 | 5160 | 5317 | 5473 |
| 6.4 | 5630 | 5786 | 5942 | 6097 | 6253 | 6408 | 6563 | 6718 | 6872 | 7026 |
| 6.5 | 7180 | 7334 | 7487 | 7641 | 7794 | 7947 | 8099 | 8251 | 8403 | 8555 |
| 6.6 | 8707 | 8858 | 9010 | 9160 | 9311 | 9462 | 9612 | 9762 | 9912 | *0061 |
| 6.7 | 1.9 0211 | 0360 | 0509 | 0658 | 0806 | 0954 | 1102 | 1250 | 1398 | 1545 |
| 6.8 | 1692 | 1839 | 1986 | 2132 | 2279 | 2425 | 2571 | 2716 | 2862 | 3007 |
| 6.9 | 3152 | 3297 | 3442 | 3586 | 3730 | 3874 | 4018 | 4162 | 4305 | 4448 |
| **7.0** | 4591 | 4734 | 4876 | 5019 | 5161 | 5303 | 5445 | 5586 | 5727 | 5869 |
| 7.1 | 6009 | 6150 | 6291 | 6431 | 6571 | 6711 | 6851 | 6991 | 7130 | 7269 |
| 7.2 | 7408 | 7547 | 7685 | 7824 | 7962 | 8100 | 8238 | 8376 | 8513 | 8650 |
| 7.3 | 8787 | 8924 | 9061 | 9198 | 9334 | 9470 | 9606 | 9742 | 9877 | *0013 |
| 7.4 | 2.0 0148 | 0283 | 0418 | 0553 | 0687 | 0821 | 0956 | 1089 | 1223 | 1357 |
| 7.5 | 1490 | 1624 | 1757 | 1890 | 2022 | 2155 | 2287 | 2419 | 2551 | 2683 |
| 7.6 | 2815 | 2946 | 3078 | 3209 | 3340 | 3471 | 3601 | 3732 | 3862 | 3992 |
| 7.7 | 4122 | 4252 | 4381 | 4511 | 4640 | 4769 | 4898 | 5027 | 5156 | 5284 |
| 7.8 | 5412 | 5540 | 5668 | 5796 | 5924 | 6051 | 6179 | 6306 | 6433 | 6560 |
| 7.9 | 6686 | 6813 | 6939 | 7065 | 7191 | 7317 | 7443 | 7568 | 7694 | 7819 |
| **8.0** | 7944 | 8069 | 8194 | 8318 | 8443 | 8567 | 8691 | 8815 | 8939 | 9063 |
| 8.1 | 9186 | 9310 | 9433 | 9556 | 9679 | 9802 | 9924 | *0047 | *0169 | *0291 |
| 8.2 | 2.1 0413 | 0535 | 0657 | 0779 | 0900 | 1021 | 1142 | 1263 | 1384 | 1505 |
| 8.3 | 1626 | 1746 | 1866 | 1986 | 2106 | 2226 | 2346 | 2465 | 2585 | 2704 |
| 8.4 | 2823 | 2942 | 3061 | 3180 | 3298 | 3417 | 3535 | 3653 | 3771 | 3889 |
| 8.5 | 4007 | 4124 | 4242 | 4359 | 4476 | 4593 | 4710 | 4827 | 4943 | 5060 |
| 8.6 | 5176 | 5292 | 5409 | 5524 | 5640 | 5756 | 5871 | 5987 | 6102 | 6217 |
| 8.7 | 6332 | 6447 | 6562 | 6677 | 6791 | 6905 | 7020 | 7134 | 7248 | 7361 |
| 8.8 | 7475 | 7589 | 7702 | 7816 | 7929 | 8042 | 8155 | 8267 | 8380 | 8493 |
| 8.9 | 8605 | 8717 | 8830 | 8942 | 9054 | 9165 | 9277 | 9389 | 9500 | 9611 |
| **9.0** | 9722 | 9834 | 9944 | *0055 | *0166 | *0276 | *0387 | *0497 | *0607 | *0717 |
| 9.1 | 2.2 0827 | 0937 | 1047 | 1157 | 1266 | 1375 | 1485 | 1594 | 1703 | 1812 |
| 9.2 | 1920 | 2029 | 2138 | 2246 | 2354 | 2462 | 2570 | 2678 | 2786 | 2894 |
| 9.3 | 3001 | 3109 | 3216 | 3324 | 3431 | 3538 | 3645 | 3751 | 3858 | 3965 |
| 9.4 | 4071 | 4177 | 4284 | 4390 | 4496 | 4601 | 4707 | 4813 | 4918 | 5024 |
| 9.5 | 5129 | 5234 | 5339 | 5444 | 5549 | 5654 | 5759 | 5863 | 5968 | 6072 |
| 9.6 | 6176 | 6280 | 6384 | 6488 | 6592 | 6696 | 6799 | 6903 | 7006 | 7109 |
| 9.7 | 7213 | 7316 | 7419 | 7521 | 7624 | 7727 | 7829 | 7932 | 8034 | 8136 |
| 9.8 | 8238 | 8340 | 8442 | 8544 | 8646 | 8747 | 8849 | 8950 | 9051 | 9152 |
| 9.9 | 9253 | 9354 | 9455 | 9556 | 9657 | 9757 | 9858 | 9958 | *0058 | *0158 |
| **10.0** | 2.3 0259 | 0358 | 0458 | 0558 | 0658 | 0757 | 0857 | 0956 | 1055 | 1154 |
| **N** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |